REG-CP2A-2425-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. C	3. B	4. C	5. B
6. B	7. A	8. D	9. A	10. B
11. A	12. D	13. C	14. B	15. C
16. D	17. D	18. A	19. B	20. B
21. B	22. B	23. C	24. B	25. A
26. B	27. D	28. D	29. D	30. A
31. A	32. B	33. A	34. A	35. C
36. C	37. C	38. C	39. A	40. C
41. B	42. C	43. C	44. D	45. C
46. D	47. C			

1. A

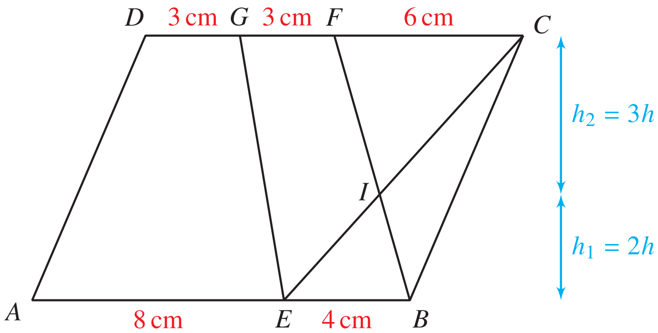
Let $DG = 3\,\text{cm}$. Then we have the lengths as shown in the figure.

$\triangle FIC \sim \triangle BIE$

$$\frac{h_2}{h_1} = \frac{FC}{EB} = \frac{3}{2}$$

Consider the area of $\triangle BIC$,

$$\frac{(4)(5h)}{2} - \frac{(4)(2h)}{2} = 6$$
$$h = 1$$



Required area

$$= \frac{(9)(5)}{2} - \frac{(6)(3)}{2}$$
$$= 13.5\,\text{cm}^2$$

2. C

Let F be the intersection of AE and CD produced.

Let $AB : BC = 1 : r$.

Since $\triangle CBD \sim \triangle CAF$, $FD : DC = 1 : r$.

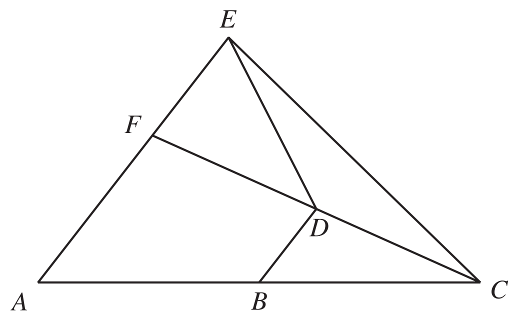
Area of $\triangle DEF = \frac{8}{r}$ cm^2

Area of $\triangle CAF = \frac{4(1+r)^2}{r^2}$ cm^2

$$8 + \frac{8}{r} + \frac{4(1+r)^2}{r^2} = 45$$

$$r = \frac{2}{3} \quad \text{or} \quad -\frac{2}{11} \text{ (rejected)}$$

So, $AB : BC = 3 : 2$.

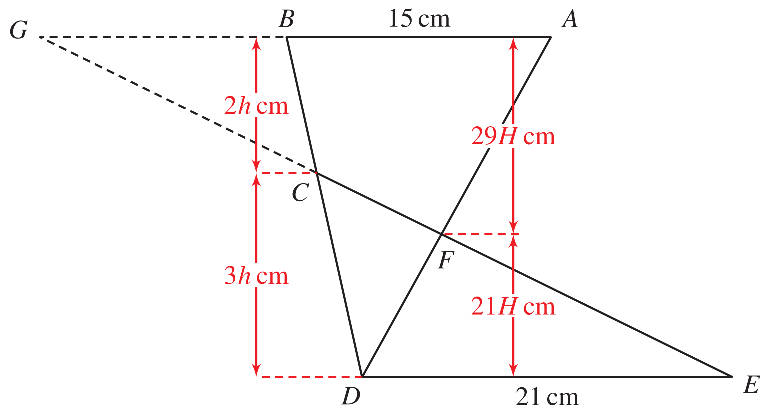


3. B

Let $AB = 15$ cm. Then $DE = 21$ cm.

