REG-CP2B-2425-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. A	3. B	4. A	5. D
6. C	7. C	8. D	9. C	10. D
11. A	12. C	13. B	14. A	15. A
16. B	17. C	18. C	19. C	20. B
21. B	22. C	23. C	24. A	25. A
26. C	27. C	28. D	29. D	30. D
31. D	32. A	33. A	34. B	35. D
36. A	37. C	38. C	39. B	40. D
41. A				

1. $\boxed{\text{C}}$

$\angle AOC + \angle OAC + \angle OCA = 180°$

$\angle AOC + 2(27°) = 180°$

$\angle AOC = 126°$

$\angle BOC = \angle AOC \times \frac{2}{1+2} = 84°$

$\angle BAC = \frac{\angle BOC}{2} = 42°$

$\angle BCD = \angle BAC = 42°$

2. $\boxed{\text{A}}$

$\angle PBA = \angle PAB = \frac{180° - 42°}{2} = 69°$

$\angle ABD = 180° - 69° - 35° = 76°$

$\angle ACD = 180° - 76° = 104°$

3. $\boxed{\text{B}}$

Let O be the centre of the semi-circle $ABCD$.

We have $OC \perp EF$ and $OB \perp BE$.

Thus, $\angle BOC = 360° - 90° - 90° - 90° = 90°$.

$\angle AOC = \angle AFE + 90° = 132°$

$\angle AOB = 132° - 90° = 42°$

$\angle ACB = \frac{\angle AOB}{2} = 21°$

4. $\boxed{\text{A}}$

$\angle AEB = \angle SAB = 60°$

$\angle BCD = \angle AEB = 60°$

$\angle BCE : \angle ECD = 15° : 60° - 15° = 1 : 3$

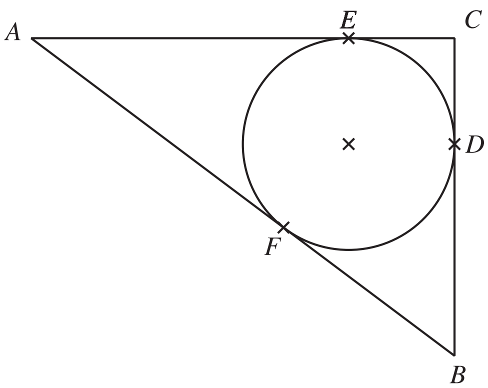
5. D

$EB = EC$
$\angle EBC = \dfrac{180° - 72°}{2} = 54°$
$\angle BAC = \angle EBC = 54°$
$\angle ACF = \angle ABC = 63°$
$AB = AC$
$\angle ABC = \dfrac{180° - 54°}{2} = 63°$
$\angle DBE = 90°$
$\angle CBD = 90° - 54° = 36°$
$\angle CAF = \angle CBD = 36°$
$\angle AFC = 180° - 36° - 63° = 81°$

6. C

Let D, E, F be the point of contact of the inscribed circle and BC, AC, AB respectively.

Let $CE = x$. Then $AE = 12 - x$, $CD = x$ and $BD = 5 - x$.
By tangent properties, $AF = 12 - x$ and $BF = 5 - x$.
Since $AB = \sqrt{5^2 + 12^2} = 13$, we have $x = 2$.
Thus, the coordinates of incentre are $(10,\ 3)$.



7. C

$\angle OBC + \angle OCB + \angle BOC = 180°$
$\angle BOC = 180° - 2 \times 55°$
$= 70°$
$\angle OCT = 90°$
$\angle OTC + \angle OCT + \angle COT = 180°$
$\angle OTC = 180° - 70° - 90°$
$\angle ATC = 20°$

8. D

$\angle BCE = \angle CEG = 70°$
$\angle CBE = \angle CEG = 70°$
$\angle BEC = 180° - 70° - 70° = 40°$
$\angle ABE = \angle BEC = 40°$
$\angle ADE = \angle ABE = 40°$

9. C

Join OE and let the radius of circle be r.

$OD = 8 - r$.

