REG-CP2B-2425-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. D	3. B	4. A	5. C
6. B	7. C	8. D	9. C	10. A
11. D	12. B	13. C	14. C	15. C
16. B	17. B	18. D	19. C	20. B
21. D	22. D	23. B	24. B	25. A
26. C	27. D	28. C	29. A	30. B
31. A	32. C	33. B	34. B	35. D
36. D	37. B	38. A	39. A	40. C
41. D	42. D	43. B	44. D	45. C
46. B	47. C	48. A	49. A	50. B
51. A	52. B	53. D	54. C	55. C
56. A	57. A	58. B	59. B	60. A
61. C	62. C	63. D	64. C	65. D
66. C	67. D	68. A	69. D	70. D
71. D	72. A			

1. D

$11010_2 = 26$ and the '1' has a place value 2^9.
So, the answer is D.

2. D

$512 = 200_{16}$
$11 \times 8^{16} = 11 \times 2^{48} = 11 \times 16^{12} = B000000000000_{16}$
The answer is D.

3. B

$C0000000000000040_{16} = 12 \times 16^{17} + 4 \times 16^{1} = 12 \times 2^{68} + 64$

4. A

$11 \times 8^4 + 2^7 - 3 \times 2^4$

$= 11 \times 2^{12} + 2^4(8-3)$

$= 11 \times 2^{12} + 5 \times 2^4$

$= (2^3 + 2 + 1)2^{12} + (2^2 + 1)2^4$

$= 2^{15} + 2^{13} + 2^{12} + 2^6 + 2^4$

The answer is A.

5. C

From binary to hexadecimal, divide the number with 4 digits in a group, counted from the rightmost digit: 1/1101/1110/0011/1100

By calculator, the tail part $00111100_2 = 3C_{16} \leftarrow$ The answer is C

6. B

$15 = 1111_2$

$2^9 = 1000000000_2$ (count from degree 0 from the right)

So, $2^9 + 15 = 1000001111_2$

7. C

$90000_{16} \rightarrow 9 \times 16^4$ and $D_{16} = 13$

Only option C satisfies this.

8. D

$8^2 + 4 \times 8^{16} = 64 + 4 \times 2^{48}$

$= 4 \times 16 + 4 \times 16^{12}$

$= 4000000000040_{16}$

9. C

$2 - 2^3 + 2^4 + 4 \times 2^5 = 138 = 10001010_2$ and $5 \times 2^{10} = 1010000000000_2$.

Only option C satisfies this.

10. A

I. ✓. $1234_{16} = 1 \times 16^3 + 2 \times 16^2 + 3 \times 16 + 4$

$= 2^{12} + 2^9 + (1+2)2^4 + 2^2$

$= 2^{12} + 2^9 + 2^5 + 2^4 + 2^2$

$= 1001000110100_2$

II. ✓. $1234_{16} = 2^{12} + 2^9 + 2^5 + 2^4 + 2^2 = 2^{12} + 2^9 + 52$

III. ✗. $4^8 + 4^6 = 16^4 + 16^3 = 11000_{16} > 1234_{16}$

11. D

110001011001_2

$= 1 \times 2^{11} + 1 \times 2^{10} + 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^0$

$= 2^{11} + 2^{10} + 89$

12. B

$160_{10} = A0_{16}$

$13 \times 16^8 = D00000000_{16}$ (count from degree 0 to degree 8, total 9 digits)

13. C

$$\begin{aligned}(2-i)z &= (2-i)(1+i)(2x-yi)\\ &= (3+i)(2x-yi)\\ &= (6x+y)+(2x-3y)i\end{aligned}$$

Since x and y are positive integers such that $6x + y = 25$ and $x > y$,

we have $x = 4$ and $y = 1$.

Thus, $x - y = 4 - 1 = 3$.

14. C

Using calculator CMPLX mode, $\dfrac{i}{1+2i} = \dfrac{2}{5} + \dfrac{1}{5}i$.

