

REG-2425-MOCK-SET 6-MATH-CP 1**Suggested solutions**

$$1. \quad x = \frac{2y}{y-1}$$

$$xy - x = 2y$$

$$y(x-2) = x$$

$$y = \frac{x}{x-2}$$

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$$2. \quad \frac{(a^2b^{-3})^4}{a^5b^6} = \frac{a^8b^{-12}}{a^5b^6}$$

$$= \frac{a^{8-5}}{b^{6+12}}$$

$$= \frac{a^3}{b^{18}}$$

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3. Let the number of headphones bought by non-members be x .

Then the number of headphones bought by members is $2x + 5$.

1A

$$85(2x + 5) + 110x = 1825$$

1M

$$x = 5$$

$$\text{Required number} = x + (2x + 5) = 20$$

1A

$$4. \quad (a) \quad 7x^2 + 6xy - y^2 = (7x - y)(x + y)$$

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$$(b) \quad 14x^2 + 12xy - 2y^2 - 7x + y = 2(7x^2 + 6xy - y^2) - (7x - y)(x + y)$$

$$= 2(7x - y)(x + y) - (7x - y)(x + y)$$

$$= (7x - y)(2x + 2y - 1)$$

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5. (a) Note that $\angle AOC = 243^\circ - 63^\circ = 180^\circ$. A, O, C are collinear.

Required coordinates are $(1, 63^\circ)$.

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(b) $\angle AOB = 153^\circ - 63^\circ = 90^\circ$

$$\text{Required area} = \frac{1}{2}(8 + 6)(4)$$

$$= 28$$

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$$6. \quad (a) \quad \frac{1}{3}(x - 5) \leq \frac{x}{2}$$

$$-\frac{x}{6} \leq \frac{5}{3}$$

$$x \geq -10$$

$$2x + 13 > 3$$

$$x > -5$$

Thus, $x \geq -10$.

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(b) 1

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7. (a) Marked price = $\frac{4860}{1 - 20\%}$	1M
= \$6075	1A
(b) Cost = $\frac{4860}{1 - 10\%}$	1M
= \$5400	1A
8. (a) 0.05 m	1A
(b) Least possible area = (28.65)(15.15)	1M
= 434.0475 m ²	1A
(c) Upper limit of area = (28.75)(15.25)	
= 438.4375 m ²	1A
Since 434.0475 ≤ 435 < 438.4375, the actual area can be 435 m ² .	
The claim is agreed.	1A
9. (a) Let ∠AOE = x.	
∠AEO = ∠AOE = x	
∠DBA = $\frac{x}{2}$	1M
∠BDC = $x + \frac{x}{2}$	
x = 32°	1A
(b) ∠BOA = 180° - 2 × 48° - 32° = 52°	1A
$\widehat{AB} = 2\pi(OA) \times \frac{52^\circ}{360^\circ}$	1M
= 2π(AE) × $\frac{13}{90}$	
≈ 0.908AE < AE	
The claim is agreed.	1A
10. (a) $\frac{k}{5 + 12 + 10 + k + 15} = 0.16$	1M
k = 0.16k + 6.72	1M
k = 8	1A
(b) Let x be the number of new students.	
$\frac{5 + x}{50 + x} = \frac{72^\circ}{360^\circ}$	1M
25 + 5x = 50 + x	
x = $\frac{25}{4}$ (rejected)	1A
The angle will not be 72°.	1A

11. (a) Let $C = a + \frac{b}{n^2}$, where a and b are non-zero constants.

$$\begin{cases} 82\,200 = a + b \\ 22\,200 = a + \frac{b}{4} \end{cases}$$

Solving, we have $a = 2200$ and $b = 80\,000$.

$$\text{Required cost} = 2200 + \frac{80\,000}{100^2} = \$2208$$

- (b) $\frac{80\,000}{n^2} > 0$ for all positive integers n .

$$C = 2200 + \frac{80\,000}{n^2} > 2200 \text{ for all positive integers } n.$$

The claim is disagreed.

12. (a) We have

$$\begin{cases} -7 = [3(3) + p](3 - 2)^2 + q \\ 0 = (12 + p)(2)^2 + q \end{cases}$$

Solving, we have $p = \frac{-32}{3}$ and $q = \frac{-16}{3}$.

$$(b) 0 = \left(3x - \frac{32}{3}\right)(x - 2)^2 - \frac{16}{3}$$

$$0 = (9x - 32)(x^2 - 4x + 4) - 16$$

$$0 = 9x^3 - 68x^2 + 164x - 144$$

$$0 = (x - 4)(9x^2 - 32x + 36)$$

$$x = 4 \quad \text{or} \quad 9x^2 - 32x + 36 = 0$$

$$\text{For } 9x^2 - 32x + 36 = 0, \Delta = 32^2 - 4(9)(36) = -272 < 0.$$

The equation has no real roots.