Let G be the intersection of AB produced and EC produced.

Rotate the figure such that the parallel lines appear horizontal as shown.



Point C

We have $\triangle GBC \sim \triangle EDC$ (ratio 2 : 3).

Let $2h$ cm and $3h$ cm be the heights of two triangles respectively.

$GB = \frac{2}{3} \times DE = 14$ cm

Point F

We have $\triangle AFG \sim \triangle DFE$ (ratio 29 : 21).

Let $29H$ cm and $21H$ cm be the heights of two triangles respectively.

Consider the area of $\triangle CDF$.

$$\frac{21(3h)}{2} - \frac{21(21H)}{2} = 63$$

$$3h - 21H = 6$$

Consider the total height of the figure.

$$\begin{cases} 3h - 21H = 6 \\ 2h + 3h = 29H + 21H \end{cases}$$

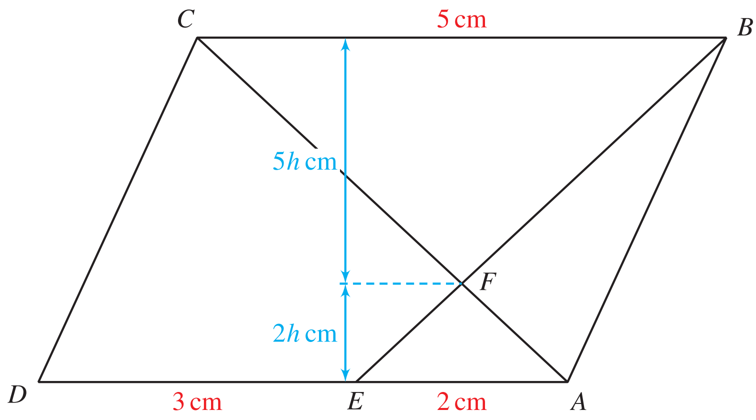
Solving, we have $h = \frac{20}{3}$ and $H = \frac{2}{3}$.

$$\text{Required area} = \frac{29(29H)}{2} - \frac{14(2h)}{2}$$

$$= 187 \text{ cm}^2$$

4. C

Let $BC = 5$ cm. Then we have the lengths as shown in the figure.



Point F Note that $\triangle AEF \sim \triangle BCF$ (ratio 2 : 5).
Consider the area of $\triangle DEF$.

$$\frac{(2)(2h)}{2} = 4$$

$$h = 2$$

$$\text{Required area} = 5(5h + 2h) = 70\text{ cm}^2$$

5. B

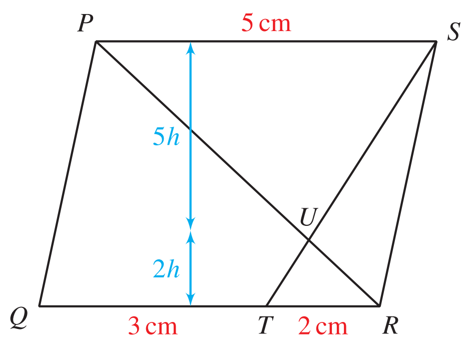
Let $QT = 3$ cm. Then $TR = 2$ cm and $PS = 5$ cm.

$\triangle PSU \sim \triangle RTU$ (ratio 5 : 2)

$$\frac{(2)(2h)}{2} = 12$$

$$h = 6$$

$$\text{Required area} = \frac{(5)(7h)}{2} - \frac{(2)(2h)}{2} = 93\text{ cm}^2$$



6. B

Let $BF = 1$ cm. Then $FC = 2$ cm and $AD = 3$ cm.

