REG-CP2A-2425-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. A	3. D	4. C	5. A
6. A	7. D	8. D	9. B	10. D
11. B	12. B	13. D	14. A	15. C
16. D	17. A	18. D	19. A	20. D
21. A	22. C	23. D	24. A	25. B
26. A	27. D	28. B	29. C	30. C
31. C	32. A	33. D	34. D	35. D
36. B	37. D	38. B	39. A	40. A
41. C	42. B	43. D	44. B	45. B
46. D	47. A	48. B	49. C	50. D
51. B	52. A	53. C	54. B	55. C
56. B	57. A	58. D	59. B	60. D
61. C	62. D	63. C	64. B	65. B
66. D	67. C	68. A	69. A	70. C
71. A	72. A	73. D	74. C	75. C
76. D	77. A	78. C		

1. C

$$(2x-k)^2-(2x-k)(x-k)=0$$
$$(2x-k)(2x-k-x+k)=0$$
$$x=\frac{k}{2}\quad\text{or}\quad 0$$

2. A

$$x^2+4x+k=2$$
$$x^2+4x+(k-2)=0$$

The equation has two distinct real roots.

$$\Delta=4^2-4(1)(k-2)>0$$
$$-4k+24>0$$
$$k<6$$

3. D

The equation holds for $x = k$.

When $x = k - 1$,

$$(x-1)(x-2) = (k-2)(k-3)$$
$$\neq (k-1)(k-2)$$

When $x = 3 - k$,

$$(x-1)(x-2) = (2-k)(1-k)$$
$$= (k-1)(k-2)$$

Thus, $x = k$ or $3 - k$.

4. C

$$(x-k)^2 = 4k^2$$
$$x - k = \pm 2k$$
$$x = -k \quad \text{or} \quad 3k$$

5. A

β is a root.

$$4\beta^2 - 5\beta - 1 = 0$$
$$4\beta^2 - 5\beta = 1$$

$$7 + 10\beta - 8\beta^2 = 7 - 2(4\beta^2 - 5\beta)$$
$$= 7 - 2(1)$$
$$= 5$$

6. A

β is a root.

$$2\beta^2 - 3\beta - k = 0$$
$$2\beta^2 - 3\beta = k$$

We have $6\beta - 4\beta^2 = -2(2\beta^2 - 3\beta) = -2k$.

$$(6\beta - 4\beta^2)^2 + 2\beta^2 - 3\beta - 14 = 0$$
$$(-2k)^2 + k - 14 = 0$$
$$4k^2 + k - 14 = 0$$
$$k = -2 \quad \text{or} \quad \frac{7}{4}$$

7. D

$2x^2 + 5x + k - 3 = 0$

$$\Delta = 5^2 - 4(2)(k-3) \ge 0$$
$$-8k + 49 \ge 0$$
$$k \le \frac{49}{8}$$

8. D

$$(x-2a)(a-x-1) = 3(2a-x)^2$$
$$(x-2a)(a-x-1) - 3(x-2a)^2 = 0$$
$$(x-2a)[(a-x-1) - 3(x-2a)] = 0$$
$$(x-2a)(-4x+7a-1) = 0$$
$$x = 2a \quad \text{or} \quad \frac{7a-1}{4}$$

9. B

$(x-b)^2 = a^2 - 2ab + b^2$

$(x-b)^2 = (a-b)^2$

$x - b = \pm(a-b)$

$x = a \quad \text{or} \quad -a + 2b$

10. D

$x - a = (x-a)(x-b)$

$x = a \quad \text{or} \quad 1 = x - b$

$x = a \quad \text{or} \quad x = b + 1$

11. B

Let $BE = x$ cm.

