1. (a) $\frac{1}{2}\times\frac{4}{3}\pi r^3=\frac{3}{2}\times\frac{1}{3}\pi r^2(r+6)$ 1M+1M

$4r^3=3r^3+18r^2$

$r^3-18r^2=0$

$r=18$ or 0 (rejected) 1A

(b) Original total surface area

$=\pi(18)\sqrt{18^2+24^2}+\frac{1}{2}\times 4\pi(18)^2$ 1M+1M

$=1188\pi\text{ cm}^2$

Total surface area of the 8 smaller solids

$=8\times 1188\pi\times\left(\sqrt[3]{\frac{1}{8}}\right)^2$ 1M

$=2376\pi\text{ cm}^2$

$\approx 7460\text{ cm}^2$

$<7500\text{ cm}^2$

The claim is disagreed. 1A

2. (a) $\cos\frac{\angle AOB}{2}=\frac{10-2}{10}$ 1M

$\angle AOB\approx 73.7°$

Required area

$=\pi(10)^2\times\frac{\angle AOB}{360°}$ 1M

$\approx 64.4\text{ cm}^2$ 1A

(b) $AB=2\sqrt{10^2-8^2}=12\text{ cm}$

Volume of water

$=\left(\pi(10)^2\times\frac{\angle AOB}{360°}-\frac{(12)(8)}{2}\right)(30)$ 1M

$\approx 491\text{ cm}^3$

$CD=12\times\frac{5}{4}=15\text{ cm}$ 1M

$\sin\frac{\angle COD}{2}=\frac{\left(\frac{15}{2}\right)}{10}$

$\angle COD\approx 97.2°$

Volume of the oil

$=\left[\pi(10)^2\times\frac{\angle COD}{360°}-\frac{(15)(\sqrt{10^2-7.5^2})}{2}\right](30)-(\text{volume of water})$ 1M

$\approx 565\text{ cm}^3$

$>491\text{ cm}^3$

The volume of the oil is greater than the volume of the water. 1A

3. (a) $\frac{1}{3}\pi r^2 h_1 = \pi r^2 h_2$ 1M

$h_1 = 3h_2$

The claim is agreed. 1A

(b) Let R cm be the radius of the sphere.

$4\pi R^2 = 576\pi$ 1M

$R^2 = 144$

$R = 12$ or -12 (rejected)

The volume of the cone is half the volume of the sphere.

$\frac{1}{3}\pi(8^2)h_1 = \frac{1}{2} \times \frac{4}{3}\pi(12)^3$ 1M

$h_1 = 54$

The height of the cone is 54 cm. 1A

4. (a) Volume of milk $= \frac{1}{3}\pi(78)^2\sqrt{130^2 - 78^2}$ 1M

$= 210\,912\pi \text{ cm}^3$ 1A

(b) Let the height of the frustum inside the hemispherical vessel be h cm.

$h^2 + \left(\frac{AC}{2}\right)^2 = 75^2$ 1M

$h^2 + 60^2 = 75^2$

$h = 45$

Volume of the frustum

$= \frac{1}{3}(96)(72)(150)\left[1 - \left(\frac{150 - 45}{150}\right)^3\right]$ 1M

$= 227\,059.2 \text{ cm}^3$

Capacity of the vessel

$= \frac{2}{3}\pi(75)^3 - 227\,059.2$ 1M

$\approx 656\,514 \text{ cm}^3$ 1A

$< 210\,912\pi \text{ cm}^3$

The milk will overflow.

The claim is agreed. 1A

5. (a) Let the base radius be r cm.

$80\pi = 10(2r\pi)$ 1M

$r = 4$ 1A

The base radius is 4 cm.

(b) Capacity $= (4)^2\pi(10)$ 1M

$= 160\pi \text{ cm}^3$ 1A

(c) Base radius of bucket $B = \dfrac{8\pi}{2\pi} = 4\text{ cm}$.

$\dfrac{\text{base radius of } A}{\text{base radius of } B} = 1 \neq \dfrac{\text{height of } A}{\text{height of } B} = \dfrac{10}{12}$ 1M

Thus, they are not similar. 1A

6. (a) Let r cm be the base radius of P.

$\pi r(39) = 585\pi$ 1M

$r = 15$

Required height

$= \sqrt{39^2 - 15^2}$ 1M

$= 36\text{ cm}$ 1A

(b) (i) $\dfrac{\text{volume of } P}{\text{volume of } Q} = (\sqrt{9})^3$ 1M

$= 27$

Required volume

$= \dfrac{1}{3}\pi(15)^2(36)\left(1 - \dfrac{1}{27}\right)$ 1M

$= 2600\pi \text{ cm}^3$ 1A

(ii) Let x cm be the length of a side of the cubes.

$4x^3 = 2600\pi$ 1M

$x \approx 12.69$

< 15

The claim is disagreed. 1A

7. (a) Let r cm be the radius of the upper base and x cm be the height of the upper part.

$\dfrac{x}{x+9} = \dfrac{r}{2r}$

$x = 9$ 1A

Consider the volume of X.

$\dfrac{1}{3}\pi(2r)^2(18) - \dfrac{1}{3}\pi r^2(9) = 3024\pi$ 1M

$r^2 = 144$

$r = 12$ or -12 (rejected)

The radius of the lower base is 24 cm. 1A

(b) Curved surface area of the cone

$= \pi(24)\sqrt{24^2 + 18^2}$

$= 720\pi \text{ cm}^2$

Total curved surface area of Y

$= 2\pi(24)^2 + 720\pi \times \left[1 - \left(\frac{1}{2}\right)^2\right]$ 1M

$= 1692\pi \text{ cm}^2$ 1A

Total curved surface area of Z

$= \left(\sqrt[3]{\frac{8}{27}}\right)^2 \times 1692\pi$ 1M

$= 752\pi \text{ cm}^2$

$\approx 2360 \text{ cm}^2$

$< 2400 \text{ cm}^2$

The claim is agreed. 1A

8. (a) Let r cm be the base radius of the cylinder.

$\pi r^2 = 9\pi$ 1M

$r = 3$ or -3 (rejected)

Height of the solid $= 3 + 4$

$= 7$ m 1A

(b) Total surface area of the solid

$= \frac{1}{2} \times 4\pi(3)^2 + 2\pi(3)(4) + 9\pi$ 1M

$= 51\pi \text{ m}^2$

Required cost

$= 15 \times 51\pi$ 1M

$\approx \$2403$

$< \$2500$

The required cost of painting will not exceed the budget of the worker. 1A

9. (a) $BC = \sqrt{24^2 - 15^2}$

$= \sqrt{351}$ cm

Height of the pyramid

$= \sqrt{37^2 - \left(\frac{24}{2}\right)^2}$

$= 35$ cm

Required volume

$= \frac{1}{3} \times (15 \times \sqrt{351}) \times 35$ 1M

$= 525\sqrt{39}\ \text{cm}^3$ 1A

$\approx 3280\ \text{cm}^3$

(b) (i) Let r cm and h cm be the radius and the height of the cylinder respectively.

$$4\pi r^2 = 2\pi r h + 2\pi r^2$$ 1M

$$2r = h + r$$

$$r = h$$

Consider the volume of the solids.

$$\frac{4}{3}\pi r^3 + \pi r^2 h = 525\sqrt{39}$$

$$\frac{4}{3}\pi r^3 + \pi r^3 = 525\sqrt{39}$$

$$\frac{7}{3}\pi r^3 = 525\sqrt{39}$$

$$\frac{4}{3}\pi r^3 = 300\sqrt{39}$$

$$\approx 1870$$

The volume of the sphere is $300\sqrt{39}\ \text{cm}^3$. 1A

(ii) Total surface area of the right pyramid

$$= 15(\sqrt{351}) + 2 \times \frac{1}{2}\left[15 \times \sqrt{35^2 + \left(\frac{\sqrt{351}}{2}\right)^2} + \sqrt{351} \times \sqrt{35^2 + \left(\frac{15}{2}\right)^2}\right]$$ 1M

$\approx 1495.11\ \text{cm}^2$

Total surface area of the sphere and the cylinder

$= 2 \times 4\pi r^2$

$\approx 1469.88\ \text{cm}^2$

Change in total surface area

$\approx 1495.11 - 1469.88$ 1M

$\approx 25.2\ \text{cm}^2$ 1A

$> 25\ \text{cm}^2$

The claim is disagreed. 1A

10. (a) $BG = \sqrt{13^2 - (29 - 24)^2}$ 1M

$= 12\ \text{cm}$

Capacity $= \frac{(29 + 24)(12)}{2}(24)$ 1M

$= 7632\ \text{cm}^3$ 1A

(b) (i) Base radius of the cone is 12 cm.

