1. (a) $AD = CB$ *(property of square)*

$\angle ADR = 90°$ *(property of square)*

$\angle CBP = 90°$ *(property of square)*

$= \angle ADR$

$\angle CPB = \angle RAB$ *(corr. ∠s, AR//PC)*

$\angle ARD = \angle RAB$ *(alt. ∠s, AB//DC)*

$= \angle CPB$

$\triangle ADR \cong \triangle CBP$ *(AAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\angle ARD + \angle DAR + 90° = 180°$

$$\angle ARD + \frac{\angle ARD}{4} = 90°$$

$$\angle ARD = 72°$$

Since $\triangle ADR \cong \triangle CBP$, we have $\angle CPB = \angle ARD = 72°$. 1M

Note that $QC = AR = PC$. $\triangle CPQ$ is an isosceles triangle. 1M

$\angle CQB = \angle CPB = 72°$ 1A

2. (a) $\triangle ABE \sim \triangle CDE$ *(given)*

$\angle BAE = \angle DCE$ *(corr. ∠s, ~ △s)*

$AB // DC$ *(alt. ∠s equal)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\angle BDC = \angle ABD$ 1M

$\angle BDC + \angle ACD = \angle BEC$

$\angle BDC + 2\angle BDC = 75°$

$\angle BDC = 25°$

$$\angle ADC = \frac{180° - \angle ACD}{2}$$ 1M

$= 65°$

$\angle ADB = 65° - 25°$

$= 40°$ 1A

3. (a) $\angle ABC = 90^\circ$ *(given)*

$\angle BCD + 90^\circ = 180^\circ$ *(int. ∠s, AB//DC)*

$\angle BCD = 90^\circ = \angle ABC$

$\angle BAE = \angle CED$ *(given)*

$\triangle ABE \sim \triangle ECD$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $\angle AEB = \angle EDC$ 1M

$\angle AED = 180^\circ - \angle AEB - \angle DEC$

$= 180^\circ - \angle EDC - \angle DEC$

$= \angle DCE$

$= 90^\circ$

$\triangle ADE$ is a right-angled triangle. 1A

(ii) $\dfrac{AE}{DE} = \dfrac{AB}{CE}$ 1M

$\dfrac{AE}{28} = \dfrac{12}{16}$

$AE = 21\text{ cm}$

Required area $= \dfrac{(21)(28)}{2}$

$= 294\text{ cm}^2$ 1A

4. (a) (i) $AB = AD$ *(given)*

$\angle ABC = \angle ADC = 90^\circ$ *(given)*

$AC = AC$ *(common side)*

$\triangle ABC \cong \triangle ADC$ *(RHS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) $\triangle ABC \cong \triangle ADC$ *(proved)*

$CD = BC$ *(corr. sides, ≅ △s)*

$\angle ECD = \angle ECB$ *(corr. ∠s, ≅ △s)*

$CE = CE$ *(common side)*

$\triangle BCE \cong \triangle DCE$ *(SAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\angle BFD + \angle FBC = 90° + 90° = 180°$

Therefore, $DE // BC$. 1M

We have $\angle DEC = \angle BCE$ and $\angle DEC = \angle BEC$.

Thus, we have $\angle BEC = \angle BCE$ and $BE = BC$. 1M

Then $DE = CD = CB = BE$ and $BCDE$ is a rhombus.

The claim is correct. 1A

5. (a) $\angle ABE = \angle CDE$ *(alt. ∠s, AB//DC)*

$\angle BAE = \angle DCE$ *(alt. ∠s, AB//DC)*

$\triangle ABE \sim \triangle CDE$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\frac{CE}{CD} = \frac{AE}{AB}$

$\frac{CE}{74} = \frac{175 - CE}{111}$

$111CE = 12\,950 - 74CE$

$CE = 70\,\text{cm}$

$DC^2 = 74^2 = 5476\,\text{cm}^2$

$DE^2 + CE^2 = 24^2 + 70^2 = 5476\,\text{cm}^2 = DC^2$ 1A

Thus, $\angle DEC = 90°$.