$AD = \sqrt{100} = 10$ cm

$$42 = 100 - \frac{(10)(10-x)}{2} - \frac{(x)(x)}{2} - \frac{(10-x)(10)}{2}$$
$$0 = -\frac{x^2}{2} + 10x - 42$$
$$x = 6 \quad \text{or} \quad 14 \text{ (rejected)}$$

12. B

$x^2 + ax + (a-1) = 0.$

$$\Delta = a^2 - 4(a-1) = 0$$
$$a^2 - 4a + 4 = 0$$
$$a = 2$$

13. D

$$(x-c)(x-4c)=(3c-x)(x-4c)$$
$$(x-4c)[(x-c)-(3c-x)]=0$$
$$(x-4c)(2x-4c)=0$$
$$x=2c \quad \text{or} \quad 4c$$

14. A

$$2x^2+8x=p+5$$
$$2x^2+8x-(p+5)=0$$

The equation has two distinct real roots.

$$\Delta=8^2-4(2)[-(p+5)]>0$$
$$8p+104>0$$
$$p>-13$$

15. C

$$\Delta=k^2-4(8k+36)=0$$
$$k^2-32k-144=0$$
$$k=-4 \quad \text{or} \quad 36$$

16. D

Consider $\begin{cases} 6a-3b+9c=9 \\ 2a+3b+5c=5 \end{cases}$ such that the ratio condition is satisfied.

c is a free variable. Solving the system by putting $c=2$, we have $a=-\frac{7}{4}$ and $b=-\frac{1}{2}$.

So, $a:b=7:2$.

(Note: you will get $a=b=0$ if you put $c=1$. In this case the ratio cannot be obtained. Substitute other values for the answer.)

17. A

$$\frac{3}{2a}=\frac{4}{3b}=\frac{7}{5c} \quad \Rightarrow \quad \frac{\left(\frac{3}{2}\right)}{a}=\frac{\left(\frac{4}{3}\right)}{b}=\frac{\left(\frac{7}{5}\right)}{c}$$

So, $a:b:c=\frac{3}{2}:\frac{4}{3}:\frac{7}{5}$. Since they are positive numbers, $a>c>b$.

18. D

Required ratio $=\frac{1}{1-30\%}:\frac{1}{1-25\%}=15:14$

19. A

Put $x=6$. Then $y=8$ and $x=3$, and $\frac{y-z}{y+z}=\frac{8-3}{8+3}=\frac{5}{11}$

20. D

Let $a = 6$, then $b = 3$ and $c = 2$.

$(a + b) : (b + c) : (c + a) = 9 : 5 : 8$

21. A

Let the required cost be $\$b$/L.

$$3(42) + 2b = 36(2 + 3)$$
$$b = 27$$

22. C

Let $c = 4$. Then $a = 3$ and $b = 10$.

$(3a + c) : (a + 3b) = 13 : 33$

23. D

Let $p = 9k$ and $q = 8k$, where k is a non-zero constant.

$$\frac{3r - p}{q + r} = \frac{3}{23}$$
$$\frac{3r - 9k}{8k + r} = \frac{3}{23}$$
$$69r - 207k = 24k + 3r$$
$$r = \frac{7k}{2}$$

Thus, we have $p : r = 9k : \frac{7k}{2} = 18 : 7$.

24. A

$$3x = 8y$$
$$\frac{3}{\left(\frac{1}{x}\right)} = \frac{8}{\left(\frac{1}{y}\right)}$$

So, $\frac{1}{x} : \frac{1}{y} = 3 : 8$ and only option A satisfies this.

25. B

Let $\begin{cases} 2u + 4v = 2 \\ 5u - 2v = 3 \end{cases}$. Then $u = \frac{2}{3}$ and $v = \frac{1}{6}$.

We have $u : v = \frac{2}{3} : \frac{1}{6} = 4 : 1$.

26. $\boxed{\text{A}}$

Compare coefficients of x.

$$2q = r$$
$$\frac{q}{r} = \frac{1}{2}$$

Compare constant term.

$$6p = 5r$$
$$\frac{p}{r} = \frac{5}{6}$$

Thus, $p : q : r = 5 : 3 : 6$.

