Solution	Marks
ELITE-2425-MOCK-SET 14-MATH-CP 1	
Suggested solutions	
1. $\frac{5a-3b}{4}=1-\frac{b}{2}$	
$5a-3b=4-2b$	1M
$5a=4+b$	1M
$a=\frac{4+b}{5}$	1A
2. $\frac{2}{2x-3}-\frac{3}{x-1}=\frac{2(x-1)-3(2x-3)}{(2x-3)(x-1)}$	1M+1A
$=\frac{7-4x}{(2x-3)(x-1)}$	1A
3. Required probability $=\frac{8}{4\times 3}$	1M+1A
$=\frac{2}{3}$	1A
4. (a) $8x^2-8x+2=2(2x-1)^2$	1A
(b) $18xy^2-8x^3+8x^2-2x=2x(3y)^2-2x(2x-1)^2$	1M
$=2x(3y+2x-1)(3y-2x+1)$	1A
5. Marked price $=\frac{2000}{1-20\%}=\$2500$	1A
Let the cost of each watch be $\$x$.	
$6(2000)+4(2500)=10x+7000$	1M+1A
$x=1500$	1A
Required cost is \$1500.	
6. (a) $5x-11<\frac{2(x-3)}{4}$	
$\frac{9x}{2}<\frac{19}{2}$	1M
$x<\frac{19}{9}$	1A
(b) $5x-41\leq 0$	
$x\leq\frac{41}{5}$	1A
Thus, $x\leq\frac{41}{5}$.	
There are 8 positive integers.	1A

Solution	Marks
7. (a) $\frac{2x}{2+3}+\frac{1(3)}{3+2}=\frac{x+1}{2}$	1M+1A
$-\frac{x}{10}=-\frac{1}{10}$	
$x=1$	1A
(b) Suppose y litre beverage B is added to the mixture.	
$1+\frac{3y}{5}=(2+y)\times 60\%$	1M
$1=1.2$ (contradiction)	
It is impossible.	1A
8. Total distance travelled = $(2x+8)$ km	1A
Consider the total time taken.	
$\frac{x}{60}+\frac{4}{60}+\frac{x+8}{80}=\frac{2x+8}{66}$	2M+1A
$-\frac{x}{880}=-\frac{1}{22}$	
$x=40$	1A
9. (a) Let the base radius and height of the cylinder be r cm and h cm respectively.	
$2\pi rh=\frac{1}{2}(2\pi rh+2\pi r^2)$	1M
$h=r$	
$\pi r^2h=216\pi$	
$r^3=216$	
$r=6$	1A
Base radius = 6 cm	
(b) Percentage change = $\frac{2(6)(12)}{2\pi(6)^2+2\pi(6)(6)}\times 100\%$	1M+1M
$\approx 31.8\%$	1A
10. (a) Let $p(x)=ax+b(x+1)^2$, where a and b are non-zero constants.	1A
$\begin{cases}7=-3a+4b\\3=a+4b\end{cases}$	1M
Solving, we have $a=-1$ and $b=1$.	1A
Thus, $p(x)=-x+(x+1)^2$.	
(b) $-x+(x+1)^2=7-x^2$	1M
$2x^2+x-6=0$	
$x=-2$ or $\frac{3}{2}$	1A