Slant height $= \sqrt{12^2 + 16^2}$ 1M

$= 20\,\text{cm}$

Curved surface area of the cone $= \pi(12)(20)$ 1M

$= 240\pi\,\text{cm}^2$

Required area $= 240\pi\left[1 - \left(\dfrac{16-12}{16}\right)^2\right]$ 1M

$= 225\pi\,\text{cm}^2$ 1A

(ii) Required volume $= 7632 - \left[\dfrac{1}{3}\pi(12)^2(16) - \dfrac{1}{3}\pi(3)^2(16-12)\right]$ 1M

$\approx 5257\,\text{cm}^3$ 1A

11. (a) Radius of the sphere is 9 cm.

Inner radius of the container is 9 cm. 1A

Required volume

$= \pi(9)^2(18) - \dfrac{4}{3}\pi(9)^3$ 1M

$= 486\pi\,\text{cm}^3$ 1A

(b) Let h cm be the height of water in the container.

$\pi(9)^2(h) = 486\pi$ 1M

$h = 6$ 1A

12. (a) Let x cm be the radius of the larger hemispherical vessel.

$\left(\dfrac{10}{x}\right)^2 = \dfrac{16}{25}$ 1M

$x = 12.5$ or -12.5 (rejected)

Total volume of water

$= \dfrac{1}{2} \times \dfrac{4}{3}\pi(12.5^3 + 10^3)$ 1M

$= \dfrac{7875\pi}{4}\,\text{cm}^3$ 1A

(b) Let H cm and h cm be the heights of the upper part and the lower part respectively.

$\dfrac{H}{H+h} = \dfrac{10}{15}$ 1M

$H = 2h$

Consider the volume of the frustum.

$\dfrac{1}{3}\pi(15)^2(H+h) - \dfrac{1}{3}\pi(10)^2 H = \dfrac{7875\pi}{4}$ 1M

$225(3h) - 100(2h) = \dfrac{23\,625}{4}$

$h \approx 12.4$ 1A

The height of the frustum is 12.4 cm.

13. (a) Let h cm be the height of the cylinder.

$$2\pi(40)h = 6000\pi \qquad \text{1M}$$

$$h = 75$$

Required volume $= \pi(40)^2(75)$ 1M

$= 120\,000\pi \text{ cm}^3$ 1A

(b) Let H cm be the height of B.

$$\frac{1}{3}\pi(40)^2(60) + \frac{1}{3}\pi(60)^2 H = 120\,000\pi \qquad \text{1M+1M}$$

$$H = \frac{220}{3}$$

$$\frac{\text{height of } B}{\text{height of } A} = \frac{\left(\frac{220}{3}\right)}{60} = \frac{11}{9} \qquad \text{1M}$$

$$\frac{\text{base radius of } B}{\text{base radius of } A} = \frac{60}{40} = \frac{3}{2} \neq \frac{11}{9}$$

Thus, A and B are not similar.

The claim is not correct. 1A

14. (a) Let θ be the required angle.

$$10^2\pi \times \frac{\theta}{360^\circ} = 2\pi \qquad \text{1M}$$

$$\theta = 7.2^\circ \qquad \text{1A}$$

(b) Perimeter $= 2(10)\pi \times \dfrac{7.2^\circ}{360^\circ} + 2(10)$ 1M

$= (0.4\pi + 20)$ cm 1A

15. (a) Let h cm be the required height.

Note that the length of the square base of the smaller square pyramid is also h cm. 1M

$$27 \times \frac{1}{3} \times h^2 \times h = 4 \times \left[12^3 - \frac{1}{3} \times 12^2 \times 12\right] \qquad \text{1M}$$

$$h^3 = 512$$

$$h = 8 \qquad \text{1A}$$

(b) Required ratio $= 8^2 : 12^2$ 1M

$= 4 : 9$ 1A

(c) Let k cm be the length of the metal cube and H cm be the height of the smaller square pyramid.

The height of the largest square pyramid is k cm.

$$27 \times \frac{1}{3} \times H^2 \times H = 4 \times \left[k^3 - \frac{1}{3} \times k^2 \times k\right] \qquad \text{1M}$$

$$\left(\frac{H}{k}\right)^3 = \frac{8}{27}$$

$$\frac{H}{k} = \frac{2}{3}$$

The ratio in (b) becomes $2^2 : 3^2 = 4 : 9$, which is the same as before. 1M

The claim is disagreed. 1A

16. (a) $2 \times \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$ 1M

$h = 4r$ 1A

(b) Volume of the liquid

$= \frac{1}{2} \times \frac{4}{3}\pi(2)^3$ 1M

$= \frac{16\pi}{3}$ cm^3

Volume of the cone

$= \frac{1}{3}\pi(2)^2(4 \times 2)$ 1M

$= \frac{32\pi}{3}$ cm^3

Let h cm be the depth of liquid in the cone.

$\frac{16\pi}{3} \div \frac{32\pi}{3} = \left(\frac{h}{8}\right)^3$ 1M

$h^3 = 256$

$h = \sqrt[3]{256}$

≈ 6.35

The depth of the liquid in the cone is 6.35 cm. 1A

17. (a) Let h cm be the height of the vessel.

$\pi(18)\sqrt{18^2 + h^2} = 540\pi$ 1M

$18^2 + h^2 = 30^2$

$h = 24$ or -24 (rejected) 1A

(b) Capacity of the vessel $= \frac{1}{3}\pi(18)^2(24)$ 1M

$= 2592\pi$ cm^2

Let V cm^3 be the required volume.

$\sqrt[3]{\frac{V}{2592\pi}} = \sqrt{\frac{375\pi}{540\pi}}$ 1M+1M

$V = 1500\pi$ 1A

18. (a) Let h cm be the height of the circular cone.

$\frac{1}{3}\pi(24)^2 h = 30 \times \frac{4}{3}\pi\left(\frac{12}{2}\right)^3$ 1M

$h = 45$ 1A

The height of the circular cone is 45 cm.

(b) (i) Curved surface area of the original cone

$= \pi(24)(\sqrt{24^2 + 45^2})$ 1M+1M

$= 1224\pi \text{ cm}^2$

Volume ratio of X to the original cone is $1 : (1 + 7) = 1 : 8$.

Height ratio of X to the origianl cone is $1 : 2$.

Required area $= 1224\pi \left[1 - \left(\frac{1}{2}\right)^2\right]$ 1M

$= 918\pi \text{ cm}^2$ 1A

(ii) Total surface area of X

$= 1224\pi - 918\pi + \pi\left(\frac{24}{2}\right)^2$ 1M

$= 450\pi \text{ cm}^2$

Total surface area of Y

$= 918\pi + \pi\left(\frac{24}{2}\right)^2 + \pi(24)^2$

$= 1638\pi \text{ cm}^2$

Required ratio $= 450\pi : 1638\pi$

$= 25 : 91$ 1A

19. (a) Let $V \text{ cm}^3$ be the volume of Y.