27. $\boxed{\text{D}}$

Let $\begin{cases} 3b - 4c = 1 \\ 4b - 3c = 2 \end{cases}$. Then $b = \dfrac{5}{7}$ and $c = \dfrac{2}{7}$.

We have $a = c \times \dfrac{2}{1} = \dfrac{4}{7}$.

$(a + b) : (b + c) = \dfrac{9}{7} : \dfrac{7}{7} = 9 : 7$

28. $\boxed{\text{B}}$

Rewrite the equation into $\dfrac{\left(\frac{7}{11}\right)}{p} = \dfrac{\left(\frac{11}{13}\right)}{q} = \dfrac{\left(\frac{13}{17}\right)}{r}$

So, $p : q : r = \dfrac{7}{11} : \dfrac{11}{13} : \dfrac{13}{17} \approx 0.636 : 0.846 : 0.765$.

Since p, q and r are negative, $q < r < p$

29. $\boxed{\text{C}}$

Let x L be the required amount of water.

$$\frac{2(0.25) + x}{2 + x} = 0.75$$
$$x = 4$$

30. $\boxed{\text{C}}$

Let $y = 20$, then we have $x = 35$ and $z = 16$.

$$(x - y) : (y - z) = (35 - 20) : (20 - 16)$$
$$= 15 : 4$$

31. C

Let $2p = 4q = 5r = k$, where k is a non-zero constant.
Then $p = \frac{k}{2}$, $q = \frac{k}{4}$ and $r = \frac{k}{5}$.

$$(p+q) : (q+r) = \left(\frac{k}{2} + \frac{k}{4}\right) : \left(\frac{k}{4} + \frac{k}{5}\right)$$
$$= \frac{3k}{4} : \frac{9k}{20}$$
$$= 5 : 3$$

Let $2p = 4q = 5r = 20$. Then $p = 10$, $q = 5$ and $r = 4$.

$$(p+q) : (q+r) = (10+5) : (5+4)$$
$$= 5 : 3$$

32. A

$$\frac{81(5) + x(4)}{5+4} = 73$$
$$x = 63$$

33. D

Let $b = 12k$. Then $a = 8k$ and $c = 9k$.

$k = \frac{87}{12+8+9} = 3$ and $b = 12(3) = 36$

34. D

$$\text{Total cost} = 3(22) + 4(36)$$
$$= \$210$$
$$\text{Average cost} = \frac{210}{3+4}$$
$$= \$30/\text{kg}$$

35. D

Let $a = 3k$ and $b = 2k$, where k is a non-zero number.

$$5(3k) = 4c$$
$$c = \frac{15k}{4}$$
$$\frac{a+b}{b+c} = \frac{3k+2k}{2k+\frac{15k}{4}}$$
$$= \frac{20}{23}$$

Alternative solution

Let $a = 12$. Then $b = 8$ and $c = 15$.

$$\frac{a+b}{b+c} = \frac{12+8}{8+15}$$
$$= \frac{20}{23}$$

36. B

Ratio of number of purple : pink : yellow = 20 : 24 : 9

Number of pink dresses = $15\,741 \times \dfrac{24}{20+24+9} = 7128$

37. D

Let $z = as + \dfrac{b}{t^2}$, where a and b are non-zero constants.

$$\begin{cases} 13 = a + b \\ -5 = 3a + \dfrac{b}{4} \end{cases}$$

Solving, we have $a = -3$ and $b = 16$.

$z = -3(-2) + \dfrac{16}{(-4)^2} = 7$

38. B

Let $y = a + bx^2$, where a and b are non-zero constants.

$$\begin{cases} 5 = a + b \\ 8 = a + 4b \end{cases}$$

Solving, we have $a = 4$ and $b = 1$.

When $x = 3$, $y = 4 + 1(3)^2 = 13$.

39. A

Let $z = \dfrac{k}{\sqrt[3]{y}}$, where k is a non-zero constant.