We have $\angle BEC = 180° - 90° = 90°$.

$\triangle BCE$ is a right-angled triangle. 1A

6. (a) $\angle BEC = \angle ECD$ *(given)*

$BE // CD$ *(alt. ∠s equal)*

$\angle BEA = \angle CDE$ *(corr. ∠s, BE//CD)*

$\angle BAE = \angle CED$ *(corr. ∠s, AB//EC)*

$\triangle ABE \sim \triangle ECD$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\frac{AB}{EC} = \frac{AE}{ED}$ 1M

$\frac{36}{12} = \frac{80 - ED}{ED}$

$36ED = 960 - 12ED$

$ED = 20\text{ cm}$

$CE^2 + CD^2 = 12^2 + 16^2 = 400\text{ cm}^2$

$ED^2 = 20^2 = 400\text{ cm}^2 = CE^2 + CD^2$ 1M

We have $\angle ECD = 90°$.

Since $\triangle ABE \sim \triangle ECD$, we have $\angle ABE = \angle ECD = 90°$.

Thus, $\triangle ABE$ is a right-angled triangle. 1A

7. (a) $\angle BAC = \angle ADC$ *(given)*

$\angle DAC = \angle ACB$ *(alt. ∠s, AD // BC)*

$\triangle ADC \sim \triangle CAB$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\frac{AC}{CB} = \frac{DC}{AB}$

$\frac{AC}{625} = \frac{168}{175}$ 1M

$AC = 600\text{ cm}$

$AB^2 + AC^2 = 175^2 + 600^2 = 390\,625\text{ cm}^2$

$BC^2 = 625^2 = 390\,625\text{ cm}^2 = AB^2 + AC^2$ 1M

We have $\angle BAC = 90°$.

Also, $\angle ADC = \angle BAC = 90°$.

The claim is agreed. 1A

8. (a) $\angle ABE = \angle ECF = 90°$ *(property of square)*

$\angle BEA + \angle ABE + \angle BAE = 180°$ *(∠ sum of △)*

$\angle BAE = 90° - \angle BEA$

$\angle AEF = 90°$ *(given)*

$\angle BEA + \angle AEF + \angle CEF = 180°$ *(adj. ∠s on st. line)*

$\angle CEF = 90° - \angle BEA$

$= \angle BAE$

$\triangle ABE \sim \triangle ECF$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\frac{BE}{CF} = \frac{AB}{EC}$

$\frac{48-12}{CF} = \frac{48}{12}$ 1M

$CF = 9\text{ cm}$

Required area

$= 48^2 - \frac{(48)(36)}{2} - \frac{(12)(9)}{2} - \frac{(48)(48-9)}{2}$ 1M

$= 450\text{ cm}^2$ 1A

9. (a) $\angle EFD = \angle AFB$ *(vert. opp. ∠s)*

$\angle EDF = 90° - \angle CBE$ *(given)*

$\angle ABC = 90°$ *(given)*

$\angle ABF = 90° - \angle CBE$

$= \angle EDF$

$\triangle DEF \sim \triangle BAF$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $\angle DEF = \angle BAF = 90°$ 1M

$\triangle BDE$ is a right-angled triangle. 1A

(ii) Let $DF = x$ cm. Then $AF = (25 - x)$ cm.