$DE = \sqrt{(8-r)^2 - r^2} = \sqrt{64 - 16r}$

As $\triangle OED \sim \triangle ABD$ (AA),

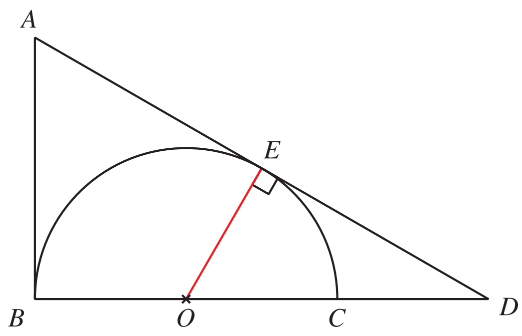
$$\frac{AB}{BD} = \frac{6}{8} = \frac{r}{\sqrt{64-16r}} = \frac{OE}{ED}$$

$$\frac{9}{16} = \frac{r^2}{64-16r}$$

$$9 = \frac{r^2}{4-r}$$

$$0 = r^2 + 9r - 36$$

$$r = 3 \text{ or } -12 \text{ (rejected)}$$



10. D

Denote the intersections of BC and the circle by C and F.

$\angle CAF = \angle ECB = 30°$

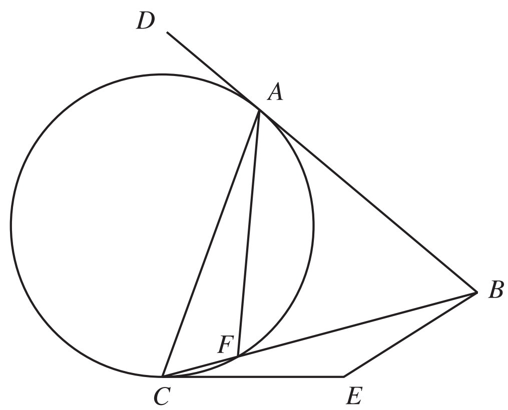
$\angle BAF = \angle FCA$

$$\angle FCA + \angle BAF + 30° + 40° = 180°$$

$$2\angle BAF = 110°$$

$$\angle BAF = 55°$$

$\angle CAD = 180° - 30° - 55° = 95°$



11. A

Note that $\angle ABC = 90°$.

Let the radius be r cm.

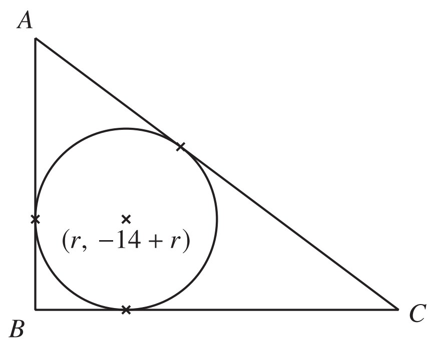
The coordinates of centre are $(r,\ -14 + r)$.

Consider the length of AC.

$$(24 - r) + (18 - r) = \sqrt{18^2 + 24^2}$$

$$r = 6$$

The coordinates of centre are $(6,\ -8)$.



12. C

Let $\angle CBD = \angle CDB = x$ and $\angle DAS = y$.
$\angle BCD = 180° - 2x$ and $\angle BAD = 2x$.
$\angle ABD = \angle DAS = y$.

$$y + (2x + y) + 40° = 180°$$
$$x + y = 70°$$

$\angle ABC = x + y = 70°$.

13. B

Let $\angle CAP = x$. Then $\angle ABC = \angle CAP = x$.

$$\angle DBA + \angle BAP = 180°$$
$$40° + x + x + 90° = 180°$$
$$x = 25°$$

14. A

BC is a diameter, $\angle BDC = 90°$.
Since $\triangle BDC \sim \triangle CDA$, $\dfrac{BD}{BC} = \dfrac{CD}{AC}$

$$\frac{BD}{CD} = \frac{3}{4}$$

$\angle BCD = \tan^{-1}\dfrac{3}{4}$ and $BD = 9\sin\angle BCD = 5.4\,\text{cm}$.

