REG-2425-MOCK-SET 7-MATH-CP 2

Answers:

1. D	2. B	3. A	4. A	5. D	6. D	7. C	8. C	9. D	10. D
11. C	12. A	13. A	14. C	15. A	16. B	17. D	18. D	19. B	20. C
21. D	22. A	23. B	24. A	25. C	26. C	27. C	28. B	29. B	30. B
31. D	32. B	33. A	34. A	35. B	36. D	37. A	38. C	39. C	40. B
41. D	42. B	43. C	44. D	45. A					

Suggested Solutions:

1. D

$$\begin{aligned}4p^4 - 64 &= 4(p^2+4)(p^2-4)\\ &= 4(p^2+4)(p+2)(p-2)\end{aligned}$$

2. B

$$5^{2020}\left(-\frac{1}{5}\right)^{2021} = (-1)^{2021}(5)^{2020-2021} = -5^{-1} = -0.2$$

3. A

$$\begin{aligned}x &= \frac{1}{2+\frac{3}{a}}\\ 2x + \frac{3x}{a} &= 1\\ \frac{3x}{a} &= 1-2x\\ a &= \frac{3x}{1-2x}\end{aligned}$$

4. A

$11 - 2x < 13$ and $\frac{x}{2} + 1 > \frac{x}{4}$

$x > -1$ $\qquad\qquad \frac{x}{4} > -1$

$\qquad\qquad\qquad x > -4$

Therefore, $x > -4$.

5. D

$\frac{1}{\pi^2+1} \approx 0.091999668$

A. ✗. It should be 0.092.

B. ✗. It should be 0.092.

C. ✗. It should be 0.09200.

D. ✓.

6. D

Required ratio $= \frac{1}{1-30\%} : \frac{1}{1-25\%} = 15 : 14$

7. C

$h(1) = 1 + c + d = 0$

Remainder $= (-1)^{2020} + c(-1)^{2019} + d$

$= 1 - c + d$

$= 1 - c + (-1 - c)$

$= -2c$

8. C

I. ✗. y-intercept $= c$.

II. ✗. Take $a = c = 1$ and $b = 10$ (or any applicable examples). $4ac = 4$ and $b^2 = 100 > 4ac$.
The roots of $f(x) = 0$ are $\frac{-1 \pm \sqrt{10^2 - 4(1)(1)}}{2(1)} = -5 \pm 2\sqrt{6}$, which are not rational.

III. ✓.

9. D

Lower limit $= (20.4 - 0.1)^2$

$= 412.09\,\text{cm}^2$

10. D

Interest $= 200\,000\left(1 + \frac{2\%}{12}\right)^{3\times12} - 200\,000$

$\approx \$12357$

11. C

Let $c = 4$. Then $a = 3$ and $b = 10$.

$(3a + c) : (a + 3b) = 13 : 33$

12. A

Let $y = \frac{kx}{\sqrt{z}}$, where k is a non-zero constant.

Then $k = \frac{y\sqrt{z}}{x}$.

Therefore, $\frac{x}{y\sqrt{z}} = \frac{1}{k}$ must be a constant.

13. A

Let the required cost be $\$b$/L.

$$3(42) + 2b = 36(2 + 3)$$
$$b = 27$$

14. C

Denote the nth term of the sequence by a_n.

We have $a_4 = 34$, $a_7 = 144$ and $a_{n+2} = a_n + a_{n+1}$ for any positive integers n.

Let $a_3 = x$.

Put $n = 3$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_5 = a_3 + a_4$$
$$a_5 = x + 34$$

Put $n = 4$ and $n = 5$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_6 = a_4 + a_5 \qquad a_7 = a_5 + a_6$$
$$a_6 = 34 + (x + 34) \qquad 144 = (x + 34) + (x + 68)$$
$$a_6 = x + 68 \qquad x = 21$$

15. A

Let the base radius of solid A be r_A and radius of solid B be r_B.

$$2 \times \frac{1}{3}\pi(r_A)^2(r_A) = \frac{1}{2} \times \frac{4}{3}\pi(r_B)^3$$
$$r_A^3 = r_B^3$$
$$r_A = r_B$$

$$\text{Ratio of curved surface area} = \frac{\pi r_A\sqrt{r_A^2 + r_A^2}}{2\pi r_B^2}$$
$$= \frac{\sqrt{2}}{2}$$

Required ratio is $\sqrt{2} : 2$.