$\frac{DF}{BF} = \frac{EF}{AF}$

$\frac{x}{25} = \frac{6}{25-x}$ 1M

$25x - x^2 = 150$

$-x^2 + 25x - 150 = 0$

$x = 10$ or 15 (rejected)

$DE = \sqrt{10^2 - 6^2} = 8\text{ cm}$ 1M

$BD = \sqrt{(25+6)^2 + 8^2} = \sqrt{1025}\text{ cm}$

Required perimeter $= (25+6) + 8 + \sqrt{1025}$

$= (39 + 5\sqrt{41})\text{ cm}$ 1A

$\approx 71.0\text{ cm}$

10. (a) $\angle ABH = 90^\circ$ *(property of square)*

$\angle BCG = 90^\circ$ *(property of square)*

$= \angle ABH$

$AB = BC = CD$ *(property of square)*

$DG = HC$ *(given)*

$BH = BC - HC$

$= CD - DG$

$= CG$

$\triangle ABH \cong \triangle BCG$ *(SAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\triangle BHF$, $\triangle AHB$, $\triangle BGC$ 1A

(c) Note that $\triangle AHB \sim \triangle ABF$.

$$\frac{AB}{AF} = \frac{AH}{AB} \quad \textit{(corr. sides, } \sim \triangle\textit{s)}$$ 1M

$$AB^2 = AF(AF + AH)$$

$$AB^2 - AF^2 = AF \times FH$$ 1

11. (a) $DE = DE$ *(common side)*

$\angle DFE = 90^\circ$ *(given)*

$\angle DCE = 90^\circ$ *(property of rectangle)*

$= \angle DFE$

$AD = AE$ *(given)*

$\angle AED = \angle ADE$ *(base $\angle$s, isos. $\triangle$)*

$AD // BC$ *(property of rectangle)*

$\angle CED = \angle ADE$ *(alt. $\angle$s, $AD // BC$)*

$= \angle AED$

$\triangle CDE \cong \triangle FDE$ *(AAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) $AF = AE - FE = AE - CE = 5 - 1 = 4\,\text{cm}$ 1M

$DF = \sqrt{AD^2 - AF^2} = \sqrt{5^2 - 4^2} = 3\,\text{cm}$ 1M

Area of $\triangle ADF = \frac{1}{2}(4)(3) = 6\,\text{cm}^2$ 1A

12. (a) We have $CE = AD = 7\,\text{cm}$ and $DF = CG$. 1M

$CG = DF$

$= 10 - CE + EF$

$= 10 - 7 + 3$

$= 6\,\text{cm}$ 1A

(b) $\angle HEF = \angle CEG$ *(common ∠)*

$\triangle ADF \cong \triangle ECG$ *(given)*

$\angle EFH = \angle EGC$ *(corr. ∠s, ≅ △s)*

$\triangle EHF \sim \triangle ECG$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(c) $EG = \sqrt{6^2 + 7^2} = \sqrt{85}\,\text{cm}$ 1M

Let h be the perpendicular distance from H to EF.

Consider the area ratio of $\triangle EFH$ and $\triangle CEG$.

$$\frac{\left(\frac{(3)(h)}{2}\right)}{\left(\frac{(7)(6)}{2}\right)} = \left(\frac{3}{\sqrt{85}}\right)^2$$ 1M

$$h = \frac{126}{85}$$

Minimum length of $HT = 7 - h$ 1M

$\approx 5.52\,\text{cm}$

$< 5.6\,\text{cm}$

The claim is agreed. 1A

13. (a) $\angle DCE = \angle DCB$ *(given)*

$\angle EDC = \angle DCB$ *(alt. ∠s, BC//DE)*

$= \angle DCE$

$DE = CE$ *(sides opp. equal ∠s)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) Note that $\triangle ADE \sim \triangle ABC$.

$AE : AC = AD : AB$

$= 5 : (5 + 3)$

$AE : EC = 5 : (8 - 5)$

$= 5 : 3$ 1A

$\sin \angle BAC = \dfrac{DE}{AE}$ 1M

$= \dfrac{CE}{AE}$

$= \dfrac{3}{5}$

$\angle BAC \approx 36.9^\circ$ 1A

14. (a) $CB = CD$ *(property of square)*

$BQ = DP$ *(given)*

$\angle CDP = 90^\circ$ *(property of square)*

$\angle CBA = 90^\circ$ *(property of square)*

$\angle CBQ + \angle CBA = 180^\circ$ *(adj. ∠s on st. line)*

$\angle CBQ + 90^\circ = 180^\circ$

$\angle CBQ = 90^\circ$

$= \angle CDP$

$\triangle CBQ \cong \triangle CDP$ *(SAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) We have $\angle BCQ = \angle DCP$. 1M