Consider the area of $\triangle BEF$,

$$\frac{(1)(BE)}{2} = 2$$
$$BE = 4\text{ cm}$$

$AE = BE = 4$ cm

$$\text{Required area} = (3)(8) - \frac{1}{2}(3)(4) - 2 - \frac{1}{2}(8)(2)$$
$$= 8\text{ cm}^2$$

7. A

Lower base of the trapezoidal cross section

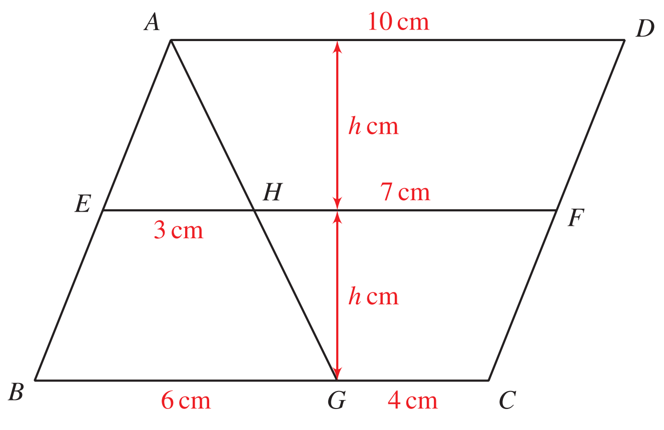
$= 24 - \sqrt{25^2 - 24^2}$

$= 17$ cm

$$\text{Required volume} = \frac{(17+24)(24)}{2} \times 10$$
$$= 4920\text{ cm}^3$$

8. D

Let $BG = 6$ cm. Then $GC = 4$ cm and $AD = 10$ cm.



Note that $\triangle AEH \sim \triangle ABG$ (ratio 1 : 2).

We have $EH = 3$ cm and $HF = 7$ cm.

Consider the area of $\triangle AEH$.

$$\frac{3(h)}{2} = 36$$
$$h = 24$$

$$\text{Required area} = \frac{(7+4)h}{2}$$
$$= 132\text{ cm}^2$$

9. $\boxed{\text{A}}$

$CM = \sqrt{26^2 - 10^2}$

$= 24 \text{ cm}$

$BM = \sqrt{20^2 - \left(\frac{24}{2}\right)^2}$

$= 16 \text{ cm}$

Required area $= \frac{(12)(16)}{2}$

$= 96 \text{ cm}^2$

10. $\boxed{\text{B}}$

$AE : EC = 24 : 36 = 2 : 3$

$\boxed{\text{Point } E}$ Note that $\triangle CDE \sim \triangle AFE$ (ratio 3 : 2).

Let $CD = 3$ cm and the heights of two triangles be $3h$ cm and $2h$ cm respectively.

We have $AF = BF = 2$ cm.

Consider $\triangle CDE$.

$\frac{3(3h)}{2} = 36$

$h = 8$

Required area $= \frac{(3+4)(5h)}{2}$

$= 140 \text{ cm}^2$

11. $\boxed{\text{A}}$

Let the slant height of the cone be ℓ.

$4\pi r^2 = \pi r \ell$

$\ell = 4r$

Volume $= \frac{1}{3}\pi r^2 \sqrt{\ell^2 - r^2}$

$= \frac{\sqrt{15}}{3}\pi r^3$

12. D

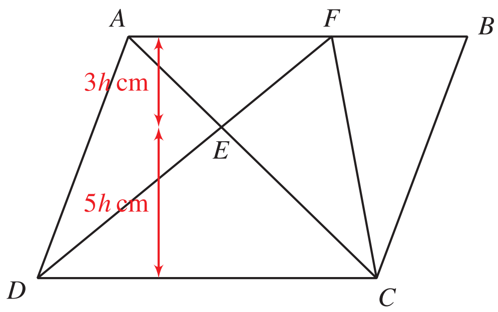
$\triangle AFE \sim \triangle CDE$ (ratio 3 : 5)

Let $AF = 3$ cm, then $CD = 5$ cm and $BF = 2$ cm.