$\sqrt[3]{\frac{V}{6144\pi}} = \sqrt{\frac{25}{16}}$ 1M+1M

$V = 12\,000\pi$ 1A

(b) Volume of $A = \frac{1}{3}\pi(12)^2(42)$

$= 2016\pi \text{ cm}^3$

Volume of $B = 6144\pi + 12\,000\pi - 2016\pi$ 1M

$= 16\,128\pi \text{ cm}^3$

Let r cm be the base radius of B.

$\frac{1}{3}\pi r^2(84) = 16\,128\pi$ 1M

$r = 24 \quad \text{or} \quad -24 \text{ (rejected)}$

$\frac{\text{Base radius of } A}{\text{Base radius of } B} = \frac{12}{24} = \frac{1}{2}$

$\frac{\text{Height of } A}{\text{Height of } B} = \frac{42}{84} = \frac{1}{2}$ 1M

Thus, A and B are similar. 1A

20. (a) Let r cm be the radius of the metal sphere.

$$4\pi r^2 = 36\pi$$ 1M

$$r = 3 \quad \text{or} \quad -3 \text{ (rejected)}$$

Volume of the metal sphere

$= \frac{4}{3}\pi(3)^3$

$= 36\pi \text{ cm}^3$ 1A

(b) Let h cm be the original depth of water in the container.

$$36\pi + \pi(8)^2 h = \pi(8)^2(7)$$ 1M+1M

$$h = \frac{103}{16}$$ 1A

(c) Let R cm and ℓ cm be the base radius and the slant height of the circular conical vessel respectively.

$$2\pi R = 24\pi$$

$$R = 12$$

Consider the curved surface area.

$$\pi R\ell = 180\pi$$

$$\ell = 15$$

Height of the vessel $= \sqrt{15^2 - 12^2}$ 1M

$= 9 \text{ cm}$

Capacity of the vessel $= \frac{1}{3}\pi(12)^2(9)$ 1M

$= 432\pi \text{ cm}^3$

Volume of water in the container

$= \pi(8)^2\left(\frac{103}{16}\right)$

$= 412\pi \text{ cm}^3$

$< 432\pi \text{ cm}^3$

The water will not overflow. 1A

21. (a) Let r cm be the base radius of X.

$$\pi r(85) = 3060\pi$$ 1M

$$r = 36$$

Height of $X = \sqrt{85^2 - 36^2}$

$= 77 \text{ cm}$

Volume of $X = \frac{1}{3}\pi(36)^2(77)$ 1M

$= 33\,264\pi \text{ cm}^3$ 1A

(b) (i) Volume ratio of Y to $Z = \left(\sqrt{\frac{16}{25}}\right)^3$ 1M

$= \frac{64}{125}$

Volume of $Y = 33\,264\pi \times \frac{64}{64 + 125}$ 1M

$= 11\,264\pi \text{ cm}^3$ 1A

(ii) $\frac{\text{Volume of } X}{\text{Volume of } Y} = \frac{33\,264\pi}{11\,264\pi}$

$= \frac{189}{64}$

$\frac{\text{Total surface area of } X}{\text{Total surface area of } Y} = \frac{3060\pi + 36^2\pi}{3025\pi}$ 1M

$= \frac{36}{25}$

Note that $\sqrt{\frac{36}{25}} = \frac{6}{5}$, while $\sqrt[3]{\frac{189}{64}} \approx 1.43 \neq \frac{6}{5}$. 1M

X and Y are not similar. 1A

22. (a) Volume of the cone $= \frac{1}{3}\pi(64)^2(48)$ 1M

$= 65\,536\pi \text{ cm}^3$

Required volume $= 65\,536 \times \left[1 - \left(\frac{18}{48}\right)^3\right]$ 1M

$= 62\,080\pi \text{ cm}^3$ 1A

(b) Curved surface area of X

$= \pi(64)\sqrt{48^2 + 64^2}\left[1 - \left(\frac{18}{48}\right)^2\right]$ 1M

$= 4400\pi \text{ cm}^2$

Total surface area of X

$= 4400\pi + \pi(64)^2\left[1 + \left(\frac{18}{48}\right)^2\right]$ 1M

$= 9072\pi \text{ cm}^2$

$\left(\frac{\text{Height of } X}{\text{Height of } Z}\right)^2 = \left(\frac{48 - 18}{10}\right)^2 = 9$

$\frac{\text{Total surface area of } X}{\text{Total surface area of } Z} = \frac{9072\pi}{1296\pi} = 7 \neq 9$ 1M

X and Z are not similar. 1A

23. (a) $\pi OX^2 \times \frac{216°}{360°} = 540\pi$ 1M

$OX = 30 \text{ cm}$ or -30 cm (rejected)

The length of OX is 30 cm. 1A

(b) Let R cm be the base radius of the container.

$$2\pi R = 2\pi(30) \times \frac{216°}{360°}$$ 1M

$$R = 18$$

Required capacity

$= \frac{1}{3}\pi(18)^2\sqrt{30^2 - 18^2}$ 1M

$= 2592\pi \text{ cm}^3$ 1A

(c) Let A cm^2 be the area of the wet curved surface of the container.

$$\sqrt{\frac{A}{540\pi}} = \sqrt[3]{\frac{768\pi}{2592\pi}}$$ 1M

$$A = 240\pi$$

Area of inner dry curved surface of the container

$= 540\pi - 240\pi$ 1M

$\approx 942 \text{ cm}^2$

$< 950 \text{ cm}^2$

The claim is disagreed. 1A

24. (a)
$$\frac{OM}{ON} = \frac{AM}{ND}$$

$$\frac{13.5 - ON}{ON} = \frac{5}{4}$$ 1M

$$ON = 6 \text{ cm}$$

Required volume $= \frac{1}{3}\pi(4)^2(6)$ 1M

$= 32\pi \text{ cm}^3$ 1A

(b) Since the volume of milk is greater than the volume of the lower cone,
there are $36\pi - 32\pi = 4\pi$ cm^3 of milk in the upper cone.
Let the height of milk in the upper cone be h cm.

$$\left(\frac{h}{6}\right)^3 = \frac{4\pi}{32\pi}$$ 1M

$$h = 3$$

Curved surface area of the lower cone

$= \pi(4)(\sqrt{4^2 + 6^2})$ 1M

$= 8\sqrt{13}\pi$

Total wet curved surface area

$= 8\sqrt{13}\pi \times \left(\frac{3}{6}\right)^2 + 8\sqrt{13}\pi$ 1M

$= 10\sqrt{13}\pi \text{ cm}^2$

$> 36\pi \text{ cm}^2$

The claim is agreed. 1A

25. (a) Volume of the cylinder $= \pi(4)^2\left(\frac{70}{9}\right)$

$= \frac{1120\pi}{9}\text{ cm}^3$

Length ratio of the cones $= \sqrt{4} : \sqrt{9} = 2 : 3$

Volume ratio of the cones $= 2^3 : 3^3 = 8 : 27$ 1M

Required volume $= \frac{1120\pi}{9} \times \frac{27}{8+27}$ 1M

$= 96\pi\text{ cm}^3$ 1A

(b) Height of the larger cone $= \frac{96\pi}{\frac{1}{3}\pi(6)^2}$

$= 8\text{ cm}$ 1A

$\frac{\text{curved surface area of the larger cone}}{\text{curved surface area of } A}$

$= \frac{\pi(6)(\sqrt{6^2+8^2})}{240\pi}$ 1M

$= \frac{1}{4}$

$\left(\frac{\text{base radius of the larger cone}}{\text{base radius of } A}\right)^2$

$= \left(\frac{6}{15}\right)^2$

$= \frac{4}{25}$

$\neq \frac{1}{4}$ 1M

They are not similar.