$\angle PCQ = \angle BCQ + \angle BCP$

$= \angle DCP + \angle BCP$

$= \angle BCD$

$= 90^\circ$

Thus, $\triangle CPQ$ is a right-angled triangle. 1A

(c) $CP = CQ = 8\,\text{cm}$

$\triangle CPQ$ is a right-angled isosceles triangle.

We have $\angle CPQ = 45^\circ$.

Let d be the shortest distance between C and PQ.

$\sin 45^\circ = \dfrac{d}{PC}$ 1M

$d = 4\sqrt{2}\,\text{cm}$

$> 5\,\text{cm}$

There is no point F on PQ such that the distance between F and C is less than 5 cm. 1A

15. (a)

$$\angle CFE = 90^\circ \quad \textit{(given)}$$
$$\angle CFB = 90^\circ \quad \textit{(given)}$$
$$\angle DFE = \angle CFB = 90^\circ \quad \textit{(vert. opp. } \angle s\textit{)}$$
$$\angle CED = 90^\circ \quad \textit{(prop. of rectangle)}$$
$$\angle FED = 90^\circ - \angle CEF$$
$$\angle CEF + \angle EFC + \angle FCE = 180^\circ \quad (\angle \textit{ sum of } \triangle)$$
$$\angle CEF + 90^\circ + \angle FCE = 180^\circ$$
$$\angle FCE = 90^\circ - \angle CEF$$
$$= \angle FED$$
$$\triangle CEF \sim \triangle EDF \quad \textit{(AA)}$$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) Note that $CE = AD = BC$ and $CF \perp BE$. 1M

We have $BF = EF$.

F is the mid-point of BE. 1

(ii) $EF = BF = 12\text{ cm}$

$CF = \sqrt{15^2 - 12^2} = 9\text{ cm}$ 1M

Since $\triangle CEF \sim \triangle EDF$.

$$\frac{ED}{CE} = \frac{EF}{CF}$$
$$\frac{ED}{15} = \frac{12}{9} \quad \text{1M}$$
$$ED = 20\text{ cm} \quad \text{1A}$$

16. D

$DE = \frac{1}{2}AD = 1\text{ cm}$

$DG = \frac{1}{2}CD = 1\text{ cm}$

Note that $\triangle CDE \cong \triangle ADG$.

We have $\angle DAG = \angle DCE$.

Consider $\triangle ADG$.

$$AG^2 = AD^2 + DG^2$$
$$AG = \sqrt{2^2 + 1^2}$$
$$= \sqrt{5}\text{ cm}$$

Note that $\triangle CFG \sim \triangle ADG$.

$$\frac{\text{area of } \triangle CFG}{\text{area of } \triangle ADG} = \left(\frac{CG}{AG}\right)^2$$
$$\frac{\text{area of } \triangle CFG}{\frac{1}{2}(2)(1)} = \left(\frac{1}{\sqrt{5}}\right)^2$$
$$\text{area of } \triangle CFG = \frac{1}{5}\text{ cm}^2$$

17. B

$$\angle BAC + 2\angle CBD + 2\angle BCD = 180°$$
$$\angle CBD + \angle BCD = 55°$$
$$\angle BDC + \angle CBD + \angle BCD = 180°$$
$$\angle BDC = 125°$$

18. D

I. ✓.

$$(n-2)180° = 360°$$
$$n = 4$$

The regular polygon is a square.

