REG-CP1B-2425-ASM-SET 4-MATH
Suggested solutions

Conventional Questions

1. (a) $f(x) = 3x^2 - 12kx + 11k^2 - 15$

$= 3[x - 2(2k)x + (2k)^2] - k^2 - 15$ 1M

$= 3(x - 2k)^2 - k^2 - 15$

Required coordinates are $\left(2k,\ -k^2 - 15\right)$. 1A

(b) The coordinates of the vertex of the graph of $y = g(x)$ are $\left(3k,\ k^2 + 30\right)$. 1A

The maximum value of $g(x)$ is $k^2 + 30$, and $k^2 + 30 > 0$.

The claim is disagreed. 1A

2. (a) $f(x) = x^2 - 8kx + 6x + 16k^2 - 19k + 11$

$= [x^2 - 2(x)(4k - 3) + (4k - 3)^2] + 5k + 2$ 1M

$= [x - (4k - 3)]^2 + 5k + 2$

The coordinates of the vertex are $(4k - 3,\ 5k + 2)$. 1A

(b) The coordinates of P and Q are $(4k - 3,\ 5k + 2)$ and $(4k + 6,\ 5k + 8)$ respectively. 1A

(Slope of OH)(slope of PQ)

$= \dfrac{12 - 0}{-9 - 0} \times \dfrac{(5k + 8) - (5k + 2)}{(4k + 6) - (4k - 3)}$ 1M

$= -\dfrac{4}{3} \times \dfrac{2}{3}$

$= -\dfrac{8}{9} \neq -1$

It is not possible. 1A

3. (a) $p(x) = 4x^2 - 48ax + 146a^2 - 5$

$= 4[x^2 - 2(x)(6a) + (6a)^2] + 2a^2 - 5$ 1M

$= 4(x - 6a)^2 + 2a^2 - 5$

The coordinates of vertex are $\left(6a,\ 2a^2 - 5\right)$. 1A

(b) The coordinates of H are $\left(6a,\ 2a^2 - 5\right)$.

The coordinates of K are $\left(-2a,\ -2a^2 + 5\right)$. 1A

Note that $HR : RK = (6a - 0) : (0 + 2a) = 3 : 1$.

Let $(0,\ r)$ be the coordinates of R.

$\dfrac{(2a^2 - 5) - r}{r - (-2a^2 + 5)} = \dfrac{3}{1}$ 1M

$2a^2 - 5 - r = 3r + 6a^2 - 15$

$r = \dfrac{5}{2} - a^2$

The coordinates of R are $\left(0,\ \frac{5}{2} - a^2\right)$. 1A

4. (a) $g(x) = f(kx - 1)$ 1M

$= 2k^2x^2 - (k^2 + 4k)x + k + 2$ 1A

(b) $-2x + 3 = 2k^2x^2 - (k^2 + 4k)x + k + 2$

$0 = 2k^2x^2 - (k^2 + 4k - 2)x + k - 1$

$\Delta = (k^2 + 4k - 2)^2 - 4(2k^2)(k - 1)$ 1M

$= k^4 + 20k^2 - 16k + 4$

$= k^4 + 4k^2 + (16k^2 - 16k + 4)$

$= k^4 + 4k^2 + 4(2k - 1)^2$ 1M

> 0

Thus, L and Γ intersect at two distinct points. 1

5. (a) $f(x) = 3x^2 - 24kx + 200$

$= 3[x^2 - 2(x)(4k) + (4k)^2] + 200 - 48k^2$ 1M

$= 3(x - 4k)^2 + 200 - 48k^2$

The coordinates of P are $\left(4k,\ 200 - 48k^2\right)$. 1A

(b) (i) The coordinates of Q are $\left(4k - 8,\ 200 - 48k^2\right)$. 1A

The circumcentre lies on the perpendicular bisector of PQ.

$\frac{4k + (4k - 8)}{2} = 4$

$k = 2$ 1A

(ii) We have $\angle QPR = 90^\circ$ and QR is a diameter of circle PQR. 1M

The coordinates of Q are $(0,\ 8)$.

Let the coordinates of R be $(a,\ b)$.

$\frac{a + 0}{2} = 4$ and $\frac{b + 8}{2} = -8$ 1M

$a = 8$ $\quad$ $b = -24$

The coordinates of R are $(8,\ -24)$. 1A

6. (a) $f(x) = 2x^2 - (4k + 8)x + 2k^2 + 4k + 9$

$= 2[x^2 - 2(x)(k + 2) + (k + 2)^2] - 4k + 1$ 1M

$= 2[x - (k + 2)]^2 - 4k + 1$

The coordinates of the vertex are $(k + 2,\ -4k + 1)$. 1A

(b) (i) The coordinates of A and B are $(k + 2,\ -4k + 1)$ and $(3k + 6,\ 4k - 1)$ respectively. 1A

The coordinates of S are $(2k + 4,\ 0)$. 1A

(ii) Note that area of $\triangle OAS$ is half the area of $\triangle OAB$.

$$110 = 2 \times \frac{(4k-1)(2k+4)}{2}$$ 1M

$$0 = 8k^2 + 14k - 114$$

$$k = 3 \quad \text{or} \quad -\frac{19}{4} \text{ (rejected)}$$ 1A

7. (a) Let $f(x) = ax^2 + b(2x-7)$, where a and b are non-zero constants. 1A

$$\begin{cases} 10 = a(4)^2 + b(8-7) \\ 7 = a(7)^2 + b(14-7) \end{cases}$$ 1M

Solving, we have $a = 1$ and $b = -6$. 1A+1A

Thus, $f(x) = x^2 - 6(2x-7) = x^2 - 12x + 42$.