Solution	Marks
11. (a) $f(x) = 2(x-2)(x^2-4x+1)+ax+b$	1M
$= 2x^3 - 12x^2 + (18+a)x + (b-4)$	
We have $2b = -12$, $18 + a = 7a$ and $c = b - 4$.	1M
Solving, we have $a = 3$, $b = -6$ and $c = -10$.	1A
(b) $0 = 2(x-2)(x^2-4x+1)+3x-6$	
$= (x-2)[2(x^2-4x+1)+3]$	1M
$= (x-2)(2x^2-8x+5)$	
$x = 2$ or $\dfrac{8 \pm \sqrt{8^2-4(2)(5)}}{2(2)}$	
$= 2$ or $\dfrac{4 \pm \sqrt{6}}{2}$	1A
There is 1 rational root.	1A
12. (a) (i) Mode = 39	
Thus, $a = b = 9$.	1A
(ii) $\dfrac{(50+c)+51}{2} - \dfrac{(30+d)+30}{2} = 21$	1M
$c - d = 1$	
Range $= (60+d)-(20+c)$	
$= 40-(c-d)$	
$= 39$	1A
(b) Mean $= \dfrac{(20+c)+25+26+\ldots+(60+d)}{20}$	1M
$= \dfrac{830+2(c+d)}{20}$	
Since $c - d = 1$, $1 \le c \le 5$ and $2 \le d \le 5$, we have $3 \le c + d \le 9$.	1M
$\dfrac{830+2(3)}{20} = 41.8 \le \text{mean} \le \dfrac{830+2(9)}{20} = 42.4$	
Thus, mean = 42 and $c + d = 5$.	1A
Solving, we have $c = 3$ and $d = 2$.	
Standard deviation ≈ 11.9	1A

Solution	Marks
13. (a) $\angle ADC = 90°$ and $\angle CDE = 90° - 60° = 30°$	1A
$\angle DCE = 180° - 60° = 120°$	1A
$\angle CED = 180° - 120° - 30° = 30° = \angle CDE$	
Thus, $CD = CE$ and $\triangle CDE$ is isosceles.	1A
(b) In $\triangle BEF$, $\angle EBF = 90°$ and $\angle BEF = 30°$.	
Thus, $EF = \dfrac{BE}{\cos 30°}$.	
In $\triangle CDE$, $DE = 2CE \cos 30°$.	
In $\triangle ADF$, $DF = AD = 5\sqrt{3}$ cm.	
$2CE \cos 30° + \dfrac{BE}{\cos 30°} = 5\sqrt{3}$	1M
$BC \cos 30° + \dfrac{BC}{2 \cos 30°} = 5\sqrt{3}$	
$BC = 6$ cm	1A
(c) Let M be the mid-point of DF.	
$GM = \dfrac{DF}{2} \times \tan 30° = \dfrac{5}{2}$ cm	1A
$CE = \dfrac{BC}{2} = 3$ cm	
Area of $\triangle GDE = \dfrac{1}{2}(GM)(DE)$	
Area of $\triangle CDE = \dfrac{1}{2}(CE \sin 30°)(DE)$	
$= \dfrac{1}{2}(1.5)(DE)$	
$< \dfrac{1}{2}(GM)(DE)$	
Area of $\triangle CDE <$ area of $\triangle GDE$	
The claim is agreed.	1A

Solution	Marks
14. (a) (i) $\angle EHG = 90^\circ$ *(property of rectangle)*	
$\angle EHA = 180^\circ - 90^\circ = 90^\circ$ *(adj. ∠s on st. line)*	
$\angle ADN = 90^\circ = \angle EHA$ *(property of square)*	
$CD // AB$ *(property of square)*	
$\angle DNA = \angle EAH$ *(alt. ∠s, CD // AB)*	
$\triangle EHA \sim \triangle ADN$ *(AA)*	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

Solution	Marks
(ii) $\angle CGN = 90^\circ$ *(property of rectangle)*	
$\angle ADN = 90^\circ = \angle CGN$ *(property of square)*	
$\angle DNA = \angle GNC$ *(vert. opp. ∠s)*	
$\triangle CGN \sim \triangle ADN$ *(AA)*	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

Solution	Marks
(b) (i) From (a), $\triangle EHA \sim \triangle CGN$.	1M
$\frac{GC}{EH} = \frac{CN}{AE}$	
$\frac{p}{p+1} = \frac{6-k}{4}$	1M+1M
$\frac{4p}{p+1} - 6 = -k$	
$k = \frac{2p+6}{p+1}$	1A
(ii) $\frac{2p+6}{p+1} > 3$ and $\frac{2p+6}{p+1} < 6$	1M
$2p+6 > 3p+3$ $\quad$ $2p+6 < 6p+6$	
$p < 3$ $\quad$ $p > 0$	
Thus, $0 < p < 3$.	1A
15. Required number $= 8 \times 7 \times C_3^{13} + 7 \times 6 \times C_3^{13}$	1M+1A
$= 28\,028$	1A