II. ✓.

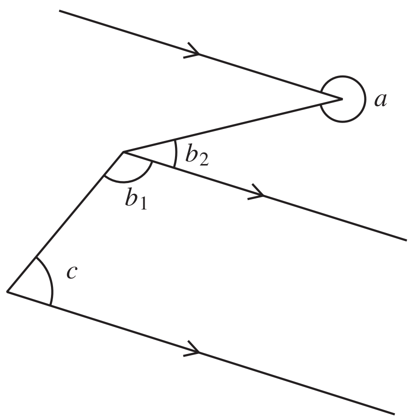
Property of square.

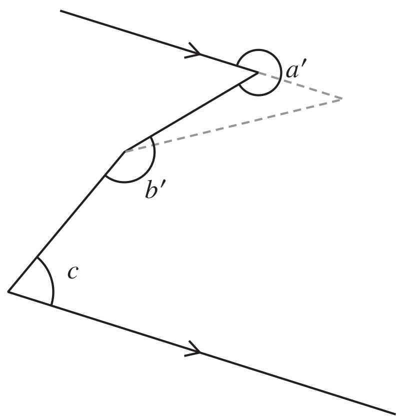
III. ✓.

Property of square.

19. A

(Left figure) Draw a parallel line passing through the vertex corresponding to angle b.
(Right figure) Move the vertex corresponding to angle a to the left.





I. ✓.

Consider the left figure.

We have $b_2 + a = 360°$ and $b_1 + c = 180°$.

$$\begin{aligned} a + b + c &= a + (b_1 + b_2) + c \\ &= (a + b_2) + (b_1 + c) \\ &= 360° + 180° \\ &= 540° \end{aligned}$$

II. ✗.

Consider the right figure. We have $a' < a$ and $b' > b$.

We have $a' - b' + c < a - b' + c < a - b + c$.

The equation $a - b + c = 270°$ cannot hold in both cases.

III. ✗. Consider the right figure. We have $a' < a$ and $b' > b$.

We have $a' - b' - c < a - b' - c < a - b - c$.

The equation $a - b - c = 90°$ cannot hold in both cases.

20. C

Let M be the mid-point of BC.

We have $AB = AC = 2a$ and $BM = CM = \frac{a}{2}$.

Note that $\triangle ACM \sim \triangle BCD$.

$$\frac{CD}{CM} = \frac{BC}{AC}$$

$$\frac{CD}{\left(\frac{a}{2}\right)} = \frac{a}{2a}$$

$$CD = \frac{a}{4}$$

$$BD = \sqrt{BC^2 - CD^2} = \sqrt{a^2 - \left(\frac{a}{4}\right)^2} = \frac{\sqrt{15}a}{4}$$

$$\begin{aligned} \text{Required area} &= \frac{1}{2} \times BD \times CD \\ &= \frac{1}{2} \times \frac{\sqrt{15}a}{4} \times \frac{a}{4} \\ &= \frac{\sqrt{15}a^2}{32} \end{aligned}$$

21. A

Since $\triangle DEF$ and $\triangle EFG$ are equilateral, we have $\angle DEF = \angle FEG = 60°$.

Consider the regular pentagon $ABCDE$.

$$5\angle AED = (5 - 2)180°$$

$$\angle AED = 108°$$

Consider the angles at point E.

$$\begin{aligned}\angle AED + \angle DEF + \angle FEG + \angle GEA &= 360^\circ \\ 108^\circ + 60^\circ + 60^\circ + \angle GEA &= 360^\circ \\ \angle GEA &= 132^\circ\end{aligned}$$

Note that $EG = EF = ED = EA$, we have $\angle EAG = \angle AGE$.

$$\begin{aligned}\angle EAG + \angle AGE + \angle GEA &= 180^\circ \\ 2\angle EAG + 132^\circ &= 180^\circ \\ \angle EAG &= 24^\circ\end{aligned}$$

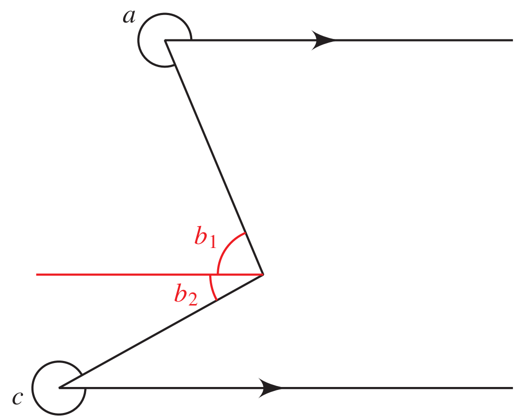
22. A

I. ✓.

Refer to the figure.