$$\frac{(2)(3h+5h)}{2} = 16$$

$$h = 2$$

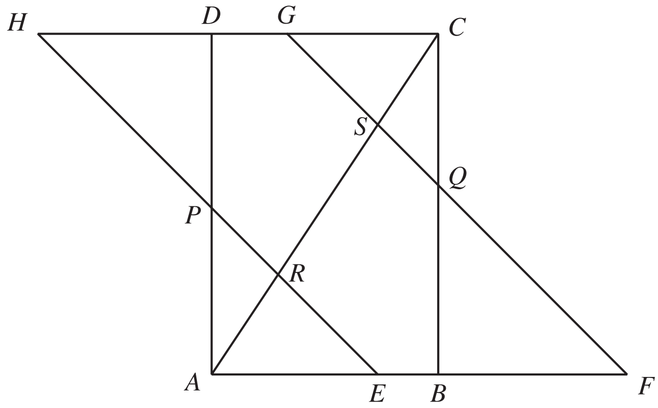
Area of $\triangle CDE = \frac{(5)(5\times 2)}{2} = 25\text{ cm}^2$



13. C

I. ✗.

Take $\angle GCQ = 90^\circ$ such that $ABCD$ is a rectangle, where $CD \neq CB$.



Note that $\angle GCS \neq 45^\circ$ and $\angle GCS \neq \angle QCS$.

Thus, $\triangle CGS$ and $\triangle CQS$ are not congruent triangles.

II. ✓.

Let K be the intersection of FG produced and AD produced.

$\angle ABC = \angle ADC$	*(opp. ∠s, //gram)*
$\angle FBQ = 180^\circ - \angle ABC$	*(adj. ∠s on st. line)*
$\angle PDH = 180^\circ - \angle ADC$	*(adj. ∠s on st. line)*
$= \angle FBQ$	
$\angle BFQ = \angle CGQ$	*(alt. ∠s, HC//AF)*
$CG = CQ$	*(given)*
$\angle CGQ = \angle\angle CQG$	*(base. ∠s, isos. △)*
$\angle CQG = \angle AKF$	*(alt. ∠s, AK//BC)*
$\angle DPH = \angle AKF$	*(alt. ∠s, EH//FK)*
$= \angle BFQ$	
$\triangle BFQ \sim \triangle DPH$	*(AA)*

III. ✓.

We have $\triangle BFQ \sim \triangle DPH$.

$\angle AEP = \angle BFQ$	*(corr. ∠s, EH//FG)*
$\angle BFQ = \angle DPH$	*(corr. ∠s, ≅ △s)*
$\angle APE = \angle DPH$	*(vert. opp. ∠s)*
$= \angle AEP$	
$AE = AP$	*(sides opp. equal ∠s)*

14. B

I. ✓.

Since $TQ = TR$, we have $\angle RQT = \angle TRQ$.

$$\begin{aligned} \angle RQT + \angle TRQ + \angle QTR &= 180^\circ \\ 2\angle RQT + 44^\circ &= 180^\circ \\ \angle RQT &= 68^\circ \end{aligned}$$

Since $TP = TS$, we have $\angle TPS + \angle PST$.

$$\begin{aligned} \angle TPS + \angle PST + \angle STP &= 180^\circ \\ 2\angle TPS + 44^\circ &= 180^\circ \\ \angle TPS &= 68^\circ \end{aligned}$$

Since $\angle RQT = \angle TPS$, we have $PS // QR$.

II. ✗.

Consider $\triangle RST$.

$$\begin{aligned} \angle SRT + \angle TSR &= \angle QTR \\ \angle SRT + 32^\circ &= 44^\circ \\ \angle SRT &= 12^\circ \end{aligned}$$

We have $\angle SRQ = \angle SRT + \angle TRQ = 12^\circ + 68^\circ = 80^\circ \neq 78^\circ$.