Percentage change

$= \dfrac{\dfrac{k}{\sqrt[3]{1.08y}} - \dfrac{k}{\sqrt[3]{y}}}{\dfrac{k}{\sqrt[3]{y}}} \times 100\%$

$\approx -2.53\%$

Let $z = \dfrac{k}{\sqrt[3]{y}}$, where k is a non-zero constant.

$z_r = \dfrac{k}{\sqrt[3]{1.08}}$

$z_r \approx 0.9747$

Percentage change

$= (z_r - 1) \times 100\%$

$\approx -2.53\%$

40. A

Let $y = \dfrac{kx}{\sqrt{z}}$, where k is a non-zero constant.

Then $k = \dfrac{y\sqrt{z}}{x}$.

Therefore, $\dfrac{x}{y\sqrt{z}} = \dfrac{1}{k}$ must be a constant.

41. C

Let $z = ay + \dfrac{b}{y^2}$, where a and b are non-zero constants.

$$\begin{cases} -1 = 2a + \dfrac{b}{4} \\ 5 = -a + b \end{cases}$$

Solving, we have $a = -1$ and $b = 4$.

When $y = 1$, $z = -1 + \dfrac{4}{(-1)^2} = 3$

42. B

Let $a = mb^2$ and $\sqrt{b} = \dfrac{n}{c}$, where m and n are non-zero constants.

Then $c = \dfrac{n}{\sqrt{b}} = \dfrac{nm^{\frac{1}{4}}}{a^{\frac{1}{4}}} = \dfrac{p}{a^{\frac{1}{4}}}$, where $p = nm^{\frac{1}{4}}$ is a constant.

Percentage change $= \dfrac{\dfrac{p}{(1.3a)^{\frac{1}{4}}} - \dfrac{p}{a^{\frac{1}{4}}}}{\dfrac{p}{a^{\frac{1}{4}}}} \approx -6.35\%$

43. D

Let $z = \dfrac{kx^2}{\sqrt{y}}$, where k is a non-zero constant.

$$\begin{aligned} z' &= \frac{(1.1)^2}{\sqrt{1 - 0.36}} \\ &= 1.5125 \end{aligned}$$

z is increased by 51.25%.

44. B

Let $y = \dfrac{kx^2}{\sqrt{w}}$, where k is a non-zero constant.

Then $k = \dfrac{y\sqrt{w}}{x^2}$.

Thus, $\dfrac{x^4}{wy^2} = \dfrac{1}{k^2}$ is a constant.

45. B

Let $z = \dfrac{ky}{x^2}$, where k is a non-zero constant.

Percentage change $= \dfrac{\dfrac{k(1.35y)}{(1.25x)^2} - \dfrac{ky}{x^2}}{\dfrac{ky}{x^2}} \times 100\% = -13.6\%$

46. D

Let $x = \frac{ky^3}{\sqrt{z}}$, where k is a non-zero constant.

$$k = \frac{x\sqrt{z}}{y^3}$$

$$k^2 = \frac{x^2 z}{y^6}$$

$$\frac{y^6}{x^2 z} = \frac{1}{k^2} = \text{constant}$$

47. A

Let $p = \frac{kr}{q^2}$, where k is a non-zero constant.

Percentage change

$$= \frac{\frac{k(0.9r)}{(1.2q)^2} - \frac{kr}{q^2}}{\frac{kr}{q^2}} \times 100\%$$

$$= -37.5\%$$

> Let $p = \frac{kr}{q^2}$, where k is a non-zero constant.
>
> $$\frac{p_2}{p_1} = \frac{1 - 10\%}{(1 + 20\%)^2}$$
>
> $$= 0.625$$
>
> p is decreased by 37.5%.

48. B

Let $w = \frac{k\sqrt{u}}{v}$, where k is a non-zero constant. Then $k = \frac{vw}{\sqrt{u}}$.

So, $\frac{wv}{\sqrt{u}} = k$ is a constant.