Note that $a + b_1 = 360^\circ$ and $c + b_2 = 360^\circ$.

$$\begin{aligned}a + b + c &= (a + b_1) + (b_2 + c) \\ &= 360^\circ + 360^\circ \\ &= 720^\circ\end{aligned}$$



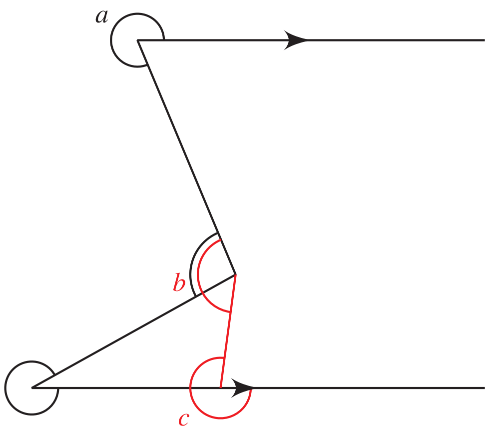
II. ✓.

Note that $a + b_1 = 360^\circ$.

$$\begin{aligned}a + b &= (a + b_1) + b_2 \\ &= 360^\circ + b_2 \\ &> 360^\circ\end{aligned}$$

III. ✗.

Refer to the figure.



Note that b becomes larger and c becomes smaller.

The result of $a + b - c$ becomes larger.

The equation cannot hold in both cases.

23. D

$360^\circ - 44^\circ = 316^\circ$

Required bearing is 316°.

24. B

Note that $\triangle ADE \sim \triangle ACF$.

$$\begin{aligned} \frac{AD}{AC} &= \frac{AE}{AF} \\ \frac{AD}{AD + 10} &= \frac{6}{6 + 6} \\ AD &= 10\,\text{cm} \end{aligned}$$

In $\triangle ADE$, we have $DE = \sqrt{10^2 - 6^2} = 8\,\text{cm}$.

$$\begin{aligned} \frac{CF}{DE} &= \frac{AF}{AE} \\ \frac{CF}{8} &= \frac{6 + 6}{6} \\ CF &= 16\,\text{cm} \end{aligned}$$

In $\triangle BCF$, we have $BC = \sqrt{16^2 + 12^2} = 20\,\text{cm}$.

25. A

Since $BA // CE$, we have $\angle BAC = \angle ACD = 65^\circ$.

$$\begin{aligned} \angle ACB + 65^\circ + 38^\circ &= 180^\circ \\ \angle ACB &= 77^\circ \end{aligned}$$

In $\triangle EAC$, we have $\angle AEC = \angle ACE = 77°$.

$$\begin{aligned}\angle EAC + \angle AEC + \angle ACE &= 180° \\ \angle EAC + 77° + 77° &= 180° \\ \angle EAC &= 26°\end{aligned}$$

26. D

I. ✗.
The sum of each pair of interior angle and exterior angle is 180°.
Each exterior angle $= 180° \times \dfrac{1}{1+5}$
$= 30°$
Sum of exterior angle of polygon is 360°.
We have $n = \dfrac{360°}{30} = 12 \neq 10$.