The claim is disagreed. 1A

26. (a) (i) Volume $= \frac{1}{3}(60)^2(72)$ 1M

$= 86\,400\text{ cm}^3$ 1A

(ii) Height of the lateral face $= \sqrt{\left(\frac{60}{2}\right)^2 + 72^2}$ 1M

$= 78\text{ cm}$

Required area $= \frac{1}{2}(60)(78) \times 4$

$= 9360\text{ cm}^2$ 1A

(b) (i) Required volume $= 86\,400 - 86\,400\left(\sqrt{\frac{1}{9}}\right)^3$ 1M

$= 83\,200\text{ cm}^3$ 1A

(ii) Let h cm be the height of the cone.

$$\frac{1}{3}\pi(10)^2 h = 83\,200 \qquad \text{1M}$$

$$h = \frac{2496}{\pi}$$

$$\text{Required area} = \pi(10)(\sqrt{10^2 + h^2})$$

$$\approx 25\,000\text{ cm}^2 \qquad \text{1A}$$

27. (a) $(10 + 6\pi) = 2(5) + 2\pi(5) \times \dfrac{m^\circ}{360^\circ}$ 1M

$m = 216$ 1A

(b) Let r cm be the base radius of the cone.

$2\pi r = (10 + 6\pi) - 2(5)$ 1M

$r = 3$

Required height $= \sqrt{5^2 - 3^2}$ 1M

$= 4$ cm 1A

28. (a) $\dfrac{\text{Volume of } Y}{\text{Volume of } X} = \left(\sqrt{\dfrac{9}{4}}\right)^3$ 1M

$= \dfrac{27}{8}$

Required volume $= \pi(10)^2(4.2) \times \dfrac{27}{27 + 8}$

$= 324\pi\text{ cm}^3$ 1A

(b) Let r cm be the base radius of Y.

$\dfrac{1}{3}\pi r^2 \left(\dfrac{4r}{3}\right) = 324\pi$ 1M

$r^3 = 729$

$r = 9$

The base radius and the height of Y are 9 cm and 12 cm respectively.

Required area $= \pi(9)(\sqrt{9^2 + 12^2})$ 1M

$= 135\pi\text{ cm}^2$ 1A

(c) Curved surface area of $X = 135\pi \times \dfrac{4}{9}$ 1M

$= 60\pi\text{ cm}^2$

Curved surface area of Z

$= 135\pi - 135\pi\left(1 - \dfrac{2}{3}\right)^2$ 1M

$= 120\pi\text{ cm}^2$

$= 2 \times (\text{curved surface area of } X)$

The claim is correct. 1A

29. (a) Note that $h : r = 3 : 1$ and $h + r = 12$.

We have $h = 9$ and $r = 3$. 1A

Volume of the upper part of the solid

$= \frac{1}{2}\left[\frac{4}{3}\pi(3)^3\right]$ 1M

$= 18\pi \text{ cm}^3$

Volume of the lower part of the solid

$= \pi(3)^2(9)$ 1M

$= 81\pi \text{ cm}^3$

Required difference $= 81\pi - 18\pi$

$= 63\pi \text{ cm}^3$ 1A

(b) Total surface area of the solid

$= \pi(3)^2 + 2\pi(3)(9) + \frac{1}{2}(4\pi)(3)^2$ 1M

$= 81\pi \text{ cm}^2$ 1A

30. (a) $BC = \sqrt{40^2 - 24^2} = 32 \text{ cm}$ 1A

Height of the pyramid $= \sqrt{29^2 - \left(\frac{40}{2}\right)^2} = 21 \text{ cm}$

Volume of the pyramid $= \frac{1}{3}(24)(32)(21)$ 1M

$= 5376 \text{ cm}^3$ 1A

(b) (i) Let r cm and h cm be the radius and the height of the cylinder.

$4\pi r^2 = 2\pi r^2 + 2\pi r h$ 1M

$r = h$

$\frac{4}{3}\pi r^3 + \pi r^2(r) = 5376$ 1M

$\pi r^3 = 2304$

Volume of the sphere $= \frac{4}{3}\pi r^3 = 3072 \text{ cm}^3$ 1A

(ii) Original total surface area

$= (24)(32) + \frac{(24)\sqrt{21^2 + 16^2}}{2} \times 2 + \frac{(32)\sqrt{21^2 + 12^2}}{2} \times 2$ 1M

$\approx 2180 \text{ cm}^2$

New total surface area

$= 4\pi r^2 \times 2$

$= 8\pi \left(\dfrac{2304}{\pi}\right)^{\frac{2}{3}}$

$\approx 2040\,\text{cm}^2 < 2180\,\text{cm}^2$ 1A

The claim is disagreed. 1A

31. (a) Radius of the larger hemisphere $= 12 \times \sqrt{\dfrac{25}{16}}$ 1M

$= 15\,\text{cm}$

Required volume

$= \dfrac{1}{2} \times \dfrac{4}{3}\pi(15)^3$ 1M

$= 2250\pi\,\text{cm}^3$ 1A

(b) (i) Volume of the smaller hemisphere

$= \dfrac{1}{2} \times \dfrac{4}{3}\pi(12)^3$

$= 1152\pi\,\text{cm}^3$

Required volume

$= 2250\pi + 1152\pi - \dfrac{1}{3}\pi(9)^2(30)$ 1M

$= 2592\pi\,\text{cm}^3$ 1A

(ii) Let h cm be the height of Y.

$\dfrac{1}{3}\pi(12)^2 h = 2592\pi$ 1M

$h = 54$

Curved surface area of Y

$= \pi(12)\sqrt{12^2 + 54^2}$ 1M

$\approx 2090\,\text{cm}^2$

Two times the curved surface area of X

$= 2 \times \pi(9)\sqrt{9^2 + 30^2}$

$\approx 1770\,\text{cm}^2$

$< 2090\,\text{cm}^2$

The claim is agreed. 1A

32. (a) Required area $= \pi(36)(\sqrt{36^2 + 48^2})$ 1M

$= 2160\pi\,\text{cm}^2$ 1A

(b) (i) Let r cm and h cm be the base radius and the height of the part of circular cone that is below the water surface respectively.

$$\left(\frac{r}{36}\right)^2 = \frac{2160\pi - 2025\pi}{2160\pi}$$ 1M

$$r^2 = 81$$

$$r = 9 \quad \text{or} \quad -9 \text{ (rejected)}$$

We have $h = 48 \times \dfrac{r}{36} = 12$.