49. C

Let $z = \frac{kx}{\sqrt{y}}$, where k is a non-zero constant.

I. ✗. The statement is false when $k \neq 1$.

II. ✓. $k = \frac{z\sqrt{y}}{x} \Rightarrow \frac{yz^2}{x^2} = k^2 = \text{constant}$

III. ✓. $\frac{0.5z\sqrt{4y}}{x} = \frac{z\sqrt{y}}{x} = k$. The percentage changes are correct.

50. D

Let $z = \frac{kx}{y^2}$, where k is a non-zero constant.

Percentage change in $z = \dfrac{\frac{k(0.9x)}{(0.75y)^2} - \frac{kx}{y^2}}{\frac{kx}{y^2}}$

$= +60\%$

51. B

Let $p = \frac{ks^2}{t}$, where k is a non-zero constant.

Then $s = \sqrt{\frac{pt}{k}}$.

Percentage change in s

$= \dfrac{\sqrt{\frac{(0.25p)(1.44t)}{k}} - \sqrt{\frac{pt}{k}}}{\sqrt{\frac{pt}{k}}} \times 100\%$

$= -40\%$

> Let $p = \frac{ks^2}{t}$, where k is a non-zero constant.
>
> $0.25 = \frac{s_r^2}{1.44}$
>
> $s_r = 0.6$
>
> Thus, s is decreased by 40%.

52. A

Let $c = \frac{ka^2}{b}$, where k is a non-zero constant.

Percentage change $= \dfrac{\frac{k(0.4a)^2}{1.6b} - \frac{ka^2}{b}}{\frac{ka^2}{b}} \times 100\%$

$= -90\%$

53. C

Let $z = \frac{ky}{x^3}$, where k is a non-zero constant.

Then $k = \frac{x^3 z}{y}$ is a constant. So, $\frac{y}{x^3 z} = k^{-1}$ is also a constant.

54. B

Let $z = \dfrac{kx^3}{\sqrt{y}}$, where k is a non-zero constant.

Then $k = \dfrac{\sqrt{y}z}{x^3}$.

So, $\dfrac{yz^2}{x^6} = k^2$ is a constant.

55. C

Let $y = \dfrac{k}{x^2}$, where k is a non-zero constant.

$$\frac{y_2}{y_1} = \frac{1}{(1.25)^2}$$
$$= 0.64$$

y is decreased by 36%.

56. B

Let $x = \dfrac{kz}{\sqrt{y}}$, where k is a non-zero constant.

$$\text{Percentage change} = \frac{\frac{k(0.88z)}{\sqrt{1.21y}} - \frac{kz}{\sqrt{y}}}{\frac{kz}{\sqrt{y}}} \times 100\%$$
$$= -20\%$$

57. A

Let $y = \dfrac{k\sqrt{x}}{z^2}$, where k is a non-zero constant.

Then $k = \dfrac{yz^2}{\sqrt{x}}$ and so $\dfrac{x}{y^2z^4} = k^{-2}$ is a constant.

58. D

Let $z = \dfrac{kx}{y^2}$, where k is a non-zero constant. Then $k = \dfrac{y^2z}{x}$ and

$$\frac{4^2(3)}{4} = \frac{9^2(2)}{x}$$
$$x = \frac{27}{2}$$

59. B

Let $x = \dfrac{k\sqrt{y}}{z^2}$, where k is a non-zero constant.

$$\text{Percentage change} = \frac{\frac{k\sqrt{1.44y}}{(0.8z)^2} - \frac{k\sqrt{y}}{z^2}}{\frac{k\sqrt{y}}{z^2}} \times 100\%$$
$$= 87.5\%$$

60. D

Let $y = ax^3$ and $z = \dfrac{b}{y^2}$, where a and b are non-zero constants.