15. A

$\angle CBP = \angle BCP = \angle BAC = 56°$
$\angle BPC = 180° - 56° - 56° = 68°$
$\angle PBD = 90°$
$\angle BQP = 180° - 90° - 68°$
$= 22°$

16. B

$EA = ED$
$\angle EDA = \dfrac{180° - 86°}{2} = 47°$
$\angle ACD = \angle EDA = 47°$
$\angle FGH = 180° - \angle FEH = 94°$
$\angle GBC = \dfrac{180° - 94°}{2} = 43°$
$\angle BDC = \angle GBC = 43°$
$\angle AID = \angle BDC + \angle ACD = 90°$

17. C

RT is the angle bisector of $\angle SRP$. So, $\angle SRP = 24°$.
$\angle QPS = \angle QSR$.

$$\angle QPS + (\angle QSR + 70°) + 24° = 180°$$
$$\angle QPS = 43°$$

18. C

$\angle ADC = 180° - 100° = 80°$
$\angle ADB = 80° \times \frac{2}{2+3} = 32°$
$\angle BCA = \angle ADB = 32°$
$\angle DAT = \angle ADC - \angle ATD = 50°$
$\angle ACT = \angle DAT = 50°$
$\angle BCT = 32° + 50° = 82°$

19. C

The graph passes through $(0,\ 3)$.

A. ✗. When $x = 0$, $y = \sin 30° + 3 = 3.5 \neq 3$

B. ✗. When $x = 0$, $y = 2\sin 30° + 1 = 2 \neq 3$

C. ✓. When $x = 0$, $y = 2\sin 30° + 2 = 3$

D. ✗. When $x = 0$, $y = 3\sin 30° + 4 = 5.5 \neq 3$

20. B

$$4 + \cos(90° + \theta) = 4\cos^2\theta$$
$$4 - \sin\theta = 4(1 - \sin^2\theta)$$
$$4\sin^2\theta - \sin\theta = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \frac{1}{4}$$

The equation $\sin\theta = 0$ has one root.
The equation $\sin\theta = \frac{1}{4}$ has two roots.
Thus, the equation $4 + \cos(90° + \theta) = 4\cos^2\theta$ has 3 roots.

21. B

We have $2\sin^2 x = 1$ or $\sin x = 0$.
When $\sin x = 0$, $x = 180°$
When $2\sin^2 x = 1$, $\sin x = \pm\frac{1}{\sqrt{2}}$ and there are four roots within the required range.
Thus, there are 5 roots.

22. C

The curve passes through $(0,\ 1)$. Only option C satisfies this.

23. C

The graph passes through $(0°, 1)$ and $(90°, 3)$.

	$(0°, 1)$	$(90°, 3)$
A.	✓	✗
B.	✗	
C.	✓	✓
D.	✗	

24. A

If p is positive, then the maximum value and the minimum value of $y = p\sin 3x° + q$ are $p + q$ and $-p + q$ respectively.

Solve $\begin{cases} p + q = 1 \\ -p + q = -3 \end{cases}$, we have $p = 2$ and $q = -1$.

25. A

$$3\tan x = 2\sin(90° - x)$$
$$\frac{3\sin x}{\cos x} = 2\cos x$$
$$3\sin x = 2\cos^2 x$$
$$3\sin x = 2(1 - \sin^2 x)$$
$$2\sin^2 x + 3\sin x - 2 = 0$$
$$\sin x = 0.5 \quad \text{or} \quad -2 \text{ (rejected)}$$
$$x = 30° \quad \text{or} \quad 150°$$

There are 2 roots.

26. C

A. 3 roots

B. 3 roots

C. 4 roots

D. 2 roots

27. C

$$4 - \sin\theta = 4\cos^2\theta$$
$$4 - \sin\theta = 4(1 - \sin^2\theta)$$
$$4\sin^2\theta - \sin\theta = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \frac{1}{4}$$

The equation $\sin\theta = 0$ has one root.

The equation $\sin\theta = \frac{1}{4}$ has two roots.

Thus, there are 3 roots.

28. D

$$3\tan^2 x = \tan x$$

$$\tan x(3\tan x - 1) = 0$$

$$\tan x = 0 \quad \text{or} \quad \frac{1}{3}$$

When $\tan x = 0$, $x = 0°$ or $180°$ or $360°$

When $\tan x = \frac{1}{3}$, there are 2 roots (in quadrants I and III)

Altogether 5 roots.

29. D

$$\sin\theta = \sin^3\theta$$

$$0 = \sin\theta(\sin^2\theta - 1)$$

$$\sin\theta = 0 \quad \text{or} \quad \pm 1$$

With reference to the graph of sine, there are 5 roots.