II. ✓.
Each interior angle $= 180° - 30°$
$= 150°$

III. ✓.

27. C

Since $\triangle ABC$ is equilateral, we have $\angle BAC = 60°$.
Since $ACDE$ is a square, we have $\angle CAE = 90°$.
Consider the regular pentagon $AEFGH$.

$$\begin{aligned}5\angle EAH &= (5-2)180° \\ \angle EAH &= 108°\end{aligned}$$

Consider the angles at point A.

$$\begin{aligned}\angle BAC + \angle CAE + \angle EAH + \angle HAB &= 360° \\ 60° + 90° + 108° + \angle HAB &= 360° \\ \angle HAB &= 102°\end{aligned}$$

Note that $AH = AE = AC = AB$, we have $\angle AHB = \angle ABH$.

$$\begin{aligned}\angle AHB + \angle ABH + \angle HAB &= 180° \\ 2\angle AHB + 102° &= 180° \\ \angle AHB &= 39°\end{aligned}$$

28. C

$\triangle AFG \sim \triangle AED$

$$\frac{FG}{DE} = \frac{AG}{AD}$$

$$\frac{FG}{5} = \frac{\frac{1}{2}\sqrt{5^2 + 12^2}}{12}$$

$$FG = \frac{65}{24}$$

29. A

$BCDE$ is a rhombus.

$\angle BCE = \angle DCE = 58°$

$$\angle ACB + \angle BCE + \angle DCE = 180°$$
$$\angle ACB + 58° + 58° = 180°$$
$$\angle ACB = 64°$$

$\angle BAC = \angle ACB = 64°$

$$\angle ACB + \angle BAC + \angle ACB = 180°$$
$$\angle ACB + 64° + 64° = 180°$$
$$\angle ACB = 52°$$

30. D

Since $AB = BE$, we have $\angle AEB = \angle BAE$.

$$\angle AEB + \angle BAE = \angle ABF$$
$$2\angle AEB = 132°$$
$$\angle AEB = 66°$$

Since $AD // FC$, we have $\angle EAD = \angle AEB = 66°$.
Since $AE = DE$, we have $\angle ADE = \angle EAD = 66°$.

$$\angle EAD + \angle ADE + \angle DEA = 180°$$
$$66° + 66° + \angle DEA = 180°$$
$$\angle DEA = 48°$$

Note that BEC is a straight line.

$$\angle AEB + \angle DEA + \angle DEC = 180°$$
$$66° + 48° + \angle DEC = 180°$$
$$\angle DEC = 66°$$

31. C

Since $BA // CD$.

$$\angle ADC + \angle BAD = 180^\circ \quad \text{and} \quad \angle CBA + \angle DCB = 180^\circ$$
$$110^\circ + \angle BAD = 180^\circ \qquad \angle CBA + 125^\circ = 180^\circ$$
$$\angle BAD = 70^\circ \qquad \angle CBA = 55^\circ$$

Since $AB = BD$, we have $\angle ADB = \angle BAD = 70^\circ$.

$$\angle DBA + \angle ADB + \angle BAD = 180^\circ$$
$$\angle DBA + 70^\circ + 70^\circ = 180^\circ$$
$$\angle DBA = 40^\circ$$

Thus, $\angle DBC = \angle CBA - \angle DBA = 55^\circ - 40^\circ = 15^\circ$.

32. B

Let $\angle BAC = x$.
Since $DE = DA$, we have $\angle AED = \angle DAE = x$.
Consider $\triangle ADE$.

$$\angle DAE + \angle AED = \angle BDE$$
$$x + x = \angle BDE$$
$$\angle BDE = 2x$$

Since $BD = BE$, we have $\angle DEB = \angle BDE = 2x$.
AEC is a straight line.

$$\angle BEC + \angle DEB + \angle AED = 180^\circ$$
$$\angle BEC + 2x + x = 180^\circ$$
$$\angle BEC = 180^\circ - 3x$$

Since $BE = BC$, we have $\angle ECB + \angle BEC = 180^\circ - 3x$.
Since $AB = AC$, we have $\angle CBA = \angle ACB = 180^\circ - 3x$.