III. ✓.

Note that $\triangle PTQ \cong \triangle STR$.

We have $PQ = SR$.

15. C

In $\triangle ABD$, we have $\angle BAD = \angle ABD$.

$$\begin{aligned} \angle ABD + \angle BAD + \angle ADB &= 180^\circ \\ 2\angle ABD + 28^\circ &= 180^\circ \\ \angle ABD &= 76^\circ \end{aligned}$$

Since $AB // DC$, we have $\angle CDB = \angle ABD = 76^\circ$.

In $\triangle BCD$, we have $\angle BCD = \angle CBD$.

$$\begin{aligned} \angle BCD + \angle CBD + \angle CDB &= 180^\circ \\ 2\angle BCD + 76^\circ &= 180^\circ \\ \angle BCD &= 52^\circ \end{aligned}$$

16. D

I. ✓. Consider $\triangle URS$.

$$\angle URS = \angle USR \quad \textit{(given)}$$

$$UR = US \quad \textit{(sides opp. equal } \angle\textit{s)}$$

II. ✓. Consider $\triangle PQS$ and $\triangle PRT$.

$$\angle PQU = \angle PTU \quad \textit{(given)}$$

$$\angle QPS = \angle TPR \quad \textit{(common } \angle\textit{)}$$

$$\angle PRT + \angle PTU + \angle TPR = 180^\circ \quad \textit{(}\angle\textit{ sum of }\triangle\textit{)}$$

$$\angle PRT = 180^\circ - \angle PTU - \angle TPR$$

$$\angle PSQ + \angle QPS + \angle PQU = 180^\circ \quad \textit{(}\angle\textit{ sum of }\triangle\textit{)}$$

$$\begin{aligned}\angle PSQ &= 180^\circ - \angle PQU - \angle QPS \\ &= 180^\circ - \angle PTU - \angle TPR \\ &= \angle PRT\end{aligned}$$

Consider $\triangle PRS$.

$$\angle URS = \angle USR \quad \textit{(given)}$$

$$\begin{aligned}\angle PRS &= \angle PRT + \angle URS \\ &= \angle PSQ + \angle USR \\ &= \angle PSR\end{aligned}$$

$$PR = PS \quad \textit{(sides opp. equal } \angle\textit{s)}$$

III. ✓. Consider $\triangle PQS$ and $\angle PRT$.

$$\angle PQU = \angle PTU \quad \textit{(given)}$$

$$\angle QPS = \angle TPR \quad \textit{(common } \angle\textit{)}$$

$$PR = PS \quad \textit{(proved)}$$

$$\triangle PQS \cong \triangle PTR \quad \textit{(AAS)}$$

$$PQ = PT \quad \textit{(corr. sides, } \cong \triangle\textit{s)}$$

17. D

Note that $\triangle AFG \cong \triangle AEG$ *(SAS)*.

I. ✓. Use the fact that $AECF$ is a rhombus (will be proved later).

$$\begin{aligned} CF &= AE && \textit{(property of rhombus)} \\ \angle FBC &= 90^\circ = \angle ADE && \textit{(property of rectangle)} \\ BC &= AD && \textit{(property of rectangle)} \\ \triangle BCF &\cong \triangle DAE && \textit{(RHS)} \end{aligned}$$

II. ✓. We have $AB // CD$.

$$\begin{aligned} \angle AFG &= \angle CEG && \textit{(alt. } \angle s, AB//CD) \\ \angle AGF &= \angle CGE && \textit{(vert. opp. } \angle s) \\ \triangle FAG &\sim \triangle ECG && \textit{(AA)} \end{aligned}$$

III. ✓. Note that $\triangle AFC \cong \triangle AEC$ *(SAS)*.

$$\begin{aligned} \angle CAE &= \angle CAB && \textit{(given)} \\ \angle CAB &= \angle ACE && \textit{(alt. } \angle s, AB//CD) \\ \angle CAE &= \angle ACE && \\ CE &= AE && \textit{(sides opp. equal } \angle s) \\ AE &= AF && \textit{(given)} \\ CE &= CF && \textit{(corr. sides, } \cong \triangle s) \end{aligned}$$

Thus, we have $AE = CE = CF = AF$ and $AECF$ is a rhombus.