30. D

$$\sin^2\theta = \cos^2\theta$$

$$\tan^2\theta = 1$$

$$\tan\theta = \pm 1$$

$$\theta = 45° \text{ or } 135° \text{ or } 225° \text{ or } 315°$$

There are 4 roots.

31. D

G is the centre of circle ABC.

Radius $= GB = \sqrt{(18-10)^2 + (3+3)^2} = 10$

Since A is a point on the circle, the y-coordinate of A lies between $3 + 10$ and $3 - 10$ inclusively.

Thus, -8 is not a possible y-coordinate of A.

32. A

Note that $\angle ABC = 90°$.

Let the radius be r cm.

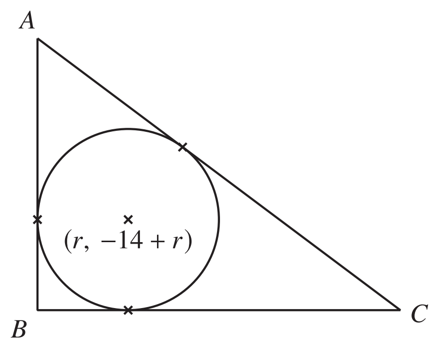
The coordinates of centre are $(r,\ -14 + r)$.

Consider the length of AC.

$$(24 - r) + (18 - r) = \sqrt{18^2 + 24^2}$$

$$r = 6$$

The coordinates of centre are $(6,\ -8)$.



33. A

Note that $\angle POQ = 90^\circ$.

The orthocentre of $\triangle OPQ$ is O.

The circumcentre of $\triangle OPQ$ is the mid-point of PQ.

The coordinates of the circumcentre of $\triangle OPQ$ are $\left(\frac{p}{2},\ \frac{q}{2}\right)$.

The straight line $2x - y = 3k$ passes through O and $\left(\frac{p}{2},\ \frac{q}{2}\right)$.

$$\begin{cases} 2(0) - 0 = 3k \\ 2\left(\frac{p}{2}\right) - \frac{q}{2} = 3k \end{cases}$$

Solving, we have $k = 0$ and $2p = q$.

Thus, $p : q = 1 : 2$.

34. B

Denote the mid-point of AB by M.

Let G be the centroid of $\triangle ABP$.

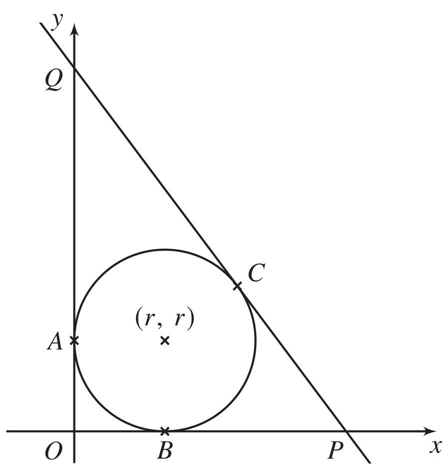
Note that $MG : GP = 1 : 2$.

The y-coordinate of G is therefore constant.

The locus of centroid of $\triangle ABP$ is a straight line parallel to L.

35. D

The coordinates of P and Q are $(6,\ 0)$ and $(0,\ 8)$ respectively.



Let the radius of the inscribed circle be r.

Consider the area of $\triangle OPQ$.

$$\frac{(6)(8)}{2} = \frac{(6)(r)}{2} + \frac{(8)(r)}{2} + \frac{(\sqrt{6^2 + 8^2})(r)}{2}$$

$$r = 2$$

Required coordinates are $(2,\ 2)$.

Refer to the figure.

$OA = OB = r$

$BP = CP = 6 - r$

$AQ = CQ = 8 - r$

Consider the length of PQ.

