

**REG-2425-MOCK-SET 7-MATH-EP(M2)****Suggested solutions**

$$1. \quad (a) \quad \overrightarrow{OC} = \frac{3}{4}\overrightarrow{OA} + \frac{1}{4}\overrightarrow{OB}$$

$$= \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$$

1A

$$(b) \quad |\overrightarrow{OC}|^2 = \overrightarrow{OC} \cdot \overrightarrow{OC}$$

$$= \frac{9}{16}|\mathbf{a}|^2 + \frac{3}{8}\mathbf{a} \cdot \mathbf{b} + \frac{1}{16}|\mathbf{b}|^2$$

$$= \frac{9}{16}(1)^2 + \frac{3}{8}(1)(5) \cos \frac{\pi}{3} + \frac{1}{16}(5)^2$$

$$= \frac{49}{16}$$

$$|\overrightarrow{OC}| = \frac{7}{4}$$

1M

1A

$$2. \quad (a) \quad \lim_{t \rightarrow 0} \frac{1}{t} \sin \frac{\sqrt{1+t}-1}{2} = \lim_{t \rightarrow 0} \frac{\sin \frac{\sqrt{1+t}-1}{2}}{\frac{\sqrt{1+t}-1}{2}} \times \frac{\sqrt{1+t}-1}{2t}$$

$$= \lim_{t \rightarrow 0} \left( \frac{\sin \frac{\sqrt{1+t}-1}{2}}{\frac{\sqrt{1+t}-1}{2}} \times \frac{\sqrt{1+t}-1}{2t} \times \frac{\sqrt{1+t}+1}{\sqrt{1+t}+1} \right)$$

$$= \lim_{t \rightarrow 0} \left( \frac{\sin \frac{\sqrt{1+t}-1}{2}}{\frac{\sqrt{1+t}-1}{2}} \times \frac{t}{2t(\sqrt{1+t}+1)} \right)$$

$$= \lim_{t \rightarrow 0} \left( \frac{\sin \frac{\sqrt{1+t}-1}{2}}{\frac{\sqrt{1+t}-1}{2}} \times \frac{1}{2(\sqrt{1+t}+1)} \right)$$

$$= 1 \cdot \frac{1}{2(1+1)}$$

$$= \frac{1}{4}$$

1M

1M

1A

$$(b) \quad f'(1) = \lim_{h \rightarrow 0} \frac{\cos \sqrt{1+h} - \cos \sqrt{1}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin \frac{\sqrt{1+h}+1}{2} \sin \frac{\sqrt{1+h}-1}{2}}{h}$$

$$= -2 \lim_{h \rightarrow 0} \left( \frac{1}{h} \sin \frac{\sqrt{1+h}-1}{2} \sin \frac{\sqrt{1+h}+1}{2} \right)$$

$$= -2 \times \frac{1}{4} \times \sin \frac{1+1}{2}$$

$$= -\frac{1}{2} \sin 1$$

1M

1M

1A

| Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Marks                                             |
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| <p>3. <math>(1 + ax)^n = 1 + C_1^n(ax) + C_2^n(ax)^2 + \dots</math></p> $= 1 + nax + \frac{n(n-1)}{2}a^2x^2 + \dots$ <p>We have <math>na = -\frac{3}{2}n</math> and <math>\frac{n(n-1)}{2}a^2 = 63</math>.</p> <p>From the first equation, we have <math>a = -\frac{3}{2}</math>.</p> $\frac{n(n-1)}{2} \left(-\frac{3}{2}\right)^2 = 63$ $\frac{n^2}{2} - \frac{n}{2} - 112 = 0$ $n = 8 \quad \text{or} \quad -7 \text{ (rejected)}$                                                                                                                                                                                                                                       | <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> |
| <p>4. (a) <math>\frac{\cos A + \cos B}{\sin A + \sin B}</math></p> $= \frac{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}$ $= \frac{\cos \frac{A+B}{2}}{\sin \frac{A+B}{2}}$ $= \cot \frac{A+B}{2}$ <p>(b) <math>5 \sin A - 3 \cos A = 3 \cos B - 5 \sin B</math></p> $5(\sin A + \sin B) = 3(\cos A + \cos B)$ $\frac{5}{3} = \frac{\cos A + \cos B}{\sin A + \sin B} \quad (\sin A + \sin B \neq 0)$ $\cot \frac{A+B}{2} = \frac{5}{3}$ $\tan \frac{A+B}{2} = \frac{3}{5}$ $\cot(A+B) = \frac{1 - \tan^2 \frac{A+B}{2}}{2 \tan \frac{A+B}{2}}$ $= \frac{1 - \left(\frac{3}{5}\right)^2}{2 \left(\frac{3}{5}\right)}$ $= \frac{8}{15}$ | <p>1M</p> <p>1</p> <p>1A</p> <p>1M</p> <p>1A</p>  |

| Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Marks                                                                                        |
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| <p>5. (a) <math>a(\sin x + 2 \cos x)b(\cos x - 2 \sin x) = (2a + b) \cos x + (a - 2b) \sin x</math><br/> We have <math>2a + b = 13</math> and <math>a - 2b = -6</math>.<br/> Solving, we have <math>a = 4</math> and <math>b = 5</math>.</p> <p>(b) Let <math>u = \sin x + 2 \cos x</math>. Then <math>du = (\cos x - 2 \sin x) dx</math>.</p> $\int_0^{\frac{\pi}{4}} \frac{13 - 6 \tan x}{\tan x + 2} dx$ $= \int_0^{\frac{\pi}{4}} \frac{13 \cos x - 6 \sin x}{\sin x + 2 \cos x} dx$ $= \int_0^{\frac{\pi}{4}} \frac{4(\sin x + 2 \cos x) + 5(\cos x - 2 \sin x)}{\sin x + 2 \cos x} dx$ $= 4 \int_0^{\frac{\pi}{4}} dx + 5 \int_0^{\frac{\pi}{4}} \frac{\cos x - 2 \sin x}{\sin x + 2 \cos x} dx$ $= 4 \left[ x \right]_0^{\frac{\pi}{4}} + 5 \int_2^{\frac{3\sqrt{2}}{2}} \frac{du}{u}$ $= \pi + 5 \left[ \ln  u  \right]_2^{\frac{3\sqrt{2}}{2}}$ $= \pi + 5 \ln \frac{3\sqrt{2}}{4}$ | <p>1M<br/>1A<br/>1M<br/><br/><br/><br/><br/><br/><br/>1M<br/>1A</p>                          |
| <p>6. <math>\frac{d}{dx}(4x + y^2) = \frac{d}{dx}(16)</math><br/> <math>4 + 2y \frac{dy}{dx} = 0</math><br/> <math>\frac{dy}{dx} = -\frac{2}{y}</math></p> <p>Let the coordinates of the point of contact of <math>L</math> and the curve be <math>(a, b)</math>.</p> $\frac{b - 0}{a - 8} = -\frac{2}{b}$ $b^2 = 16 - 2a$ $(16 - 4a) = 16 - 2a$ $a = 0$ <p>The coordinates of the point of contact are <math>(0, 4)</math> and <math>(0, -4)</math>.<br/> Equation of tangent to the curve at <math>(0, 4)</math> is</p> $y - 4 = -\frac{2}{4}(x - 0)$ $y = -\frac{x}{2} + 4$ <p>Equation of tangent to the curve at <math>(0, -4)</math> is</p> $y + 4 = -\frac{2}{(-4)}(x - 0)$ $y = \frac{x}{2} - 4$ <p>The equation of <math>L</math> is <math>y = -\frac{x}{2} + 4</math> or <math>y = \frac{x}{2} - 4</math>.</p>                                                                     | <p>1M<br/><br/><br/><br/><br/><br/><br/>1M<br/><br/><br/>1M<br/><br/>1A<br/><br/><br/>1A</p> |

| Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |  | Marks                                                    |
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| <p>7. (a) <math>\begin{vmatrix} a &amp; 1 &amp; 1 \\ 2 &amp; -1 &amp; -1 \\ 1 &amp; -3 &amp; 2 \end{vmatrix}</math></p> $= a \begin{vmatrix} -1 & -1 \\ -3 & 2 \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}$ $= -5(a + 2)$ <p>(E) has infinitely many solutions.</p> $-5(a + 2) = 0$ $a = -2$ <p>The augmented matrix of (E) becomes</p> $\left( \begin{array}{ccc c} -2 & 1 & 1 & 6 \\ 2 & -1 & -1 & b \\ 1 & -3 & 2 & 2 \end{array} \right) \sim \left( \begin{array}{ccc c} 1 & -3 & 2 & 2 \\ 2 & -1 & -1 & b \\ -2 & 1 & 1 & 6 \end{array} \right) \sim \left( \begin{array}{ccc c} 1 & -3 & 2 & 2 \\ 0 & 5 & -5 & b - 4 \\ 0 & 0 & 0 & b + 6 \end{array} \right)$ <p>(E) has infinitely many solutions.</p> $b + 6 = 0$ $b = -6$ <p>(b) When <math>b = -6</math>, the augmented matrix of (E) becomes</p> $\left( \begin{array}{ccc c} 1 & -3 & 2 & 2 \\ 0 & 5 & -5 & -10 \\ 0 & 0 & 0 & 0 \end{array} \right) \sim \left( \begin{array}{ccc c} 1 & 0 & -1 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right)$ <p>The solution set is <math>\{(t - 4, t - 2, t) : t \in \mathbb{R}\}</math>.</p> |  | <p>1M<br/>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> |

| Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks                                                              |
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| <p>8. (a) When <math>n = 1</math>,</p> $\text{L.H.S.} = \frac{1}{(1)(3)} + \frac{2^2}{(3)(5)} = \frac{3}{5}$ $\text{R.H.S.} = \frac{1(3)}{5} = \frac{3}{5} = \text{L.H.S.}$ <p>It is true for <math>n = 1</math>.</p> <p>Assume <math>\sum_{k=1}^{2p} \frac{k^2}{(2k-1)(2k+1)} = \frac{p(2p+1)}{4p+1}</math>, where <math>p \in \mathbb{Z}^+</math>.</p> $\begin{aligned} & \sum_{k=1}^{2p+2} \frac{k^2}{(2k-1)(2k+1)} \\ &= \sum_{k=1}^{2p} \frac{k^2}{(2k-1)(2k+1)} + \frac{(2p+1)^2}{(4p+1)(4p+3)} + \frac{(2p+2)^2}{(4p+3)(4p+5)} \\ &= \frac{p(2p+1)}{4p+1} + \frac{(2p+1)^2}{(4p+1)(4p+3)} + \frac{(2p+2)^2}{(4p+3)(4p+5)} \\ &= \frac{(2p+1)[p(4p+3) + (2p+1)]}{(4p+1)(4p+3)} + \frac{4(p+1)^2}{(4p+3)(4p+5)} \\ &= \frac{(2p+1)(4p^2+5p+1)}{(4p+1)(4p+3)} + \frac{4(p+1)^2}{(4p+3)(4p+5)} \\ &= \frac{(2p+1)(4p+1)(p+1)}{(4p+1)(4p+3)} + \frac{4(p+1)^2}{(4p+3)(4p+5)} \\ &= \frac{(2p+1)(p+1)}{4p+3} + \frac{4(p+1)^2}{(4p+3)(4p+5)} \\ &= \frac{(p+1)[(2p+1)(4p+5) + 4(p+1)]}{(4p+3)(4p+5)} \\ &= \frac{(p+1)(8p^2+18p+9)}{(4p+3)(4p+5)} \\ &= \frac{(p+1)((4p+3)(2p+3))}{(4p+3)(4p+5)} \\ &= \frac{(p+1)(2p+3)}{4p+5} \end{aligned}$ <p>It is true for <math>n = p + 1</math>.</p> <p>By M.I., it is true <math>\forall n \in \mathbb{Z}^+</math>.</p> <p>(b) <math display="block">\begin{aligned} &amp; \sum_{k=1}^{20} \frac{k^2}{(2k-1)(2k+1)} \\ &amp;= \sum_{k=1}^{20} \frac{k^2}{(2k-1)(2k+1)} - \sum_{k=1}^{10} \frac{k^2}{(2k-1)(2k+1)} \\ &amp;= \frac{10(21)}{41} - \frac{5(11)}{21} \\ &amp;= \frac{2155}{861} \end{aligned}</math></p> | <p>1</p> <p>1</p> <p>1M</p> <p>1</p> <p>1M</p> <p>1M</p> <p>1A</p> |

| Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Marks                                                       |
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| <p>9. (a) <math>\left(\frac{x}{2}\right)^2 + y^2 = 13^2</math><br/> <math>x^2 + 4y^2 = 676</math></p> <p>(b) When <math>y = 12</math>, we have <math>x = 10</math>.</p> $\frac{d}{dt}(x^2 + 4y^2) = \frac{d}{dt}(676)$ $2x \frac{dx}{dt} + 8y \frac{dy}{dt} = 0$ $x \frac{dx}{dt} + 4y \frac{dy}{dt} = 0$ <p>Put <math>x = 10</math>, <math>y = 12</math> and <math>\frac{dy}{dt} = \frac{1}{3}</math>.</p> $10 \frac{dx}{dt} + 4(12) \left(\frac{1}{3}\right) = 0$ $\frac{dx}{dt} = -\frac{8}{5}$ <p>Let <math>S \text{ m}^2</math> be the area of <math>\triangle ABH</math>.</p> $S = \frac{1}{2}xy$ $\frac{dS}{dt} = \frac{1}{2} \left( y \frac{dx}{dt} + x \frac{dy}{dt} \right)$ $= \frac{1}{2} \left[ 10(4) + 12 \left( -\frac{8}{5} \right) \right]$ $= \frac{52}{5}$ <p>Required rate is <math>\frac{52}{5} \text{ m}^2/\text{s}</math>.</p> | <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> |

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| Solution                                                                                                                                                                                                                                                                                                                                                                                                                           | Marks                                          |
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| 11. (a) $\int g(x) \, dx = \sin^2 x \left( \frac{\sin 2x}{2} \right) - \frac{1}{2} \int (2 \sin x \cos x) \sin 2x \, dx$<br>$= \frac{1}{2} \sin^2 x \sin 2x - \frac{1}{2} \int \sin^2 2x \, dx$                                                                                                                                                                                                                                    | 1M<br><br>1                                    |
| (b) $\int_0^\pi g(x) \, dx = \frac{1}{2} \left[ \sin^2 x \sin 2x \right]_0^\pi - \frac{1}{2} \int_0^\pi \sin^2 2x \, dx$<br>$= 0 - \frac{1}{4} \int_0^\pi (1 - \cos 4x) \, dx$<br>$= -\frac{1}{4} \left[ x - \frac{\sin 4x}{4} \right]_0^\pi$<br>$= -\frac{\pi}{4}$                                                                                                                                                                | 1M<br><br>1A                                   |
| (c) Let $u = \pi - x$ . Then $du = -dx$ .                                                                                                                                                                                                                                                                                                                                                                                          | 1M                                             |
| $\begin{aligned} \int_0^\pi xg(x) \, dx &= - \int_\pi^0 (\pi - u)g(\pi - u) \, du \\ &= \int_0^\pi (\pi - u) \sin^2(\pi - u) \cos(2\pi - 2u) \, du \\ &= \int_0^\pi (\pi - x) \sin^2 x \cos 2x \, dx \\ &= \pi \int_0^\pi g(x) \, dx - \int_0^\pi xg(x) \, dx \\ &= \frac{\pi}{2} \int_0^\pi g(x) \, dx \\ &= \frac{\pi}{2} \left( -\frac{\pi}{4} \right) \\ &= -\frac{\pi^2}{8} \end{aligned}$                                    | 1M<br><br><br><br><br><br>1M<br><br><br><br>1A |
| (d) Note that $(-x)g(-x) = -x \sin^2(-x) \cos(-2x) = -x \sin^2 x \cos 2x = -xg(x)$ .                                                                                                                                                                                                                                                                                                                                               | 1M                                             |
| Thus, $xg(x)$ is an odd function.                                                                                                                                                                                                                                                                                                                                                                                                  |                                                |
| Let $u = x - \pi$ . Then $du = dx$ .                                                                                                                                                                                                                                                                                                                                                                                               | 1M                                             |
| $\begin{aligned} \int_{-\pi}^{2\pi} xg(x) \, dx &= \int_{-\pi}^\pi xg(x) \, dx + \int_\pi^{2\pi} xg(x) \, dx \\ &= 0 + \int_\pi^{2\pi} xg(x) \, dx \\ &= \int_0^\pi (u + \pi) \sin^2(u + \pi) \cos(2u + 2\pi) \, du \\ &= \int_0^\pi (\pi + x)g(x) \, dx \\ &= \pi \int_0^\pi g(x) \, dx + \int_0^\pi xg(x) \, dx \\ &= \pi \left( -\frac{\pi}{4} \right) + \left( -\frac{\pi^2}{8} \right) \\ &= -\frac{3\pi^2}{8} \end{aligned}$ | 1M<br><br><br><br><br><br><br><br><br>1A       |