Required base radius and height are 9 cm and 12 cm respectively. 1A

(ii) Let H cm be the part of the circular cone that is in the vessel.

$$\frac{H}{48} = \frac{27}{36}$$

$$H = 36$$

Volume of water in the vessel

$= \pi(27)^2(80 - 36 + 12) - \dfrac{1}{3}\pi(9)^2(12)$ 1M

$= 40\,500\pi \text{ cm}^3$

$\approx 127\,000 \text{ cm}^3$

$\approx 0.127 \text{ m}^3$

$> 0.12 \text{ m}^3$

The claim is disagreed. 1A

33. (a) Curved surface area of the cone

$= \pi(24)(51)$ 1M

$= 1224\pi \text{ cm}^2$

Required area

$= 1224\pi \times \left(\dfrac{2^2 - 1^2}{3^2}\right)$ 1M

$= 408\pi \text{ cm}^2$ 1A

(b) Height of the cone

$= \sqrt{51^2 - 24^2}$

$= 45 \text{ cm}$

Volume of the cone

$= \dfrac{1}{3}\pi(24)^2(45)$ 1M

$= 8640\pi \text{ cm}^3$

Required volume

$= 8640\pi \times \dfrac{2^3 - 1^3}{3^3}$ 1M

$= 2240\pi \text{ cm}^3$ 1A

34. (a) $\frac{1}{3}\pi r^2(46-r) = \frac{5}{8}\pi r^3$ 1M

$$\frac{46}{3} - \frac{r}{3} = \frac{5r}{8}$$

$$r = 16$$ 1A

(b) Slant height of the cone

$= \sqrt{16^2 + 30^2}$ 1M

$= 34$ cm

Required area

$= \pi(16)(34) + \frac{1}{2} \times 4\pi(16)^2$ 1M

$= 1056\pi$ cm^2 1A

35. (a) Volume of the cylinder $= \pi(16)^2(57)$ 1M

$= 14\,592\pi$ cm^3

Let V cm^3 be the volume of the smaller cone.

$$V + \left(\sqrt{\frac{25}{9}}\right)^3 V = 14\,592\pi$$ 1M

$$V = 2592\pi$$

Required volume is 2592π cm^3. 1A

(b) Total surface area of the cylinder

$= 2\pi(16)(57) + 2\pi(16)^2$ 1M

$= 2336\pi$ cm^2

Let r cm be the base radius of the smaller cone.

$$\frac{1}{3}\pi r^2(24) = 2592\pi$$

$$r = 18 \quad \text{or} \quad -18 \text{ (rejected)}$$

Total surface area of the smaller cone

$= \pi(18)^2 + \pi(18)(\sqrt{18^2 + 24^2})$ 1M

$= 864\pi$ cm^2

Total surface area of the larger cone

$= 864\pi \times \frac{25}{9}$ 1M

$= 2400\pi$ cm^2

Total surface area of the two cones

$= 864\pi + 2400\pi$

$= 3264\pi$ cm^2

$\neq 2336\pi$ cm^2

The claim is disagreed. 1A

36. (a) Let r cm be the base radius of the cone.

$$\pi r(29) = 580\pi \qquad \text{1M}$$
$$r = 20$$

Height of the cone

$= \sqrt{29^2 - 20^2}$ 1M

$= 21$ cm 1A

(b) (i) Consider the solid formed by combining A and B, and the solid C.

Area ratio $= \dfrac{1+1}{1+1+6} = \dfrac{1}{4}$

Volume ratio $= \left(\sqrt{\dfrac{1}{4}}\right)^3 = \dfrac{1}{8}$

Volume of $C = \dfrac{7}{8} \times$ (volume of the cone) 1M

Volume of $C = \dfrac{1}{3}\pi(20)^2(21) \times \dfrac{7}{8}$ 1M

$= 2450\pi$ cm^3 1A

(ii) Let R cm be the base radius of each cylinder.

$$5\pi R^2(10) = 2450\pi \qquad \text{1M}$$
$$R = 7 \quad \text{or} \quad -7 \text{ (rejected)} \qquad \text{1A}$$

The base radius of each cylinder is 7 cm.

37. (a) Let r_X cm and r_Y cm be the base radii of X and Y respectively.

$$2\pi r_X^2 = 648\pi \quad \text{and} \quad 2\pi r_X r_Y = 432\pi \qquad \text{1M}$$
$$r_X = 18 \qquad\qquad r_Y = 12$$

Required volume

$= \pi(12)^2(18)$ 1M

$= 2592\pi$ cm^3 1A

(b) Let h cm and $2h$ cm be the heights of A and B respectively. 1M

$$\frac{1}{3}\pi(12)^2h + \frac{1}{3}\pi(24)^2(2h) = 2592\pi + \frac{2}{3}\pi(18)^3 \qquad \text{1M}$$
$$h = 15$$

Curved surface area of A

$= \pi(12)(\sqrt{15^2 + 12^2})$ 1M

≈ 724 cm^2

> 720 cm^2

The claim is agreed. 1A

38. (a) Required volume $= \frac{1}{3}\pi(9)^2(9)$ 1M

$= 243\pi \text{ cm}^3$ 1A

(b) (i) Required volume

$= 243\pi - 243\pi \times \left(\frac{9-6}{9}\right)^3$ 1M+1M

$= 234\pi \text{ cm}^3$ 1A

(ii) Capacity of the container $= 18 \times 18 \times 6$

$= 1944 \text{ cm}^3$

Total volume of water and frustum after 4 minutes

$= 5 \times 4 \times 60 + 234\pi$ 1M

$\approx 1940 \text{ cm}^3$

$< 1944 \text{ cm}^3$

The water will not overflow after 4 minutes. 1A

39. (a) Volume of cone

$= \frac{1}{3}(24)^2\pi(36)$ 1M

$= 6912\pi \text{ cm}^3$

Volume of water

$= 6912 \times \left(\frac{1}{3}\right)^3$ 1M

$= 256\pi \text{ cm}^3$ 1A

(b) (i) Let h cm be the new height of the water layer.

$$\frac{6912\pi - 256\pi}{6912\pi} = \left(\frac{36-h}{36}\right)^3$$ 1M

$$(36-h)^3 = 44\,928$$

$$36 - h = \sqrt[3]{44\,928}$$

$$h \approx 0.450$$ 1A

Required height is 0.450 cm.

(ii) $\dfrac{\text{lower base radius of oil layer}}{\text{lower base radius of water layer}} = \dfrac{24}{24} = 1$ 1M

$\dfrac{\text{volume of oil layer}}{\text{volume of water layer}} = \dfrac{6912\pi - 256\pi}{256\pi} = 26 \neq 1^3$

They are not similar.

The claim is disagreed. 1A

40. (a) Curved surface area of the original cone

$= \pi(75)(\sqrt{75^2 + 40^2})$ 1M

$= 6375\pi \text{ cm}^2$

Curved surface area of P

$= 6375\pi \times \left[1 - \left(\dfrac{16}{40}\right)^2\right]$ 1M

$= 4080\pi \text{ cm}^2$

Required area

$= 4080\pi + 75^2\pi \times \left[1 + \left(\dfrac{16}{40}\right)^2\right]$ 1M

$= 11\,730\pi \text{ cm}^2$ 1A

(b) Volume of P

$= \dfrac{1}{3}\pi(75)^2(40) \times \left[1 - \left(\dfrac{16}{40}\right)^3\right]$ 1M

$= 58\,800\pi \text{ cm}^3$

$\left(\dfrac{\text{Height of } P}{\text{Height of } R}\right)^3 = \left(\dfrac{16}{7}\right)^3 = \dfrac{4096}{343}$

$\dfrac{\text{Volume of } P}{\text{Volume of } R} = \dfrac{58\,800\pi}{7350\pi} = 8 \neq \dfrac{4096}{343}$ 1M

P and R are not similar. 1A

41. $\boxed{\text{C}}$

Height of the trapezium

$= \sqrt{15^2 - (27 - 18)^2}$

$= 12 \text{ cm}$

Consider the volumes.

$$\frac{(18 + 27)12}{2} \times h + 21\,600 = 30^3$$

$$h = 20$$

42. $\boxed{\text{B}}$

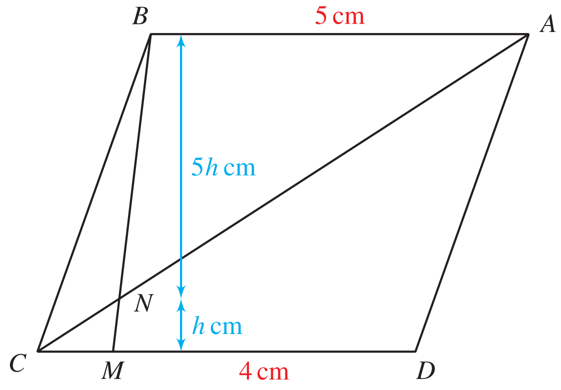
Horizontal separation of A and $F = 3 + 1 = 4 \text{ cm}$

Vertical separation of A and $F = 2 + 3 - 1 = 4 \text{ cm}$

Required distance $= \sqrt{4^2 + 4^2} \approx 5.7 \text{ cm}$

43. $\boxed{\text{C}}$

Let $CM = 1$ cm. Then we have the lengths as shown in the figure.