16. B

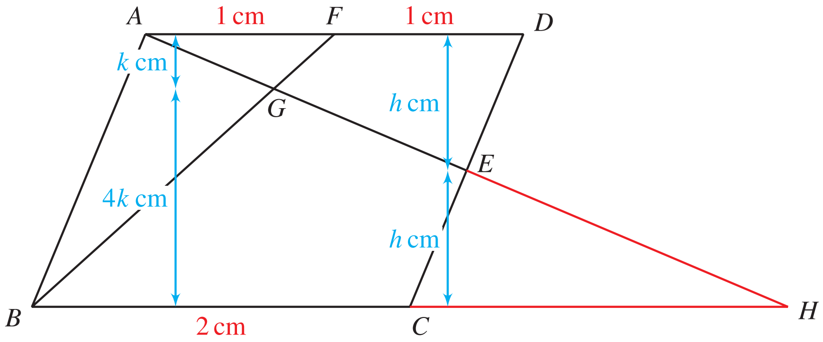
Let $AC = x$ cm.

$$4\pi = 2(18)\pi \times \frac{\angle AOB}{360^\circ}$$
$$\frac{\angle AOB}{360^\circ} = \frac{1}{9}$$
$$28\pi = (18 + x)^2\pi \times \frac{\angle AOB}{360^\circ} - (18)^2\pi \times \frac{\angle AOB}{360^\circ}$$
$$28 = (x^2 + 36x) \times \frac{1}{9}$$
$$0 = \frac{x^2}{9} + 4x - 28$$
$$x = 6$$

17. D

Suppose BC produced and AE produced intersect at H.

Let $AE = 1$ cm. Then we have the lengths as shown in the figure.



Point E Note that $\triangle ADE \sim \triangle HCE$ (ratio 1 : 1).

We have $CH = DA = 2$ cm.

Point G Note that $\triangle AFG \sim \triangle HBG$ (ratio 1 : 4).

Consider the total height of the parallelogram.

$$k + 4k = h + h$$
$$k = \frac{2h}{5}$$

Consider the area of quadrilateral $DEGF$.

$$32 = \frac{2(h)}{2} - \frac{1(k)}{2}$$
$$h = 40$$

$$\text{Required area} = \frac{4(4k)}{2} - \frac{2(h)}{2}$$
$$= 88\text{ cm}^2$$

18. D

Regular pentagons are similar. Let the hypotenuse of the triangle be x.

$$\text{Required ratio} = x^2 : (x\sin 60^\circ)^2 : (x\cos 60^\circ)^2$$
$$= 4 : 3 : 1$$

19. B

Let the centre of circle be O. Radius $= \frac{10}{2} = 5$ cm.

$\angle BOC = 2 \times 35^\circ = 70^\circ$ and $\angle AOB = 110^\circ$

$$\text{Required area} = 5^2\pi \times \frac{110^\circ}{360^\circ} - \frac{1}{2}(5)^2 \sin 110^\circ$$
$$\approx 12.3\text{ cm}^2$$

20. C

Let M, N and T be mid-points of AB, CD and EF respectively.

$AB = CD = EF \quad \rightarrow \quad OT = OM = ON$

$\triangle OMQ \cong \triangle ONQ$ and $\triangle ONR \cong \triangle OTR$

Let $\angle OQN = a$ and $\angle ORN = b$.

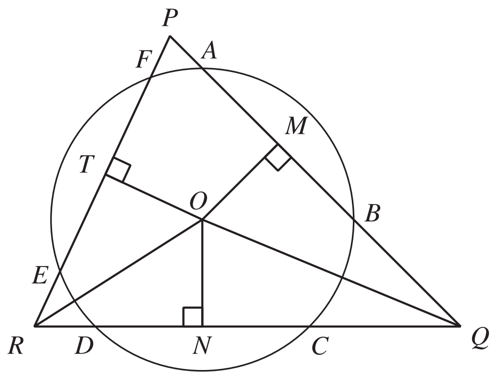
We have $\angle OQM = a$ and $ORT = b$.

$2a + 2b + 42° = 180°$

$a + b = 69°$

$\angle QOR = 180° - (a + b)$

$= 111°$



21. D

Let $AD = 4x$ cm. Then $DE = 5x$ cm and $AE = \sqrt{(5x)^2 - (4x)^2} = 3x$ cm.

$\tan \angle BEF = \dfrac{3}{4} = \dfrac{BF}{BE}$

$\dfrac{3}{4} = \dfrac{4x - 32}{124 - 3x}$

$x = 20$

$EF = \sqrt{BE^2 + BF^2} = \sqrt{48^2 + 64^2} = 80\,\text{cm}$

22. A

Bearing of B from O is $330° - 180° = 150° \quad \Rightarrow \quad \angle AOB = 215° - 150° = 65°$

$AO = AB \quad \Rightarrow \quad \angle OAB = 180° - 2 \times 65° = 50°$

Required bearing is $215° - 180° + 50° = 085°$.