| Solution |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Marks                |
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| 12. (a)  | $\overrightarrow{OC} = \frac{3}{5}\overrightarrow{OA} + \frac{2}{5}\overrightarrow{OB}$ $= 3\mathbf{i} + \mathbf{j} - \mathbf{k}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 1M<br>1A             |
| (b)      | <p>A required vector</p> $= \overrightarrow{OB} \times \overrightarrow{OC}$ $= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & 5 \\ 3 & 1 & -1 \end{vmatrix}$ $= -3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}$ <p>Required area</p> $= \frac{1}{2}  \overrightarrow{OB} \times \overrightarrow{OC} $ $= \frac{1}{2} \sqrt{3^2 + 12^2 + 3^2}$ $= \frac{9\sqrt{2}}{2}$                                                                                                                                                                                                                                                                                                                                                                                                                           | 1M<br>1A             |
| (c) (i)  | <p>Let <math>\mathbf{n} = \overrightarrow{OB} \times \overrightarrow{OC} = -3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}</math>.</p> $\overrightarrow{ED} = \frac{\overrightarrow{OD} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}} \mathbf{n}$ $= \frac{-6 - 48}{3^2 + 12^2 + 3^2} (-3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$ $= \mathbf{i} - 4\mathbf{j} - \mathbf{k}$ $\overrightarrow{BE} = \overrightarrow{BD} - \overrightarrow{ED}$ $= (5\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}) - (\mathbf{i} - 4\mathbf{j} - \mathbf{k})$ $= 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ $= 6\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ $= \frac{3}{2} \overrightarrow{BE}$ <p>Thus, <math>B</math>, <math>C</math> and <math>E</math> are collinear.</p> | 1M<br>1M<br>1A       |
| (ii)     | <p><math>\overrightarrow{OE} \cdot \overrightarrow{BC} = (\mathbf{i} + \mathbf{k}) \cdot (6\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}) = 0</math></p> <p>We have <math>OE \perp BC</math>.</p> <p>Required angle is <math>\angle ODE</math>.</p> $\sin \angle ODE = \frac{OE}{OD}$ $= \frac{\sqrt{1^2 + 1^2}}{2^2 + 4^2}$ $= \frac{1}{\sqrt{10}}$ $\angle ODE \approx 18.4^\circ$                                                                                                                                                                                                                                                                                                                                                                                                                                 | 1M<br>1M<br>1M<br>1A |

| Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Marks                                                                                                                |
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| <p>13. (a) (i) <math>f(0) = I + 0A + (1 - 1)A^2</math><br/> <math>= I</math></p> <p>(ii) <math>A(A^3 + A) = A(\mathbf{0})</math><br/> <math>A^4 + A^2 = \mathbf{0}</math><br/> <math>A^4 = -A^2</math></p> <p>(iii) <math>f(\alpha)f(\beta)</math><br/> <math>= [I + (\sin \alpha)A + (1 - \cos \alpha)A^2][I + (\sin \beta)A + (1 - \cos \beta)A^2]</math><br/> <math>= I + (\sin \alpha + \sin \beta)A + [(1 - \cos \beta) + \sin \alpha \sin \beta + (1 - \cos \alpha)]A^2</math><br/> <math>+ [\sin \alpha(1 - \cos \beta) + (1 - \cos \alpha) \sin \beta]A^3 + (1 - \cos \alpha)(1 - \cos \beta)A^4</math><br/> <math>= I + (\sin \alpha + \sin \beta)A + (2 - \cos \alpha - \cos \beta + \sin \alpha \sin \beta)A^2</math><br/> <math>+ (\sin \alpha + \sin \beta - \sin \alpha \cos \beta - \cos \alpha \sin \beta)(-A)</math><br/> <math>+ (1 - \cos \alpha - \cos \beta + \cos \alpha \cos \beta)(-A^2)</math><br/> <math>= I + (\sin \alpha \cos \beta + \cos \alpha \sin \beta)A + (1 - \cos \alpha \cos \beta + \sin \alpha \sin \beta)A^2</math><br/> <math>= I + \sin(\alpha + \beta)A + [1 - \cos(\alpha + \beta)]A^2</math><br/> <math>= f(\alpha + \beta)</math></p> <p>(iv) Note that <math>f(\theta)f(-\theta) = f(-\theta)f(\theta) = f(0) = I</math>.<br/> <math>f(\theta)</math> is invertible.<br/> <math>[f(\theta)]^{-1} = f(-\theta)</math></p> <p>(b) <math>B^2 = \begin{pmatrix} 0 &amp; -1 \\ 1 &amp; 0 \end{pmatrix} \begin{pmatrix} 0 &amp; -1 \\ 1 &amp; 0 \end{pmatrix} = \begin{pmatrix} -1 &amp; 0 \\ 0 &amp; -1 \end{pmatrix}</math><br/> <math>B^3 = \begin{pmatrix} -1 &amp; 0 \\ 0 &amp; -1 \end{pmatrix} \begin{pmatrix} 0 &amp; -1 \\ 1 &amp; 0 \end{pmatrix} = \begin{pmatrix} 0 &amp; 1 \\ -1 &amp; 0 \end{pmatrix}</math><br/> <math>B^3 + B = \begin{pmatrix} 0 &amp; 1 \\ -1 &amp; 0 \end{pmatrix} + \begin{pmatrix} 0 &amp; -1 \\ 1 &amp; 0 \end{pmatrix} = \begin{pmatrix} 0 &amp; 0 \\ 0 &amp; 0 \end{pmatrix}</math><br/> Put <math>A = B</math> and <math>\theta = \frac{\pi}{2}</math>.<br/> <math>f\left(\frac{\pi}{2}\right) = I + \left(\sin \frac{\pi}{2}\right)B + \left(1 - \cos \frac{\pi}{2}\right)B^2</math><br/> <math>= I + B + B^2</math><br/> <math>(I + B + B^2)^{-1} = f\left(-\frac{\pi}{2}\right)</math><br/> <math>= I + \left(\sin \frac{-\pi}{2}\right)B + \left(1 - \cos \frac{-\pi}{2}\right)B^2</math><br/> <math>= I - B + B^2</math><br/> We have <math>p = -1</math> and <math>q = 1</math>.<br/> The claim is correct.</p> | <p>1A</p> <p>1</p> <p>1M</p> <p>1M</p> <p>1</p> <p>1M</p> <p>1</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> |