$\boxed{\text{Point } N}$ Note that $\triangle CMN \sim \triangle ABN$ (ratio 1 : 5).

Consider the area of $\triangle CMN$.

$$10 = \frac{(1)(h)}{2}$$

$$h = 20$$

$$\text{Required area} = \frac{(5)(6h)}{2} - \frac{(1)(h)}{2}$$

$$= 290\,\text{cm}^2$$

44. $\boxed{\text{A}}$

$$\sin \angle AEB = \frac{10}{20}$$

$$\angle AEB = 30^\circ$$

$$\angle DAE = \angle AEB = 30^\circ$$

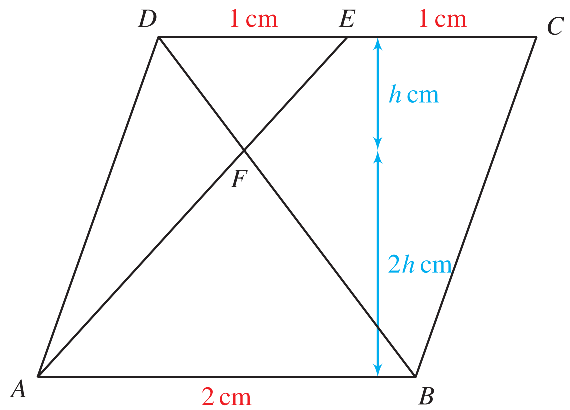
Required area

$$= 20 \times 10 - \frac{10(\sqrt{20^2 - 10^2})}{2} - \pi(20)^2 \times \frac{30^\circ}{360^\circ}$$

$$\approx 8.68\,\text{cm}^2$$

45. $\boxed{\text{C}}$

Let $AB = 2\,\text{cm}$. Then we have the lengths as shown in the figure.



$\boxed{\text{Point } F}$ Note that $\triangle ABF \sim \triangle EDF$ (ratio 2 : 1).
Consider the area of $ABCD$.

$$72 = (2)(3h)$$
$$h = 12$$

$$\text{Required area} = \frac{(2)(3h)}{2} - \frac{(1)(h)}{2} = 30\,\text{cm}^2$$

46. $\boxed{\text{B}}$

Let A km^2 be the actual area of the field.

$$\frac{A \times (1000 \times 100)^2}{15} = \left(\frac{20\,000}{1}\right)^2$$
$$A = 0.6$$

47. $\boxed{\text{B}}$

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$
$$h = \frac{4r}{3}$$

Required ratio

$= 4\pi r^2 : 2\pi r h$

$= 2r : h$

$= 2r : \frac{4r}{3}$

$= 3 : 2$

48. $\boxed{\text{B}}$

$$2\pi(OA) \times \frac{60°}{360°} = 4\pi$$
$$OA = 12\,\text{cm}$$

$OC = OA \cos 60° = 12 \cos 60° = 6\,\text{cm}$

$AC = OA \sin 60° = 12 \sin 60° = 6\sqrt{3}\,\text{cm}$

Required area

$= \pi 12^2 \times \frac{60°}{360°} - \frac{(6)(6\sqrt{3})}{2}$

$\approx 44.2\,\text{cm}^2$

49. $\boxed{\text{C}}$

Base radius of the cone $= \sqrt{20^2 - 16^2} = 12\,\text{cm}$

$$\text{Volume of the cone} = \frac{1}{3}\pi(12)^2(16)$$
$$= 768\pi \text{ cm}^3$$

Let the height of the frustum be h cm.

$$\frac{768\pi - 756\pi}{768\pi} = \left(\frac{16-h}{16}\right)^3$$
$$(16-h)^3 = 64$$
$$16 - h = 4$$
$$h = 12$$

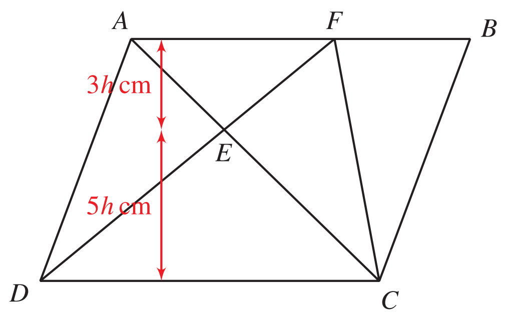
50. $\boxed{\text{D}}$

$\triangle AFE \sim \triangle CDE$ (ratio 3 : 5)

Let $AF = 3$ cm, then $CD = 5$ cm and $BF = 2$ cm.

$$\frac{(2)(3h+5h)}{2} = 16$$
$$h = 2$$

$$\text{Area of } \triangle CDE = \frac{(5)(5\times 2)}{2} = 25 \text{ cm}^2$$



51. $\boxed{\text{B}}$

$$\text{Let } O \text{ be the centre. Radius} = \frac{AD}{2} = \frac{\sqrt{10^2+24^2}}{2} = 13 \text{ cm}$$
$$AC^2 = OA^2 + OC^2 - 2(OA)(OC)\cos\angle AOC$$
$$\angle AOC \approx 135^\circ$$
$$\text{Area} = 13^2\pi \times \frac{\angle AOC}{360^\circ} - \frac{1}{2}(13)^2 \sin\angle AOC \approx 139 \text{ cm}^2$$

52. $\boxed{\text{C}}$

Let the required height be h cm.

$$\pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2(2r)$$
$$h + \frac{4r}{3} = 2r$$
$$h = \frac{2r}{3}$$

53. $\boxed{\text{C}}$

Let r cm be the radius of the smaller sphere.

$$\begin{aligned}\frac{4}{3}\pi r^3 + \frac{4}{3}\pi(5r)^3 &= 1344\pi \\ r^3 &= 8 \\ r &= 2\end{aligned}$$

Required area

$= 4\pi r^2 + 4\pi(5r)^2$

$= 416\pi \text{ cm}^2$

54. $\boxed{\text{C}}$

$DG = DB = \sqrt{10^2 + 10^2} = 10\sqrt{2}\text{ cm}$

Area $= \frac{1}{4}(10\sqrt{2} - 10)^2\pi \approx 13\text{ cm}^2$

55. $\boxed{\text{A}}$

$$\begin{aligned}BC &= \sqrt{26^2 - (28 - 18)^2} \\ &= 24\text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Required area} &= \frac{(18 + 28)24}{2} \\ &= 552\text{ cm}^2\end{aligned}$$

56. $\boxed{\text{C}}$

Note that the figure is symmetric about BD.

We have $AF = CE = \frac{24 - 8}{2} = 8\text{ cm}$.

F and C are centres of circle $ABCD$ and $BEDF$ respectively.

Note that $\triangle BCF$ is equilateral.