$\dfrac{i}{1+2i} + ai = \dfrac{2}{5} + \left(\dfrac{1}{5} + a\right)i$ is a real number.

$$\frac{1}{5} + a = 0$$

$$a = -\frac{1}{5}$$

15. C

Put $k = 1$.

$$\frac{5k+10i}{1-2i} = \frac{5+10i}{1-2i} = -3+4i$$

Imaginary part is 4.

Check the value of each option when $k = 1$.

A. 5

B. −3

C. 4

D. 0

16. B

Put $k = 1$.

$$\frac{ki}{2i-1} + \frac{4i-k}{i+2} = \frac{i}{2i-1} + \frac{4i-1}{i+2}$$
$$= \frac{4}{5} + \frac{8}{5}i$$

The imaginary part is $\frac{8}{5}$, which only option B satisfies this when $k = 1$.

17. B

$$\frac{9i^{13} + 8i^{14} + 7i^{15} + 6i^{16} + 5i^{17}}{1+i} = \frac{9i - 8 - 7i + 6 + 5i}{1+i}$$
$$= \frac{-2+7i}{1+i}$$
$$= \frac{5}{2} + \frac{9}{2}i$$

Imaginary part = $\frac{9}{2} = 4.5$

18. D

$z = (x-4) + 3i$ is purely imaginary.

$$x - 4 = 0$$
$$x = 4$$

19. C

$$\frac{i^{2024} + 2i^{2025}}{i^{2026} + i^{2027}} = \frac{1+2i}{-1-i}$$
$$= -\frac{3}{2} - \frac{1}{2}i$$

The imaginary part is $-\frac{1}{2}$.

20. B

$z = (p-4)i^{2023} + (2p+5)i^{2022}$

$= -(p-4)i - (2p+5)$

z is a purely imaginary number.

$$-(2p+5) = 0$$
$$p = -\frac{5}{2}$$

21. D

$$\frac{4i^{2020}+5i^{2019}+6i^{2018}+7i^{2017}+8i^{2016}}{1+i}$$
$$=\frac{4+5i^3+6i^2+7i+8}{1+i}$$
$$=\frac{6+2i}{1+i}$$
$$=4-2i$$

Imaginary part $= -2$

22. D

Take $m = 1$, $i^7 + \dfrac{i^5-4}{m-i} = -\dfrac{5}{2} - \dfrac{5}{2}i$.

Only option D gives $-\dfrac{5}{2}$ when $m = 1$.

23. B

I. ✓. Calculator CMPLX mode.

II. ✓. $1 + z + z^2 = 0$ by calculator. So, $z^3 + z^4 + z^5 = z^3(1 + z + z^2) = 0$.

III. ✗. $(z^{3n} + z^{3n+1} + z^{3n+2}) + \ldots + (z^{12n-3} + z^{12n-2} + z^{12n-1}) + z^{12n} = 0 + 1 = 1 \neq 0$.

24. B

$$\frac{6i^6+7i^7+8i^8+9i^9+10i^{10}}{1+i} = \frac{-6-7i+8+9i-10}{1+i}$$
$$= -3+5i$$

The real part is -3.

25. A

Let the other root be β.

Product of roots $= 3 \times \beta = \dfrac{2}{1}$

$$\beta = \frac{2}{3}$$

26. C

α and β are roots of $x^2 = 3x + 4$, i.e., $x^2 - 3x - 4 = 0$.

$$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$
$$= 3^2 - 2(-4)$$
$$= 17$$

27. D

α and β are roots of the equation $x^2 + px + 2p = 0$.
$\alpha + \beta = -p$ and $\alpha\beta = 2p$.

$$\begin{aligned}\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} &= \frac{\alpha^3 + \beta^3}{\alpha\beta} \\ &= \frac{(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]}{\alpha\beta} \\ &= \frac{-p(p^2 - 6p)}{2p} \\ &= \frac{-p^2 + 6p}{2}\end{aligned}$$

28. C

a and b are roots of the equation $2x^2 = 6x - k$, i.e., $2x^2 - 6x + k = 0$.
Therefore, $a + b = 3$ and $ab = \frac{k}{2}$.

$$\begin{aligned}a^2 + 3b &= \frac{6a - k}{2} + 3b \\ &= 3(a+b) - \frac{k}{2} \\ &= 9 - \frac{k}{2}\end{aligned}$$