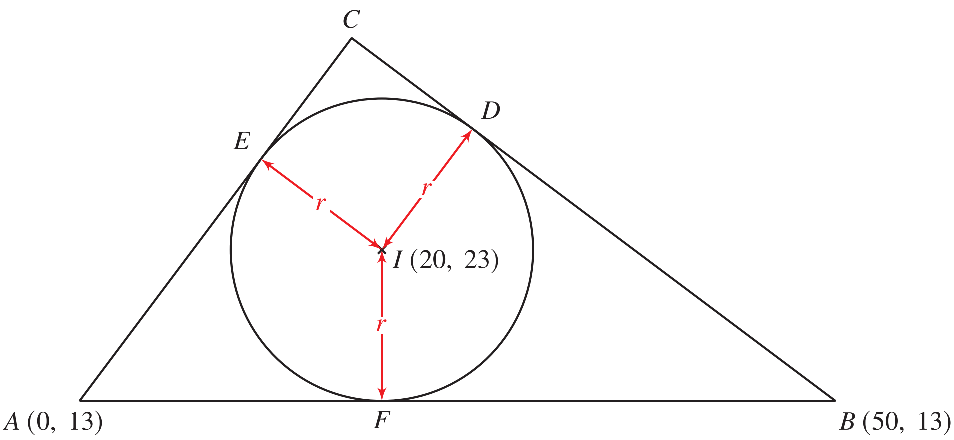
$$(6 - r) + (8 - r) = \sqrt{6^2 + 8^2}$$
$$r = 2$$

Required coordinates are $(2,\ 2)$.

36. A

Since C is the orthocentre of $\triangle ABC$, $\angle ACB = 90°$.

Let D, E and F be the points of contact as shown below. Denote the incentre of $\triangle ABC$ by I.



Consider the vertical distance from $(20,\ 23)$ to AB.

$$r = 23 - 13$$
$$= 10$$

Note that $CDIE$ is a square.

We have $AC = AE + r = AF + r = 20 + 10 = 30$.

Consider the horizontal distance between A and C.

Horizontal distance $= AC \cos \angle BAC$

$$= 30 \cos \angle BAC$$

$$= 30 \left(\frac{AC}{AB} \right) \qquad \text{(Note that } \angle ABC \text{ is a right-angled triangle.)}$$

$$= 30 \times \frac{30}{50}$$

$$= 18$$

The x-coordinate of C is $0 + 18 = 18$.

37. C

The coordinates of A and B are $\left(\frac{k}{6},\ 0\right)$ and $\left(0,\ -\frac{k}{3}\right)$ respectively.
Denote the mid-point of AB by M.
The coordinates of M are $\left(\frac{k}{12},\ -\frac{k}{6}\right)$.
Let the coordinates of C be $(c,\ 0)$.
We have $CG : GM = 2 : 1$, where G is the centroid of $\triangle ABC$.

$$\frac{CG}{GM} = \frac{c-0}{0-\frac{k}{12}}$$
$$2 = -\frac{12c}{k}$$
$$c = -\frac{k}{6}$$

The coordinates of A and B are $\left(\frac{k}{6},\ 0\right)$ and $\left(0,\ -\frac{k}{3}\right)$ respectively.
Let the coordinates of C be $(c,\ 0)$.
Note that the x-coordinate of the centroid of $\triangle ABC$ is 0.

$$\frac{\frac{k}{6}+0+c}{3} = 0$$
$$c = -\frac{k}{6}$$

38. C

I. ✓. y-coordinate of the mid-point of BC is 0.
y-coordinate of centroid of $\triangle ABC$
$= \frac{1(6)+2(0)}{1+2}$
$= 2$
The centroid of $\triangle ABC$ lies on $y = 2$.

II. ✓. Note that the y-axis passes through A and is perpendicular to BC.
Thus, the orthocentre of $\triangle ABC$ lies on the y-axis.

39. B

$$\angle BAC + 2\angle CBD + 2\angle BCD = 180^\circ$$
$$\angle CBD + \angle BCD = 55^\circ$$
$$\angle BDC + \angle CBD + \angle BCD = 180^\circ$$
$$\angle BDC = 125^\circ$$

40. D

Note that $AB = AD = AE$.

A is the centre of the circumcircle of BDE.

Since BE is a diameter of circle BDE, we have $\angle BDE = 90^\circ$.

$\angle ADE = \angle DEA = 35^\circ$

$\angle ADB = 90^\circ - 35^\circ = 55^\circ$

$\angle CBD = \angle ADB = 55^\circ$

41. A

AB is the angle bisector of $\angle CAD$.

$\angle BAC = \angle BAD = 48^\circ$

$\angle ACD = \angle BAD = 48^\circ$

$\angle ADC + \angle ACD + \angle CAD = 180^\circ$

$\angle ADC + 48^\circ + (48^\circ + 48^\circ) = 180^\circ$

$\angle ADC = 36^\circ$

END OF PAPER