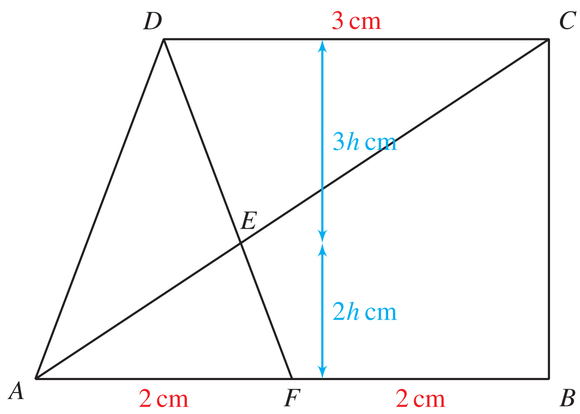
Required area

$= \pi(8)^2 \times \frac{60°}{360°} + \left(\pi(8)^2 \times \frac{60°}{360°} - \frac{1}{2}(8)^2 \sin 60°\right)$

$\approx 39.3\text{ cm}^2$

57. $\boxed{\text{B}}$

Let $DC = 3\text{ cm}$. Then we have the lengths as shown in the figure.



$\boxed{\text{Point } E}$ Note that $\triangle CDE \sim \triangle AFE$ (ratio 3 : 2).

Consider the area of $\triangle ADE$.

$$24 = \frac{(2)(5h)}{2} - \frac{(2)(2h)}{2}$$

$$h = 8$$

$$\text{Required area} = \frac{(3+4)(5h)}{2}$$

$$= 140\,\text{cm}^2$$

58. $\boxed{\text{D}}$

Let D be a point on BC such that $AD \perp BC$.

$$\frac{(BC)(AD)}{2} = 126$$

$$\frac{21(AD)}{2} = 126$$

$$AD = 12$$

$CD = \sqrt{13^2 - 12^2} = 5\,\text{cm}$

$BD = 21 - 5 = 16\,\text{cm}$

$AB = \sqrt{16^2 + 12^2} = 20\,\text{cm}$

Required perimeter $= 20 + 21 + 13$

$= 54\,\text{cm}$

59. $\boxed{\text{D}}$

Let O be the centre of the circle.

$OA = \dfrac{AB}{2} = \dfrac{24}{2} = 12\,\text{cm}$

Let M be the mid-point of AC.

$AM = \dfrac{AC}{2} = \dfrac{12}{2} = 6\,\text{cm}$

$\sin \angle AOM = \dfrac{6}{12}$

$\angle AOM = 30°$

$\angle AOC = 2\angle AOM = 2 \times 30° = 60°$

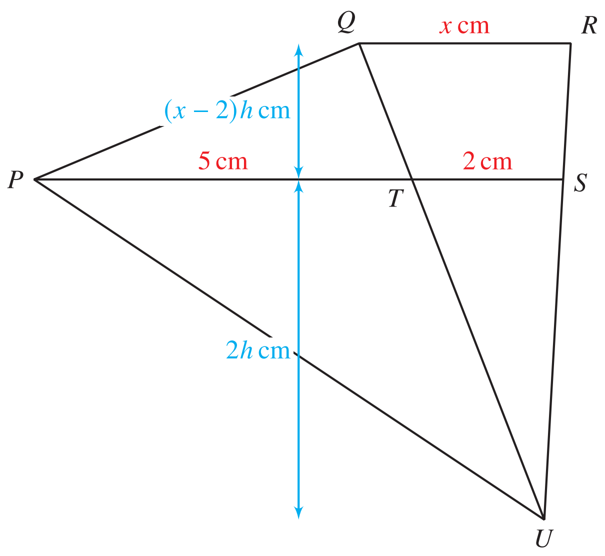
Required area

$= \pi(12)^2 \times \dfrac{1}{2} - \left[\pi(12)^2 \times \dfrac{60°}{360°} - \dfrac{1}{2} \times 12 \times \sqrt{12^2 - 6^2}\right]$

$= (36\sqrt{3} + 48\pi)\ \text{cm}^2$

60. $\boxed{\text{A}}$

Let $PT = 5$ cm and $QR = $ x cm. Then we have the lengths as shown in the figure.



$\boxed{\text{Point } U}$ Note that $\triangle STU \sim \triangle RQU$ (ratio $2 : x$).

Consider the area of $\triangle PSU$.

$$63 = \frac{(7)(2h)}{2}$$

$$h = 9$$

Consider the area of $\triangle PQT$.

$$15 = \frac{(5)(x-2)h}{2}$$

$$x = \frac{8}{3}$$

$$\text{Required area} = \frac{(x+2)[(x-2)h]}{2}$$

$$= 14\ \text{cm}^2$$

61. $\boxed{\text{A}}$

Let the height and base radius of the cone be h and r respectively.

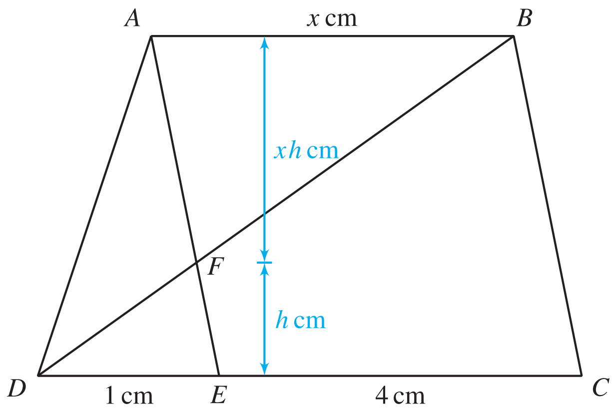
$$\frac{1}{3}\pi r^2 h = 2 \times \frac{2}{3}\pi r^3$$

$$h = 4r$$

$$h : r = 4 : 1$$

62. $\boxed{\text{C}}$

Let $DE = 1$ cm and $AB = x$ cm.
Then $CE = 4$ cm.



Consider the areas of $\triangle ADF$ and the quadrilateral $BCEF$.

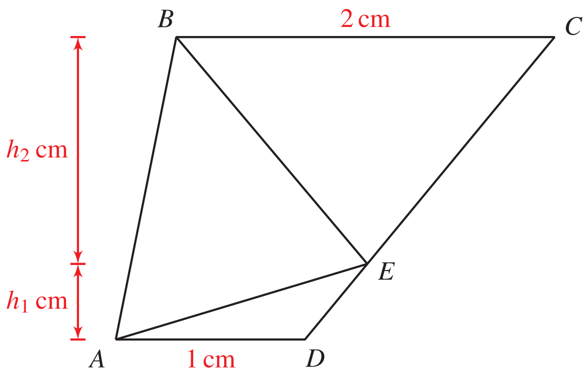
$$\begin{cases} 12 = \dfrac{(1)(h+xh)}{2} - \dfrac{(1)(h)}{2} = \dfrac{xh}{2} \\ 76 = \dfrac{(5)(h+xh)}{2} - \dfrac{(1)(h)}{2} = \dfrac{(5x+4)h}{2} \end{cases}$$

$$\frac{76}{12} = \frac{(5x+4)h}{2} \div \frac{xh}{2}$$
$$\frac{19}{3} = \frac{5x+4}{x}$$
$$19x = 15x + 12$$
$$x = 3$$

Put $x = 3$, we have $h = 8$.
Required area $= \dfrac{(x)(xh)}{2} = 36\text{ cm}^2$

63. $\boxed{\text{C}}$

Assume $AD = 1$ cm. Then $BC = 2$ cm.



Note that $h_2 : h_1 = CE : DE = 3 : 1$.