29. A

$$\beta^2 - 2\beta + k = 0$$
$$\beta^2 = 2\beta - k$$
$$\begin{aligned}2\alpha + \beta^2 &= 2\alpha + (2\beta - k) \\ &= 2(\alpha + \beta) - k \\ &= 2(2) - k \\ &= 4 - k\end{aligned}$$

30. B

α is a root of $x^2 - 3x + k = 0 \quad \Rightarrow \quad \alpha^2 = 3\alpha - k$.
$\alpha^3 = \alpha(3\alpha - k) = 3\alpha^2 - k\alpha = 3(3\alpha - k) - k\alpha = 9\alpha - k\alpha - 3k$

$$\begin{aligned}\alpha^3 + 5\beta &= 3 \\ (9\alpha - k\alpha - 3k) + 5(3 - \alpha) &= 3 \\ 4\alpha - k\alpha - 3k + 12 &= 0 \\ 4(3+\alpha) - k(\alpha+3) &= 0 \\ (4-k)(\alpha+3) &= 0 \\ \alpha = -3 \quad &\text{or} \quad k = 4\end{aligned}$$

When $\alpha = -3$, $\beta = 6$ and $k = (-3)(6) = -18$.
Sum of all possible values of $k = -18 + 4 = -14$

31. A

$\alpha + \beta = -\frac{1}{2}$ and $\alpha\beta = -\frac{k}{2}$

$$\begin{aligned}\alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{k}{2}\right) \\ &= \frac{1 + 4k}{4}\end{aligned}$$

32. C

α and β are roots of the equation $5x = 8 - 2x^2$, i.e., $2x^2 + 5x - 8 = 0$.

$$\begin{aligned}(\alpha - \beta)^2 &= (\alpha + \beta)^2 - 4\alpha\beta \\ &= \left(-\frac{5}{2}\right)^2 - 4\left(\frac{-8}{2}\right) \\ &= \frac{89}{4}\end{aligned}$$

33. B

β is a root of the equation.

$$\begin{aligned}\beta^2 - 2\beta - 1 &= 0 \\ \beta^2 &= 2\beta + 1\end{aligned}$$

We have $\alpha + \beta = 2$ and $\alpha\beta = -1$.

$$\begin{aligned}3\beta^2 + 6\alpha &= 3(2\beta + 1) + 6\alpha \\ &= 6(\alpha + \beta) + 3 \\ &= 6(2) + 3 \\ &= 15\end{aligned}$$

34. B

$a + b = 2$ and $ab = \frac{7}{2}$.

$$\begin{aligned}\frac{1}{a} + \frac{1}{b} &= \frac{a + b}{ab} \\ &= \frac{2}{\left(\frac{7}{2}\right)} \\ &= \frac{4}{7}\end{aligned}$$

35. D

c and d are roots of the equation $-9x = -x^2 + 6$.
The equation can be written as $x^2 - 9x - 6 = 0$.

$$\begin{aligned} c^2 + d^2 &= (c+d)^2 - 2cd \\ &= 9^2 - 2(-6) \\ &= 93 \end{aligned}$$

36. D

α and β are roots of $x^2 = 3x - 4$, i.e., $x^2 - 3x + 4 = 0$.
$\alpha^2\beta^2 = (\alpha\beta)^2 = 4^2 = 16$

37. B

$$\begin{aligned} 1 - \frac{ab}{a^2 - b^2} - \frac{b}{b-a} &= \frac{(a^2 - b^2) - ab + b(a+b)}{(a+b)(a-b)} \\ &= \frac{a^2}{a^2 - b^2} \end{aligned}$$

38. A

$$\begin{aligned} \frac{x^2 - 3x}{x^3 + 27} - \frac{1}{x+3} &= \frac{x^2 - 3x}{(x+3)(x^2 - 3x + 9)} - \frac{x^2 - 3x + 9}{(x+3)(x^2 - 3x + 9)} \\ &= \frac{-9}{x^3 + 27} \end{aligned}$$

39. A

$$\begin{aligned} \frac{2}{3x-4} - \frac{1}{3x+4} &= \frac{2(3x+4) - (3x-4)}{(3x-4)(3x+4)} \\ &= \frac{3x + 12}{(3x-4)(3x+4)} \\ &= \frac{3(x+4)}{(3x-4)(3x+4)} \\ &= \frac{3}{3x-4} \end{aligned}$$

40. C

Analysis the indices of the expressions.