18. A

Each interior angle $= \dfrac{(8-2)180^\circ}{8} = 135^\circ$

Consider parallelogram $EFGI$.

$$\begin{aligned} \angle FGI + \angle EFG &= 180^\circ \\ \angle FGI &= 45^\circ \end{aligned}$$

Consider $\triangle FGH$.

$$\begin{aligned} \angle GHF + \angle GFH + \angle FGH &= 180^\circ \\ \angle GHF &= \frac{180^\circ - 135^\circ}{2} \\ &= 22.5^\circ \end{aligned}$$

Consider $\triangle GHJ$.

$$\begin{aligned} \angle HJI &= \angle HGJ + \angle GHJ \\ &= (135^\circ - 45^\circ) + 22.5^\circ \\ &= 112.5^\circ \end{aligned}$$

19. B

I. ✗. $(n-2)180° = 2520°$

$n = 16$

II. ✓. Exterior angle $= \frac{360°}{16} = 22.5°$

III. ✗. Interior angle $= 180° - 22.5° = 157.5°$

20. B

$\angle FAB = \frac{(6-2)180°}{6} = 120°$

$\angle GAB = 90°$

$\angle FAG = 120° - 90° = 30°$

Note that $AF = AG$, we have $\angle AGF = \frac{180° - 30°}{2} = 75°$.

$\angle FGE = 180° - 75° = 105°$

21. B

Note that $\angle ABC = \angle ECD$ and $AB // CE$.

$$\frac{\text{area of } \triangle ACE}{\text{area of } \triangle ABC} = \frac{CE}{AB}$$

$$\frac{\text{area of } \triangle ACE}{50} = \frac{4}{5}$$

area of $\triangle ACE = 40\,\text{cm}^2$

22. B

Let G be a point on BE such that $DG // BC$.

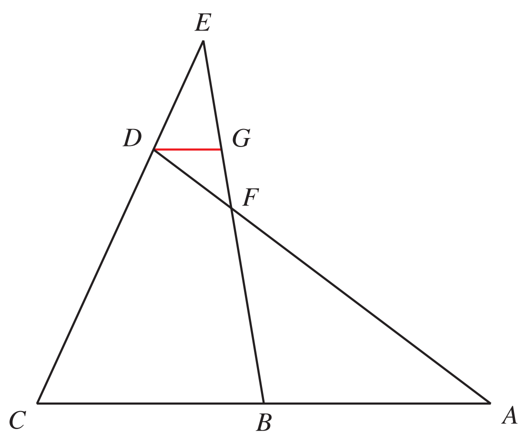
Note that $\triangle DFG \sim \triangle AFB$.

$$\frac{DG}{AB} = \frac{FD}{AF} = \frac{1}{5}$$

Note that $\triangle DEG \sim \triangle CEB$.

$$\frac{CE}{DE} = \frac{BC}{DG} = \frac{AB}{DG} = 5$$

Thus, $CD : DE = 4 : 1$.



23. C

Consider $\triangle BCM$.

$$MC^2 = BC^2 + BM^2$$
$$MC = \sqrt{12^2 + (12-7)^2}$$
$$= 13\,\text{cm}$$

Note that $\triangle CDN \sim \triangle MCB$.

$$\frac{CD}{MC} = \frac{CN}{MB}$$
$$\frac{12}{13} = \frac{CN}{12-7}$$
$$CN = \frac{60}{13}$$

$$MN : NC = \left(13 - \frac{60}{13}\right) : \frac{60}{13}$$
$$= 109 : 60$$

24. B

$$\frac{24}{1 + 3\cos^2\theta + 5\cos^2(90^\circ - \theta)} = \frac{24}{1 + 3\cos^2\theta + 5\sin^2\theta}$$
$$= \frac{24}{1 + 3(\sin^2\theta + \cos^2\theta) + 2\sin^2\theta}$$
$$= \frac{12}{2 + \sin^2\theta}$$

Since $0 \le \sin^2\theta \le 1$, the required greatest value is $\frac{12}{2+0} = 6$.