Consider the area of $\triangle ABE$.

$$\frac{(1+2)(h_1+h_2)}{2} - \frac{2h_2}{2} - \frac{1h_1}{2} = 10$$
$$\frac{3}{2}(h_1+3h_1) - 3h_1 - \frac{h_1}{2} = 10$$
$$h_1 = 4$$

$$\text{Required area} = \frac{(1+2)(4+12)}{2} = 24\,\text{cm}^2$$

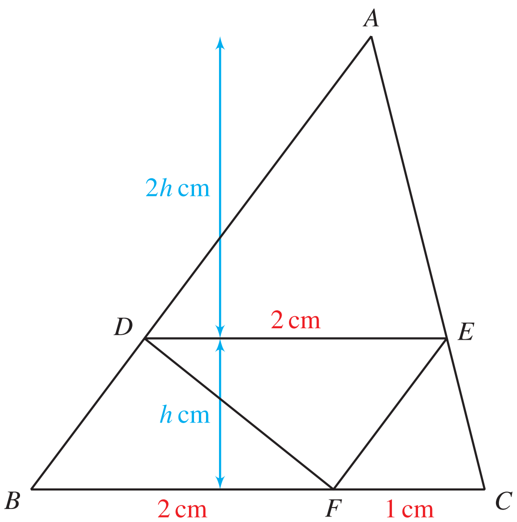
64. $\boxed{\text{B}}$

$$\text{Ratio of base radius} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$\text{Required ratio} = \frac{2^3}{3^3-2^3} = \frac{8}{19}$$

65. $\boxed{\text{D}}$

Let $DE = 2\,\text{cm}$. Then we have the lengths as shown in the figure.



$\boxed{\text{Point } A}$ Note that $\triangle ADE \sim \triangle ABC$ (ratio $2:3$).

Consider the area of $\triangle DEF$.

$$3 = \frac{(2)(h)}{2}$$
$$h = 3$$

$$\text{Required area} = \frac{(3)(3h)}{2} = 13.5\,\text{cm}^2$$

66. $\boxed{\text{A}}$

Let r cm and h cm be the base radius and the height of the cylinder respectively.

Consider the volume.

$$\pi r^2 h = 2 \times \frac{4}{3}\pi(18)^3$$
$$r^2 h = 15\,552$$

Consider the curved surface area.

$$2\pi rh = \frac{1}{2} \times 4\pi(18)^2$$
$$rh = 324$$
$$\sqrt{\frac{15\,552}{h}} \times h = 324$$
$$\sqrt{h} = \frac{324}{\sqrt{15552}}$$
$$h = 6.75$$

The height of the cylinder is 6.75 cm.

67. $\boxed{\text{B}}$

$BC = OA \sin\angle AOC = 10 \sin 30° = 5$ cm

$AB = OC - OA\cos\angle AOC = 10 - 10\cos 30° = (10 - 5\sqrt{3})$ cm

Required area

$$= \frac{(AB + OC)BC}{2} - \pi(OA)^2 \times \frac{\angle AOC}{360°}$$
$$= \frac{(AB + 10)5}{2} - \pi(10)^2 \times \frac{30°}{360°}$$
$$\approx 2.2\,\text{cm}^2$$

68. $\boxed{\text{D}}$

Let R cm and r cm be the radius of the sphere and the base radius of the cone respectively.

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2\left(\frac{r}{2}\right)$$
$$R^3 = \frac{r^3}{8}$$
$$R = \frac{r}{2}$$

Required ratio

$$= 4\pi R^2 : \left[\pi r\sqrt{r^2 + \left(\frac{r}{2}\right)^2} + \pi r^2\right]$$
$$= 4 \times \frac{1}{2^2} : \left[\sqrt{1 + \frac{1}{4}} + 1\right]$$
$$= 2 : (2 + \sqrt{5})$$

69. $\boxed{\text{D}}$

Let r cm and h cm be the base radius and the height of the cone respectively.

$$\frac{1}{2}\times\frac{4}{3}\pi r^3 = 2\times\frac{1}{3}\pi r^2 h$$
$$r = h$$
$$r : h = 1 : 1$$

70. $\boxed{\text{B}}$

$$\sin\angle ODC = \frac{OC}{OD} = \frac{1}{2}$$
$$\angle ODC = 30^\circ$$

$\angle DFE = 75^\circ - 30^\circ = 45^\circ$ and $\angle CFB = \angle DFE = 45^\circ$

$\angle CBF = 180^\circ - 90^\circ - 45^\circ = 45^\circ$ and $\angle OAB = \angle OBA = 45^\circ$

$\angle DOA = 75^\circ - 45^\circ = 30^\circ$

Area of sector $= \pi(12)^2 \times \dfrac{30^\circ}{360^\circ} = 12\pi\text{ cm}^2$

71. $\boxed{\text{D}}$

Let O be the centre of the circle.

$OC = \dfrac{BC}{2} = \dfrac{12}{2} = 6\text{ cm}$

$AC = 2\times OC\cos\angle ACB = 2\times 6\cos 30^\circ = 6\sqrt{3}\text{ cm}$

Required perimeter

$$= 6\sqrt{3} + 2\pi(6)\times\frac{180^\circ - 30^\circ - 30^\circ}{360^\circ}$$
$$= (4\pi + 6\sqrt{3})\text{ cm}$$

72. $\boxed{\text{C}}$

Volume ratio of the spheres is $1 : (\sqrt{4})^3 : (\sqrt{9})^3 = 1 : 8 : 27$.

Required volume

$$= 252\times\frac{27}{1+8+27}$$
$$= 189\text{ cm}^3$$

73. $\boxed{\text{B}}$

$$\cos\angle MON = \frac{OM}{ON}$$
$$\cos\angle MON = \frac{6}{12}$$
$$\angle MON = 60^\circ$$

Required area

$= \pi(12)^2 \times \dfrac{30°}{360°} + \dfrac{(12\sin 60°)(6)}{2}$

$= (12\pi + 18\sqrt{3})\text{ cm}^2$

74. $\boxed{\text{C}}$

Area ratio $= \dfrac{0.9 \times (1000 \times 100)^2}{250}$

$= 3.6 \times 10^7$

Required scale $= 1 : \sqrt{3.6 \times 10^7}$

$= 1 : 6000$

75. $\boxed{\text{D}}$

Area ratio $= \dfrac{78.75 \times (1000 \times 100)^2}{1260}$

$= 6.25 \times 10^8$

Required scale $= 1 : \sqrt{6.25 \times 10^8}$

$= 1 : 25\,000$

76. $\boxed{\text{A}}$

Let x cm be the length of the cube.

$$\frac{1}{2} \times \frac{4}{3}\pi 3^3 = x^3 + \frac{1}{3}x^2(2x)$$

$$x^3 = \frac{54\pi}{5}$$

Required volume

$= \dfrac{1}{3}x^2(2x)$

$= \dfrac{2}{3} \times \dfrac{54\pi}{5}$

$= 7.2\pi\text{ cm}^3$

77. $\boxed{\text{A}}$

Area ratio $= \dfrac{76.8 \times (1000 \times 100)^2}{120}$

$= 6.4 \times 10^9$

Required scale $= \sqrt{6.4 \times 10^9}$

$= 80\,000$

78. $\boxed{\text{D}}$

Let A km^2 be the actual area of the farmland.

$$\frac{A \times (1000 \times 100)^2}{15} = \left(\frac{60\,000}{1}\right)^2$$

$$A = 5.4$$

79. $\boxed{\text{B}}$

$$\begin{aligned}\text{Actual area} &= 30(40\,000)^2 \\ &= 4.8 \times 10^{10}\ \text{cm}^2 \\ &= \frac{4.8 \times 10^{10}}{(100 \times 1000)^2}\ \text{km}^2 \\ &= 4.8\ \text{km}^2\end{aligned}$$

80. $\boxed{\text{B}}$

$\angle POQ = 90°$ and $\angle QOR = 90°$. So, P, O and R are collinear.

$$18 = \frac{(4+8)q}{2}$$

$$q = 3$$

END OF PAPER