Thus, there is 1 real root.

13. (a) Let $h = 24k$, where k is a positive constant. Then $r = 5k$.

$$6.5^2 = \left(\frac{h}{2}\right)^2 + r^2$$

$$42.25 = 169k^2$$

$$k = \frac{1}{2}$$

$$\text{Required volume} = \pi r^2 h$$

$$= 75\pi \text{ cm}^3$$

$$(b) \text{ Required volume} = \frac{4}{3}\pi(6.5)^3 - 75\pi$$

$$= \frac{1747\pi}{6} \text{ cm}^3$$

14. (a) (i) Slope of $L = \frac{5-4}{5-2} = \frac{1}{3}$
Equation of L is

$$y - 5 = \frac{1}{3}(x - 5)$$

$$x - 3y + 10 = 0$$

(ii) $0 - 3y + 10 = 0$

$$y = \frac{10}{3}$$

The coordinates of C are $\left(0, \frac{10}{3}\right)$.

(b) (i) $\sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-5)^2}$
 $x^2 + y^2 - 4x - 8y + 20 = x^2 + y^2 - 10x - 10y + 50$

$$3x + y - 15 = 0$$

Required equation is $3x + y - 15 = 0$.

(ii) $3x - 0 - 15 = 0$

$$x = 5$$

The coordinates of D are $(5, 0)$.

(c) Solve $\begin{cases} x - 3y - 10 = 0 \\ 3x + y - 15 = 0 \end{cases}$, we have $(x, y) = \left(\frac{7}{2}, \frac{9}{2}\right)$.

$$\begin{aligned} \text{Area of } OCED &= \frac{1}{2} \left(\frac{10}{3}\right) \left(\frac{7}{2}\right) + \frac{1}{2}(5) \left(\frac{9}{2}\right) \\ &= \frac{205}{12} < 20 \end{aligned}$$

The claim is agreed.

15. (a) Standard deviation = $2000(1 + 10\%)$
 $= \$2200$

(b) Let $\$x$ and \bar{x} be the annual income of an employee and the mean of the annual income of all employees respectively.

$$\text{Standard score of last year} = \frac{x - \bar{x}}{2000}$$

$$\text{New standard score} = \frac{(1.1x + 8800) - (1.1\bar{x} + 8800)}{2200}$$

$$= \frac{x - \bar{x}}{2000}$$

There is no change.

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<p>16. (a) $\frac{y}{2} = \frac{18}{y}$ $y^2 = 36$ $y = 6$ or -6 (rejected) Common ratio = $\frac{6}{2} = 3$ $A(n) = 2(3)^{n-1}$</p>	1A
<p>(b) Required product $= 2(3) \times 2(3^3) \times 2(3^5) \times \dots \times 2(3^{2n-1})$ $= 2^n (3^{1+3+5+\dots+(2n-1)})$ $= 2^n \cdot 3^{\frac{1+(2n-1)n}{2}}$ $= 2^n \cdot 3^{n^2}$</p>	1A 1M 1M 1A
<p>17. (a) $\frac{\sin \angle AVB}{22} = \frac{\sin 115^\circ}{36}$ $\angle AVB \approx 33.6^\circ$ $\angle VBA = 180^\circ - 115^\circ - \angle AVB \approx 31.4^\circ$</p>	1M 1A
<p>(b) In $\triangle VAB$, $VA^2 = 22^2 + 36^2 - 2(22)(36) \cos \angle VBA$ $VA \approx 20.7 \text{ cm}$ $MP = NQ = \frac{1}{2}VA \approx 10.3 \text{ cm}$ $PQ = MN = \frac{BC}{2} = 7 \text{ cm}$ Since the solid is symmetric, $PQNM$ is a rectangle. Required area = $(PQ)(MP)$ $\approx 72.4 \text{ cm}^2$</p>	1M 1M 1M 1A

18. (a) (i) Equation of L_3 is

$$y - 80 = \frac{1}{3}(x - 240)$$

$$x - 3y = 0$$

Equation of L_4 is

$$y - 10 = \frac{80 - 10}{240 - 380}(x - 380)$$

$$x + 2y - 400 = 0$$

$$(ii) \begin{cases} x - 3y \geq 0 \\ x + 2y - 400 \leq 0 \\ x \geq 60 \\ y \geq 10 \end{cases}$$

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(b) Let x and y be the numbers of standard rooms and deluxe rooms respectively.

$$\begin{cases} x \geq 60 \\ y \geq 10 \\ x \geq 3y \\ 10x + 20y \leq 400 \\ x \text{ and } y \text{ are non-negative integers} \end{cases}$$

$$\text{Total income} = \$(4000x + 15\,000y)$$

Coordinates of the vertices of the region are (240, 80), (380, 10), (60, 10) and (60, 20).