Solution	Marks
16. $\frac{256}{2^a} = \frac{2^b}{256}$ and $3 - \log_2(a-2) = \log_2(b+20) - 3$	1M+1M
$2^{a+b} = 2^{16}$ $\quad$ $6 = \log_2[(a-2)(b+20)]$	
$a + b = 16$ $\quad$ $(a-2)(b+20) = 2^6$	
$(a-2)[(16-a)+20] = 64$	1M
$-a^2 + 38a - 136 = 0$	
$a = 34$ or 4	
When $a = 34$, $b = 16 - 34 = -18$ (rejected).	
When $a = 4$, $b = 16 - 4 = 12$.	
Thus, $a = 4$ and $b = 12$.	1A+1A
17. (a) Let M be the mid-point of AC.	
$BM = 20 \sin 60° = 10\sqrt{3}$ cm	
$BE = BM \sin 60° = 15$ cm	1A
$\angle BEC = 90°$. So, BC is a diameter of the circumcircle of $\triangle BCE$.	1M
Thus, $DE = DB = DC = \frac{20}{2} = 10$ cm	1A
(b) $AE = \sqrt{AB^2 - BE^2}$	1M
$= \sqrt{175}$ cm	
$AD = BM = 10\sqrt{3}$ cm	
$AE^2 = AD^2 + DE^2 - 2(AD)(DE)\cos\angle ADE$	1M
$\angle ADE \approx 49.5° \neq 90°$	
The claim is disagreed.	1A

Solution	Marks
18. (a) Let the radius be r. The coordinates of A are $(0,\ r)$.	
$(3-0)^2+(r-9)^2=r^2$	1M
$-18r+90=0$	
$r=5$	1A
The coordinates of A are $(0,\ 5)$.	
(b) $x^2+(y-5)^2=5^2$	1M
$x^2+y^2-10y=0$	1A
(c) (i) Γ is a pair of straight lines perpendicular to L and their perpendicular distances from AB are equal to $\dfrac{BC}{2}$.	1A+1A
(ii) Let the coordinates of C be $(t,\ 0)$.	
$\dfrac{9-0}{3-t}\times\dfrac{9-5}{3-0}=-1$	1M
$t=15$	
Required distance $=\dfrac{BC}{2}-r$	1M
$=\dfrac{OC}{2}-5$	
$=\dfrac{5}{2}$	1A

Solution	Marks
19. (a) $B(-3,\ 4)$	1A
Axis of symmetry of P is $x = \frac{6a}{2a}$, i.e., $x = 3$.	1A
The coordinates of C are $(9,\ 4)$.	1A
(b) $f(3) = 9a - 18a + 9a + b = b$	1M
The coordinates of vertex of P are $(3,\ b)$.	
$b - (-4) = 5$	
$b = 1$	1A
P passes through B.	
$a(-3)^2 - 6a(-3) + (9a + 1) = 4$	
$36a + 1 = 4$	
$a = \frac{1}{12}$	1A
(c) (i) Area of $ABDC$ is the greatest when AD is a diameter, i.e., $\angle ABD = 90°$.	1M
By symmetry, the coordinates of D are $(3,\ k)$, where k is a constant.	
$\frac{k-4}{3+3} \times \frac{4+4}{-3-3} = -1$	1M
$k = \frac{17}{2}$	1A
The coordinates of D are $\left(3,\ \frac{17}{2}\right)$.	
(ii) $AB = \sqrt{(3+3)^2 + (4+4)^2} = 10$	
$BD = \sqrt{(3+3)^2 + \left(\frac{17}{2} - 4\right)^2} = \frac{15}{2}$	1A
Let the radius of the inscribed circle be r.	
$\frac{\frac{15}{2} - r}{r} = \frac{\left(\frac{15}{2}\right)}{10}$	1M
$r = \frac{30}{7}$	
Area of the circle $= \pi\left(\frac{30}{7}\right)^2$	
$< 25\pi$	
The claim is agreed.	1A

END OF PAPER