	2	3	x	y	z
$3xy^4z^2$	0	1	1	4	2
$6x^3y^3z^2$	1	1	3	3	2
Third expression	?	?	?	?	?
Minimum index	0	0	0	2	2
Maximum index	2	1	3	4	4

Thus, the third expression is $2^2y^2z^4$ or $2^23^1y^2z^4$.
The answer is C.

41. D

$$\begin{aligned}\frac{1}{6x^2-45x+81}-\frac{1}{x-3}&=\frac{1}{3(x-3)(2x-9)}-\frac{1}{x-3}\\&=\frac{1-3(2x-9)}{3(x-3)(2x-9)}\\&=\frac{2(3x-14)}{-3(x-3)(2x-9)}\end{aligned}$$

42. D

Consider the indices of a, b and c in each expression.

Expression	a	b	c
$a^4b^2c^3$	4	2	3
$a^3b^4c^3$	3	4	3
$a^2b^5c^4$	2	5	4
Maximum degree	4	5	4

The L.C.M. is $a^4b^5c^4$.

43. B

$$\begin{aligned}\frac{5x-2}{(1-3x)^2}-\frac{2}{3x-1}&=\frac{5x-2}{(3x-1)^2}-\frac{2}{3x-1}\\&=\frac{(5x-2)-2(3x-1)}{(3x-1)^2}\\&=\frac{-x}{(3x-1)^2}\end{aligned}$$

44. D

The three expressions are $(x-3)^2$, $(x-3)(x-1)$ and $(x+3)(x-3)$.
L.C.M. $= (x-3)^2(x-1)(x+3)$

45. C

L.C.M. is obtained by taking the maximum degree of each factor.
L.C.M. is x^2y^3z.

46. B

$$\begin{aligned}\frac{5}{x^2+x-6}-\frac{6}{x^2-9} &= \frac{5}{(x+3)(x-2)}-\frac{6}{(x+3)(x-3)}\\ &= \frac{5(x-3)-6(x-2)}{(x+3)(x-3)(x-2)}\\ &= \frac{-x-3}{(x+3)(x-3)(x-2)}\\ &= -\frac{1}{(x-2)(x-3)}\end{aligned}$$

47. C

$$\begin{aligned}\frac{4}{x^2-x-12}+\frac{x}{x+3} &= \frac{4+x(x-4)}{(x+3)(x-4)}\\ &= \frac{(x-2)^2}{(x+3)(x-4)}\end{aligned}$$

48. A

The three expressions are $2^2m^2n^5$, $2\cdot 3m^3n^3$ and $2^3m^5n^4$.
The H.C.F. is $2m^2n^3$.

49. A

$$\begin{aligned}\log_2 y - 1 &= \frac{1}{3}(\log_4 x + 3)\\ \log_2 y &= \frac{1}{3}\log_4 x + 2\\ \frac{\log y}{\log 2} &= \frac{\log x}{3(2\log 2)} + 2\\ \log y &= \frac{1}{6}\log x + 2\log 2\\ &= \log 2^2 x^{\frac{1}{6}}\\ y &= 4x^{\frac{1}{6}}\end{aligned}$$

Thus, we have $n = \frac{1}{6}$.

50. B

$$\begin{aligned}5\log_2\alpha + 5\log_2\beta &= \frac{10}{3}\\ \log_2\alpha\beta &= \frac{2}{3}\\ \alpha\beta &= 2^{\frac{2}{3}}\end{aligned}$$

51. A

When $x = 1$, $y = \log_a 1 + b = b$.
The curve passes through $(1,\ b)$. We have $b < 0$.
Consider the x-intercept of the graph.

$$\begin{aligned} 0 &= \log_a x + b \\ \log_a x &= -b \\ x &= a^{-b} \end{aligned}$$

We have $a^{-b} < 1$ from the graph.
Thus, we have $0 < a < 1$.