(x , y)	(240, 80)	(380, 10)	(60, 10)	(60, 20)
Total income	\$2 160 000	\$1 670 000	\$390 000	\$540 000

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$$\text{Maximum total income} = \$2\,160\,000 < \$2\,200\,000$$

The claim is disagreed.

1A

$$\begin{aligned}
 19. \quad (a) \quad f(x) &= 3x^2 - 6kx + 12x + 3k^2 - 6k + 12 \\
 &= 3[x^2 - 2(k-2)x + (k-2)^2] + 6k \\
 &= 3[x - (k-2)]^2 + 6k
 \end{aligned}$$

1M

The coordinates of P are $(k-2, 6k)$.

1A

(b) The coordinates of Q are $(2-k, c-6k)$.

$$\frac{(c-6k) + 6k}{2} = 8$$

$$c = 16$$

1A

$$(c) \quad (i) \quad 5x^2 + 5(mx+8)^2 - 66x + 32(mx+8) - 576 = 0$$

1M

$$(5m^2 + 5)x^2 + (112m - 66)x = 0$$

$$x = 0 \quad \text{or} \quad \frac{66 - 112m}{5m^2 + 5}$$

1A

For $m > 1$, $\frac{66 - 112m}{5m^2 + 5} < 0$.

The x -coordinate of A is 0.

The x -coordinate of B is $\frac{66 - 112m}{5m^2 + 5}$.

1A

$$\begin{aligned}
 (ii) \quad AB &= \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \\
 &= \sqrt{(x_A - x_B)^2 + (mx_A - mx_B)^2} \\
 &= \sqrt{m^2 + 1}(x_A - x_B)
 \end{aligned}$$

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(iii) Let r be the radius of the inscribed circle.

$$C: x^2 + y^2 - \frac{66}{5}x + \frac{32}{5}y - \frac{576}{5} = 0$$

The coordinates of G are $\left(\frac{33}{5}, -\frac{16}{5}\right)$.

$$GA = GB = \text{radius} = \sqrt{\left(\frac{33}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \frac{576}{5}} = 13$$

Let M be the mid-point of AB .

$$AM = 5 \quad \text{and} \quad GM = \sqrt{13^2 - 5^2} = 12$$

Consider the area of $\triangle ABG$.

$$\begin{aligned}
 \frac{10(12)}{2} &= \frac{13r}{2} + \frac{13r}{2} + \frac{10r}{2} \\
 r &= \frac{10}{3}
 \end{aligned}$$

1M+1M

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$$(iv) \quad 10 = \sqrt{m^2 + 1}(x_A - x_B)$$

$$100 = (m^2 + 1) \times \left(\frac{112m - 66}{5m^2 + 5}\right)^2$$

$$625(m^2 + 1) = (56m - 33)^2$$

$$-2511m^2 + 3696m - 464 = 0$$

$$m = \frac{4}{3} \quad \text{or} \quad \frac{116}{837} \quad (\text{rejected})$$

1A

The coordinates of A and B are $(0, 8)$ and $(-6, 0)$ respectively.

The coordinates of M are $(-3, 4)$.

Let $I(a, b)$ be the in-centre of $\triangle ABG$.

Solution

Marks

We have M, I, G are collinear and $MI : IG = r : (12 - r) = 5 : 13$.

$$\frac{a+3}{\frac{33}{5}-a} = \frac{5}{13} \quad \text{and} \quad \frac{4-b}{b+\frac{16}{5}} = \frac{5}{13}$$

$$a = -\frac{1}{3} \qquad b = 2$$

If Q is the centre of the inscribed circle, then

$$2 - k = -\frac{1}{3} \quad \text{and} \quad 16 - 6k = 2$$

$$k = \frac{7}{3} \qquad k = \frac{7}{3}$$

$k = \frac{7}{3}$ satisfies both equations.

Thus, it is possible.

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