(b) (i) $f(x) = x^2 - 12x + 42$

$= [x^2 - 2(6)(x) + 6^2] + 6$ 1M

$= (x-6)^2 + 6$

Required coordinates are $(6,\ 6)$. 1A

(ii) $(-2,\ 0)$ 1A

(iii) (slope of QS)(slope of RS)

$$= \frac{7-6}{-1-6} \times \frac{7-0}{-1+2}$$ 1M

$= -1$

We have $\angle QS \perp RS$ and P is at point S.

Thus, $PQ \perp PR$.

The claim is agreed. 1A

8. (a) $f(x+k) = (x+k)^2 - 2(x+k) + 2$ and $f(-x) = (-x)^2 - 2(-x) + 2$ 1M

$= x^2 + (2k-2)x + k^2 - 2k + 2$ $= x^2 + 2x + 2$

Compare the coefficients of x and the constant terms.

We have $2k - 2 = 2$ and $k^2 - 2k + 2 = 2$.

Solving, we have $k = 2$. 1A

(b) $f(x) = f(-x)$

$x^2 - 2x + 2 = x^2 + 2x + 2$ 1M

$x = 0$

The coordinates of A are $(0,\ 2)$. 1A

(c) The coordinates of B and C are $(1,\ 1)$ and $(-1,\ 1)$ respectively. 1A

(slope of AB)(slope of AC)

$$= \frac{2-1}{0-1} \times \frac{2-1}{0+1}$$ 1M

$= -1$

We have $AB \perp AC$.

The orthocentre of $\triangle ABC$ is at point A, which is not outside the triangle.
The claim is disagreed. 1A

9. (a) $f(x) = x^2 - 4kx + 3k^2 + 25$
$= [x^2 - 2(2k)x + (2k)^2] - k^2 + 25$ 1M
$= (x - 2k)^2 - k^2 + 25$
Required coordinates are $\left(2k,\ -k^2 + 25\right)$. 1A

(b) $-k^2 + 25 > 0$ 1M
$-5 < k < 5$ 1A

(c) The coordinates of B and C are $\left(2k,\ -k^2 + 15\right)$ and $\left(2k - 24,\ -k^2 + 15\right)$ respectively. 1A
The circumcentre of $\triangle ABC$ lies on the perpendicular bisector of BC.
$$x = \frac{(2k) + (2k - 24)}{2}$$
$$x = 2k - 12$$
Since $-5 < k < 5$, we have $x = 2k - 12 < -2 < 0$. 1M
It is not possible for the circumcentre of $\triangle ABC$ to be on the right of the y-axis. 1A

10. (a) $f(x) = 3x^2 - 6(k - 1)x + 4k^2 - 6k - 6$
$= 3[x^2 - 2(k - 1)x + (k - 1)^2] + k^2 - 9$ 1M
$= 3[x - (k - 1)]^2 + k^2 - 9$
Required coordinates are $\left(k - 1,\ k^2 - 9\right)$. 1A

(b) (i) The graph of $y = f(x)$ is reflected about the x-axis;
then is translated upwards by 6 units. 1A

The graph of $y = f(x)$ is translated downwards by 6 units; then is reflected about the x-axis. 1A

(ii) The coordinates of Q are $\left(k - 1,\ -k^2 + 15\right)$.
$$-k^2 + 15 > 0$$ 1M
$$-\sqrt{15} < k < \sqrt{15}$$
Note that k is positive. We have $0 < k < \sqrt{15}$. 1A

(iii) (1) We have $k = 3$, $P\,(2,\ 0)$ and $Q\,(2,\ 6)$. 1M
Note that $OP \perp PQ$.
The coordinates of F are $(2,\ 0)$. 1A
G is the mid-point of OQ.
The coordinates of G are $(1,\ 3)$. 1A
Since FG is a median of $\triangle OPQ$, H lies on FG.
Thus, F, G and H are collinear. 1

(2) Slope of $GP = \dfrac{3-0}{1-2} = -3 \neq -1$

We have $\angle GPO \neq 45^\circ$ and GP is not the angle bisector of $\angle OPQ$. 1M

The in-centre of $\triangle OPQ$ does not lie on the line GP.

The claim is disagreed. 1A

END OF PAPER