23. B

I. ✓. Each exterior angle $= \dfrac{360°}{18} = 20°$. Each interior angle $= 180° - 20° = 160°$.

II. ✗. There are 18 axes of reflectional symmetry.

III. ✓.

24. A

Put $a = 1$. $Q\left(-1,\ -\sqrt{3}\right) \quad \longrightarrow \quad R\left(-\sqrt{3},\ 1\right) \quad \longrightarrow \quad S\left(\sqrt{3},\ 1\right)$

By calculator function (`Pol(`), the polar coordinates of S are $(2,\ 30°)$.

Only option A gives $(2,\ 30°)$ when $a = 1$.

25. $\boxed{C}$

Consider the y-intercepts.

$$\frac{15}{k} = \frac{5}{8}$$
$$k = 24$$

Consider the slopes.

$$\frac{h}{k} \times \frac{-3}{8} = -1$$
$$h = 64$$

$h - k = 64 - 24 = 40$

26. $\boxed{C}$

Equidistant from a point and a straight line $\Rightarrow$ locus of P is a parabola.

27. $\boxed{C}$

C: $x^2 + y^2 - 2x + 8y - \dfrac{534}{5} = 0$

I. ✓. Centre $(1,\ -4)$. Since $3(1) + 7(-4) + 25 = 0$, the line passes through centre of circle.

II. ✓. $2^2 + 16^2 - 2(2) + 8(-16) - \dfrac{534}{5} = \dfrac{106}{5} > 0$. $(2,\ -16)$ lies outside the circle.

III. ✗.

28. $\boxed{B}$

With \$10 banknote selected, the remaining two banknotes have to be at least \$80.

Only unfavourable case is \$20 + \$50.

Required probability $= 1 - \dfrac{1}{3} = \dfrac{2}{3}$.

29. $\boxed{B}$

I. ✓.

II. ✗.

III. ✓. Range = maximum − minimum

30. $\boxed{B}$

Mode = 72 $\Rightarrow$ frequency of 72 is at least 3 $\Rightarrow$ take $x = y = 72$

Mean = 74 $\Rightarrow$ $74 = \dfrac{33 + 72 + 76 + \ldots + z}{10}$ $\Rightarrow$ $z = 74$

Median $= \dfrac{74 + 76}{2} = 75$.

31. D

$\log_{27} y = \log_{27} a + x \log_{27} b$

$$\text{Slope} = \log_{27} b = \frac{0+1}{3-0}$$

$$b = 27^{\frac{1}{3}}$$

$$= 3$$

32. B

$C00000000000000040_{16} = 12 \times 16^{17} + 4 \times 16^{1} = 12 \times 2^{68} + 64$

33. A

I. ✓. Common difference $= 5\log x$.

II. ✓. Common difference $= x - 1$.

III. ✗. Take $x = 90$. The sequence becomes $0, -1, 0$, which is not an arithmetic sequence.

34. A

Let the first term and common ratio be a and r respectively.

$$\frac{ar^6}{ar^4} = \frac{15}{375}$$

$$r^2 = \frac{1}{25}$$

$$r = \pm\frac{1}{5}$$

I. ✓. $a_1 = \dfrac{375}{r^4} = 234\,375$.

II. ✗. When $r = -\dfrac{1}{5}$, $a_2 = \dfrac{ar^4}{r^3} < 0$ and $a_2 \neq 46\,875$.

III. ✗. When $r = -\dfrac{1}{5}$,

$$a_1 + a_2 + a_3 + \ldots = \frac{234\,375}{1 - \left(-\frac{1}{5}\right)}$$

$$= 195\,312.5 < 290\,000$$

35. B

Straight line	x-intercept	y-intercept
$x + 2y = 30$	30	15
$5x - 4y = 30$	6	-7.5
$9x + 4y = 30$	$\frac{10}{3}$	7.5

The solution region is shaded as shown.

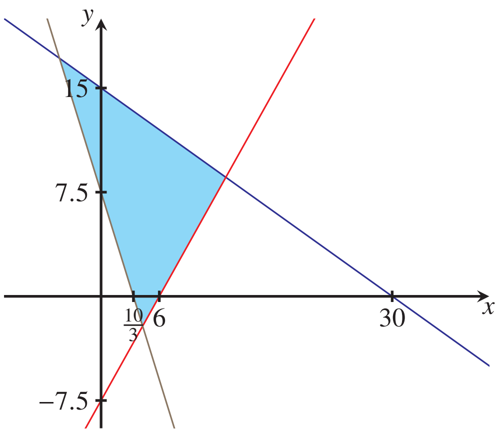
$7x + 14y + 15$ is smaller when x and y are smaller.

$7x + 14y + 15$ attains its minimum at the bottom left corner.

At $\left(\frac{30}{7}, -\frac{15}{7}\right)$, $7x + 14y + 15 = 15$.