25. A

$$\frac{\sin(90^\circ + \theta)}{\sin(180^\circ - \theta)\tan(270^\circ - \theta)} - \cos^2(180^\circ - \theta)$$
$$= \frac{\cos\theta}{\sin\theta \times \frac{1}{\tan\theta}} - \cos^2\theta$$
$$= 1 - \cos^2\theta$$
$$= \sin^2\theta$$

26. B

$$\frac{1}{3\sin^2 x + 2\cos^2 x} = \frac{1}{3(1-\cos^2 x) + 2\cos^2 x}$$
$$= \frac{1}{3 - \cos^2 x}$$

When $\cos^2 x = 0$, $\frac{1}{3-\cos^2 x} = \frac{1}{3}$.

When $\cos^2 x = 1$, $\frac{1}{3-\cos^2 x} = \frac{1}{2}$.

Required maximum value = $\frac{1}{2}$.

27. D

$$\frac{\sin(270^\circ - x)\cos(90^\circ - x)}{1 - \sin^2 x}$$

$$= \frac{-\cos x \sin x}{\cos^2 x}$$

$$= -\tan x$$

28. D

$$\frac{\tan 45^\circ}{1 - \cos\theta} + \frac{2\sin 30^\circ}{1 + \sin(90^\circ + \theta)} = \frac{1}{1 - \cos\theta} + \frac{1}{1 + \cos\theta}$$

$$= \frac{1 + \cos\theta + 1 - \cos\theta}{1 - \cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

29. D

$$\frac{2\cos\theta + 5}{\cos\theta + 2} = \frac{2(\cos\theta + 2) + 1}{\cos\theta + 2} = 2 + \frac{1}{\cos\theta + 2}$$

When $\cos\theta = -1$, $\dfrac{2\cos\theta + 5}{\cos\theta + 2} = 3$

When $\cos\theta = 1$, $\dfrac{2\cos\theta + 5}{\cos\theta + 2} = \dfrac{7}{3}$

Greatest value = 3

30. A

$$\frac{\cos(180^\circ - \theta)}{\tan(90^\circ - \theta)} - \sin(-\theta) = \frac{-\cos\theta}{\frac{1}{\tan\theta}} + \sin\theta = -\sin\theta + \sin\theta = 0$$

31. A

Since $-1 \le \cos 3x \le 1$, the maximum and minimum values of $f(x)$ are $a+b$ and $a-b$ respectively.

$$\begin{cases} a + b = 1 \\ a - b = \dfrac{1}{2} \end{cases} \quad \rightarrow \quad a = \frac{3}{4} \text{ and } b = \frac{1}{4}$$

32. B

$$[\sin(90^\circ + \theta) + 1][\cos(360^\circ - \theta) - 1]$$

$$= (\cos\theta + 1)(\cos\theta - 1)$$

$$= \cos^2\theta - 1$$

$$= -\sin^2\theta$$

33. A

$$\sin(360^\circ - \theta)\tan(270^\circ + \theta)$$

$$= (-\sin\theta)\left(-\frac{1}{\tan\theta}\right)$$

$$= \sin\theta \times \frac{\cos\theta}{\sin\theta}$$

$$= \cos\theta$$

34. $\boxed{\text{A}}$

$$\frac{\sin(-\theta)\sin(180^\circ+\theta)}{\cos(270^\circ-\theta)\cos(180^\circ-\theta)} = \frac{-\sin\theta(-\sin\theta)}{(-\sin\theta)(-\cos\theta)}$$
$$= \tan\theta$$