Then $z = \dfrac{b}{a^2x^6}$

I. ✓. $zy^2 = b$ is a constant.

II. ✓. $zx^6 = \dfrac{b}{a^2}$ is a constant.

III. ✓. $\dfrac{zy^4}{x^6} = \dfrac{(zy^2)^2}{zx^6} = \dfrac{b^2}{\left(\frac{b}{a^2}\right)}$ is a constant.

61. C

$$-2 = 8\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 - 5$$

$$a = 8$$

$$g(1) = 8(1)^3 + 8(1)^2 - 5$$

$$= 11$$

62. D

$$f(-9) = 2(-9)^2 + 17(-9) + k = 0$$

$$k = -9$$

$$\text{Remainder} = 2\left(\frac{-1}{2}\right)^2 + 17\left(\frac{-1}{2}\right) - 9$$

$$= -17$$

63. C

$$\text{Remainder} = f\left(\frac{3}{2}\right)$$

64. B

$$-2 = 2\left(-\frac{1}{2}\right)^3 - 5\left(-\frac{1}{2}\right)^2 - a^2\left(-\frac{1}{2}\right) + a$$

$$0 = \frac{a^2}{2} + a + \frac{1}{2}$$

$$a = -1$$

$$\text{Required remainder} = 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) - 1$$

$$= -7$$

65. B

$4^3 + k(4)^2 + 2k(4) + 8 = 0$

$k = -3$

$P(x) = x^3 - 3x^2 - 6x + 8$

By calculator, $P(1) = P(-2) = 0$. So, $(x - 1)(x + 2)$ is a factor of $P(x)$.

Only option B satisfies this.

66. D

$0 = 3(2)^3 - 5(2)^2 + k(2) + 20$

$k = -12$

67. C

$0 = f(-3) = 2(-3)^3 + 5(-3)^2 + k$

$k = 9$

Remainder $= f(-2) = 2(-2)^3 + 5(-2)^2 + 9 = 13$

68. A

$3^3 + a(3)^2 - 3b = 6$

$3a - b = -7$

$3a - b + 7 = (-7) + 7 = 0$

69. A

$f(3x - 2)$ is divisible by x.

$0 = f(0 - 2)$

$f(-2) = 0$

Thus, $f(x)$ is divisible by $x + 2$.

70. C

Remainder $= (-1)^{2018} + (-1)^{2017} + (-1)^{2016} + \ldots + (-1)$

$= (1 - 1) + (1 - 1) \ldots + (1 - 1)$

$= 0$

71. A

$3(2)^2 - 2(2) + k = 0$

$k = -8$

$3x^2 - 2x - 8 = (x - 2)(3x + 4)$. It is also divisible by $3x + 4$.

72. A

$0 = 1^3 + 3(1) - k$

$k = 4$

Remainder $= (-2)^3 + 3(-2) - 4$

$= -18$

73. D

$0 = k^3 - (k-1)k^2 + 2k - 3$

$0 = k^2 + 2k - 3$

$k = -3 \quad \text{or} \quad 1$

74. C

$x^2 - x - 12 = (x-4)(x+3)$

Therefore, $(x-4)$ and $(x+3)$ are factors of $P(x)$.

Thus, $P(4) = P(-3) = 0$.

75. C

$0 = k(3)^3 + 2k(3)^2 + 90$

$k = -2$

Remainder $= -2(-1)^3 - 4(-1)^2 + 90$

$= 88$

76. D

$f(5) = 0 = 2(5)^2 - 13(5) + k$

$k = 15$

Required remainder $= f\left(-\frac{1}{2}\right)$

$= 2\left(-\frac{1}{2}\right)^2 - 13\left(-\frac{1}{2}\right) + 15$

$= 22$

77. A

$p(-k) = -k^3 + k^3 - 4k - 16 = 0$

$k = -4$

Remainder $= (-2)^3 - 4(-2)^2 + 4(-2) - 16$

$= -48$

78. C

Let $f(x) = (x-2)q(x)$, where $q(x)$ is a polynomial.

$f(x-3) = (x-5)q(x-3)$ is divisible by $x - 5$.

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