At $\left(-\frac{30}{7}, \frac{120}{7}\right)$, $7x + 14y + 15 = 225$.

Least value = 15



36. D

Let $z_2 - z_1 = r$, where r is a real number.

$$r = \frac{3 + ki}{2 - i} - \frac{k + 5i}{1 - i}$$

$$r(2 - i)(1 - i) = (3 + ki)(1 - i) - (k + 5i)(2 - i)$$

$$r(1 - 3i) = -k - 2 + (2k - 13)i$$

Compare real and imaginary parts,

$$\begin{cases} r = -k - 2 \\ -3r = 2k - 13 \end{cases}$$

Solving, $r = 17$ and $k = -19$.

37. A

$$\frac{3}{\tan^2(90° - \alpha)} - \frac{5}{\sin(270° + \alpha)} = \frac{1}{\cos^2 \alpha}$$

$$3\tan^2 \alpha + \frac{5}{\cos \alpha} = \frac{1}{\cos^2 \alpha}$$

$$3\sin^2 \alpha + 5\cos \alpha = 1$$

$$-3\cos^2 \alpha + 5\cos \alpha + 2 = 0$$

$$\cos \alpha = -\frac{1}{3} \quad \text{or} \quad 2 \text{ (rejected)}$$

There are two roots for $\cos \alpha = -\frac{1}{3}$. Required number of roots is 2.

38. C

Let $PQ = 1$. Then $PR = \frac{1}{\sin 48°}$, $QR = \frac{1}{\tan 48°}$, $PS = \frac{1}{\sin 60°}$ and $QS = \frac{1}{\tan 60°}$.

$$RS^2 = QR^2 + QS^2 - 2(QR)(QS)\cos 105° = PR^2 + PS^2 - 2(PR)(PS)\cos \angle RPS$$

$$\angle RPS \approx 56°$$

39. C

Consider the system

$$\begin{cases} x - y + 13 = 0 \\ x^2 + y^2 - 14x + cy - 223 = 0 \end{cases}$$

Put $y = x + 13$ into equation of circle gives a quadratic equation.

Discriminant of the quadratic equation should be positive (2 distinct real roots) $\Rightarrow$ options A or C

When $c = 0$, by calculator program, there are two intersections $\Rightarrow$ required range contains 0

$\Rightarrow$ the answer is C.

40. B

$\angle DCA = \angle YAD = 31°$

$AX = AB \quad \Rightarrow \quad \angle XAB = \frac{180° - 64°}{2} = 58°$

$\angle ACB = \angle XAB = 58°$

$\angle BCD = 31° + 58° = 89°$

41. D

$OE = \sqrt{8^2 + 15^2} = 17$ and $EF = \sqrt{20^2 + 15^2} = 25$

Let the radius of inscribed circle be r. Consider the area of $\triangle OEF$,

$$\frac{(28 - 0)(15)}{2} = \frac{17r}{2} + \frac{25r}{2} + \frac{28r}{2}$$
$$r = 6$$

y-coordinate of G is 6. Let the x-coordinate of G be g.

$\tan \angle FOE = \text{slope of } OE \quad \Rightarrow \quad \angle FOE = \tan^{-1} \frac{15}{8}$

$$\text{Slope of } OG = \frac{6}{g} = \tan \frac{\angle FOE}{2}$$
$$= \frac{3}{5} \qquad \textit{(by calculator)}$$
$$g = 10$$

I. ✓.

II. ✓.

III. ✓. $m = \frac{6 - 0}{10 - 28} = -\frac{1}{3}$ and $n = \frac{15 - 0}{8 - 28} = -\frac{3}{4}$

$$\frac{2m}{1 - m^2} = -\frac{3}{4} = n$$

42. B

Required number = $7!6! = 3\,628\,800$

43. C

There are 4 prime numbers.

$$\text{Required probability} = \frac{6}{10} \times \frac{1}{10} + \left(\frac{6}{10}\right)^3 \times \frac{1}{10} + \left(\frac{6}{10}\right)^5 \times \frac{1}{10} + \ldots$$
$$= \frac{0.06}{1 - 0.36}$$
$$= \frac{3}{32}$$

44. D

Let the scores, mean, standard deviation, standard scores be x marks, μ marks, σ marks and z respectively.

$$z_2 - z_1 = \frac{x_2 - \mu}{\sigma} - \frac{x_1 - \mu}{\sigma}$$
$$= \frac{x_2 - x_1}{\sigma}$$
$$5 = \frac{20}{\sigma}$$
$$\sigma = 4$$

45. A

I. ✓.

II. ✗. $r' = kr \neq kr + p$.

III. ✗. $v' = k^2 v \neq kv + p$.