35. $\boxed{\text{C}}$

$$\frac{2\sin(90^\circ+\theta)-4\sin(180^\circ-\theta)}{4\cos\theta+2\sin\theta}$$
$$= \frac{2\cos\theta-4\sin\theta}{4\cos\theta+2\sin\theta}$$
$$= \frac{2-4\tan\theta}{4+2\tan\theta}$$
$$= \frac{2-4\left(-\frac{2}{5}\right)}{4+2\left(-\frac{2}{5}\right)}$$
$$= \frac{9}{8}$$

36. $\boxed{\text{C}}$

The numbers of dots are formed by +5, +7, +9, . . .
The sequence of numbers of dots is 6, 11, 18, 27, 38, 51, 66, . . .
Required number is 66.

37. $\boxed{\text{C}}$

The numbers are formed by +6, +10, +14, . . .
The sequence of numbers of dots is 1, 7, 17, 31, 49, 71, 97, . . .
Required number is 97.

38. $\boxed{\text{C}}$

The numbers of dots are formed by +3, +4, +5, . . .
The sequence of numbers of dots is 2, 5, 9, 14, 20, 27, 35, . . .
Required number is 35.

39. $\boxed{\text{A}}$

The numbers of dots are formed by +5, +7, +9, . . .
The sequence of numbers of dots is 3, 8, 15, 24, 35, 48, 63, . . .
Required number is 63.

40. C

Put $n = 3$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_5 = a_3 + a_4$$
$$70 = 4 + a_4$$
$$a_4 = 66$$

Put $n = 4$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_6 = a_4 + a_5$$
$$a_6 = 66 + 70$$
$$a_6 = 136$$

41. B

Put $n = 3$ into $a_n = 2a_{n-2} + a_{n-1}$.

$$a_3 = 2a_1 + a_2$$
$$a_3 = 2(2) + 5$$
$$a_3 = 9$$

Put $n = 4$, $n = 5$, $n = 6$ and $n = 7$ into $a_n = 2a_{n-2} + a_{n-1}$.

$a_4 = 2a_2 + a_3$	$a_5 = 2a_3 + a_4$	$a_6 = 2a_4 + a_5$	$a_7 = 2a_5 + a_6$
$a_4 = 2(5) + 9$	$a_5 = 2(9) + 19$	$a_6 = 2(19) + 37$	$a_7 = 2(37) + 75$
$a_4 = 19$	$a_5 = 37$	$a_6 = 75$	$a_7 = 149$

42. C

The numbers of dots are formed by +4, +4, +4, . . .
The sequence of numbers of dots is 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, . . .
Required number is 47.

43. C

Denote the nth term of the sequence by a_n.
We have $a_4 = 34$, $a_7 = 144$ and $a_{n+2} = a_n + a_{n+1}$ for any positive integers n.
Let $a_3 = x$.
Put $n = 3$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_5 = a_3 + a_4$$
$$a_5 = x + 34$$

Put $n = 4$ and $n = 5$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_6 = a_4 + a_5 \qquad a_7 = a_5 + a_6$$
$$a_6 = 34 + (x + 34) \qquad 144 = (x + 34) + (x + 68)$$
$$a_6 = x + 68 \qquad x = 21$$

44. D

The numbers are formed by +2, +4, +6, . . .
The sequence is 4, 6, 10, 16, 24, 34, 46, 60.
Required number is 60.

45. C

The numbers of dots are formed by +5, +5, +5, . . .
The sequence of numbers of dots is 11, 16, 21, 26, 31, 36, 41, . . .
Required number is 41.

46. D

The numbers are formed by +2, +4, +6, ...
The sequence of number of dots is 3, 5, 9, 15, 23, 33, 45, . . .
Required number is 45.

47. C

The numbers of dots are formed by +4, +7, +10, +13, . . .
The sequence of numbers of dots is 1, 5, 12, 22, 35, 51, 70, 92, . . .
Required number is 92.

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