52. B

Take logarithm on the numbers.

A. $\rightarrow 251 \log 125 \approx 526.3$

B. $\rightarrow 251 \log 152 \approx 547.6$

C. $\rightarrow 215 \log 251 \approx 515.9$

D. $\rightarrow 152 \log 521 \approx 413.0$

Number in option B is the greatest.

53. D

For the point $(0,\ -3)$.

$$\log_9 x = 0 \quad \text{and} \quad \log_3 y = -3$$
$$x = 1 \qquad\qquad y = 3^{-3} = \frac{1}{27}$$

Only option D satisfies this pair of x and y.

54. C

Bases of $\log_4 x$ and $\log_4 y$ are both 4, which is greater than 1.
The resulting graph is also decreasing.
When $\log_4 x = 0$, we have $x = 1$ and $\log_4 y = \log_4(4 \cdot 1^n) = 1 > 0$.
Value of vertical intercept is positive. Only option C satisfies all of these.

55. C

For the point $(0,\ 8)$,

$$\log_4 x = 0 \quad \text{and} \quad \log_8 y = 8$$
$$x = 1 \qquad\qquad y = 8^8 = 2^{24}$$

Only option C satisfies this.

56. $\boxed{A}$

Slope = 2 and vertical intercept = −2. We have $\log_3 y = 2\log_3 x - 2$.

$$\log_3 y = 2\log_3 x - 2$$
$$\log_3 y = \log_3 x^2 - \log_3 9$$
$$y = \frac{x^2}{9}$$

It has a shape of parabola opening upwards and passing through origin.

57. $\boxed{A}$

For the point (0, 5),

$$\log_2 x = 0 \quad \text{and} \quad \log_8 y = 5$$
$$x = 1 \qquad\qquad y = 8^5$$
$$= 2^{15}$$

Only option A satisfies this.

58. $\boxed{B}$

Base of log is 2, which is greater than 1.
The graph of y against x is also increasing.

$$x = 0 \quad \to \quad \log_2 y = -1 \quad \longrightarrow \quad y = 2^{-1} = 0.5$$

Only option B satisfies these.

59. $\boxed{B}$

$\boxed{(4,\ 0)}$

$$\log_2 x = 4 \quad \text{and} \quad \log_4 y = 0$$
$$x = 2^4 \qquad\qquad y = 1$$

$\boxed{(0,\ 5)}$

$$\log_2 x = 0 \quad \text{and} \quad \log_4 y = 5$$
$$x = 1 \qquad\qquad y = 4^5$$
$$y = 2^{10}$$

	x^5y^2	x^2y^5	x^4y^2	x^5y^4
$x = 2^4$ and $y = 1$	2^{20}	2^8	2^{16}	2^{20}
$x = 1$ and $y = 2^{10}$	2^{20}	2^{50}	2^{20}	2^{40}

The answer is B.

Slope of the linear graph $= \frac{5-0}{0-4} = -\frac{5}{4}$

$$\log_4 y - 5 = -\frac{5}{4}(\log_2 x - 0)$$

$$5\log_2 x + 4\log_4 y = 20$$

$$\frac{5\log x}{\log 2} + \frac{4\log y}{2\log 2} = 20$$

$$5\log x + 2\log y = 20\log 2$$

$$\log x^5 y^2 = \log 2^{20}$$

$$x^5 y^2 = 2^{20}$$

60. A

Consider the point (0, 2).

$\log_3 x = 0$ and $\log_3 y = 2$

$x = 1$ $\quad$ $y = 3^2 = 9$

Consider the point (4, 0).

$\log_3 x = 4$ and $\log_3 y = 0$

$x = 3^4 = 81$ $\quad$ $y = 1$

Check the relation using the values of x and y.

	$x = 1$ & $y = 9$	$x = 81$ & $y = 1$
A.	✓	✓
B.	✗	
C.	✓	✗
D.	✗	

The answer is A.

61. C

For the point (0, −2).

$\log x = 0$ and $\log y = -2$

$x = 1$ $\quad$ $y = 10^{-2} = \frac{1}{100}$

$$\frac{1}{100} = k(1)^2$$

$$k = \frac{1}{100}$$

62. C

$$y = ab^x$$

$\log y = \log a + x \log b$
Slope $= \log b < 0 \Rightarrow 0 < b < 1$

Vertical intercept $= \log a > 0 \Rightarrow a > 1$

63. D

The base of log is 6 (greater than 1), the new graph is also decreasing.

When $x = 0$, $y = 0.5$, then $\log_6 y = \log_6 0.5 < 0$.

The new graph has a negative intercept on vertical axis.

64. C

$$y = 3x^2$$

$\log y = 2 \log x + \log 3$

I. ✓.

II. ✗. There is no y-intercept indeed, we have only the log y-intercept.

III. ✓. $y = 3x^2$

$\log_3 y = 2 \log_3 x + \log_3 3$'
Slope of the line is also 2.

65. D

$\log_{27} y = \log_{27} a + x \log_{27} b$

Slope $= \log_{27} b = \dfrac{0+1}{3-0}$

$b = 27^{\frac{1}{3}}$

$= 3$

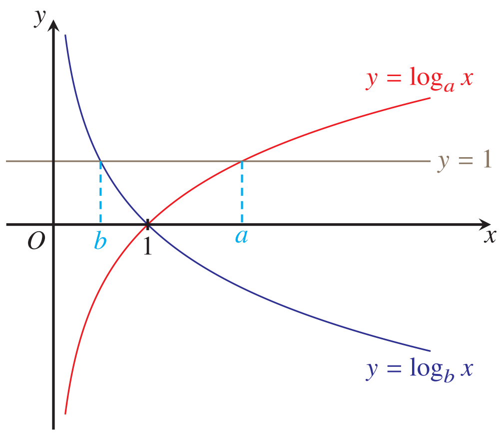
66. C

Draw the line $y = 1$.

The line intersects the graph at $(b,\ 1)$ and $(a,\ 1)$.

From the graph, we have $0 < b < 1 < a$.

The result follows.



67. D

L.H.S. = $\log q + p\log x$ and R.H.S. = $3\log x - 3\log 2 + 2\log x = 5\log x - 3\log 2 = 5\log x + \log 2^{-3}$
So, $p = 5$ and $q = 2^{-3} = \frac{1}{8}$.

68. A

$$\log x^4 = \log 4x^2 + 2$$
$$\log x^4 - \log 4x^2 = 2$$
$$\log \frac{x^4}{4x^2} = 2$$
$$\frac{x^2}{4} = 10^2$$
$$x^2 = 400$$
$$x = 20 \quad \text{or} \quad -20$$

69. D

Take logarithm on all numbers.

A. $-375\log 543 \approx -1025.5499$
B. $\frac{1}{4321}\log 867 \approx 0.000\,679\,9$
C. $349\log\frac{1}{867} \approx -1025.3687$
D. $492\log\frac{2}{243} \approx -1025.6115$

The answer is D.

70. D

$p = \log 2$ and $q = \log 3$.
$\log\frac{5}{3} = \log 10 - \log 2 - \log 3 = 1 - p - q$

71. D

I. ✗. Consider the case when $x = 1$ and $y = z = 10$. We have $\log x^2 + \log y^2 = \log z^2$ but $x^2 + y^2 \neq z^2$.

II. ✓.
$$\log x^2 + \log y^2 = \log z^2$$
$$2\log x + 2\log y = 2\log z$$
$$\log x + \log y = \log z$$

III. ✓.
$$\log x^2 + \log y^2 = \log z^2$$
$$\log x^2y^2 = \log z^2$$
$$x^2y^2 = z^2$$

72. A

$$
\begin{aligned}
f(x+1)-f(x) &= [\log(2x+2)-\log 2x]-[(x+1)-x] \\
&= \log\frac{2x+2}{2x}-1 \\
&= \log\frac{x+1}{x}-\log 10 \\
&= \log\frac{x+1}{10x}
\end{aligned}
$$

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