$$\angle BAC + \angle ACB + \angle CBA = 180^\circ$$
$$x + (180^\circ - 3x) + (180^\circ - 3x) = 180^\circ$$
$$x = 36^\circ$$

33. C

Since $CF = CD$, we have $\angle CDF = \angle CFD$.
$\angle CFD + \angle CDF + \angle DCF = 180^\circ$

$$2\angle CFD + 74^\circ = 180^\circ$$
$$\angle CFD = 53^\circ$$

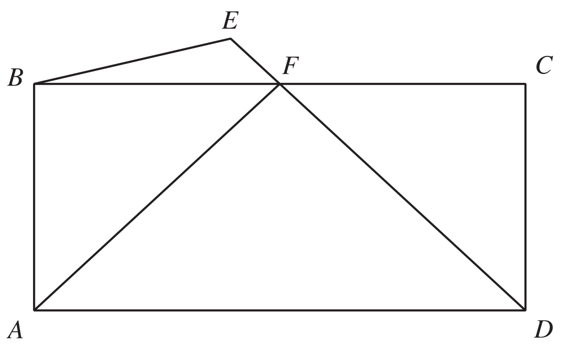
$$\angle AFD + \angle EDF = 180°$$
$$\angle AFD + 40° = 180°$$
$$\angle AFD = 140°$$
$$\angle AFB + \angle BFC + \angle CFD + \angle AFD = 360°$$
$$\angle AFB + 60° + 53° + 140° = 360°$$
$$\angle AFB = 107°$$
$$\angle ABF + \angle AFB + \angle BAF = 180°$$
$$\angle ABF + 107° + 30° = 180°$$
$$\angle ABF = 43°$$

34. A

I. ✓.
Since $AF = FD$, we have $\angle DAF = \angle FDA$.
Since $BF // AD$, we have $\angle BFE = \angle FDA$.

II. ✓.
We have $AF = FD$, $AB = CD$ and $\angle ABF = \angle FCD = 90°$.

III. ✗.
Consider the following figure. $ABCD$ is a rectangle and $AF = DF$.



Note that $\angle DAF \neq \angle FBE$.

35. B

Note that $\triangle ABC \sim \triangle CDE$.

$$\frac{CE}{AC} = \frac{CD}{AB}$$
$$\frac{CE}{\sqrt{16^2 + 30^2}} = \frac{12}{16}$$
$$CE = 25.5\,\text{cm}$$

Required perimeter $= 2(34 + 25.5)$
$= 119\,\text{cm}$

36. D

$\angle ABC = \frac{(5-2)180°}{5} = 108°$

$\angle BAC = \frac{180° - 108°}{2} = 36°$

Let F be a point due east of A.

$\angle BAF = 90° - 81° = 9°$

$\angle CAF = 36° - 9° = 27°$

Note that $AC // ED$.

Let G be a point due west of D.

$\angle GDE = \angle CAF = 27°$

$\angle GDC = 27° + 108° = 135°$

$135° - 90° = 45°$

The bearing of C from D is N45°E.

37. D

I. ✓.

Required sum $= (6-2)180°$

$= 720°$

II. ✗.

Each interior angle $= \frac{720°}{6}$

$= 120°$

Each exterior angle $= \frac{360°}{6}$

$= 60°$

$\neq \frac{120°}{3}$

III. ✓.

38. A

Consider $\triangle DEF$.

$$DF^2 = EF^2 - DE^2$$

$$DF = \sqrt{65^2 - 60^2}$$

$$= 25\text{ cm}$$

Note that $\triangle ABE \sim \triangle DFE$.

$$\frac{AB}{DF} = \frac{AE}{DE}$$

$$\frac{AB}{25} = \frac{AD + 60}{60}$$

$$AB = \frac{5AD}{12} + 25$$

The perimeter of $ABCD$ is 322 cm.

$$\begin{aligned} 2(AB+AD) &= 322 \\ 2\left[\left(\frac{5AD}{12}+25\right)+AD\right] &= 322 \\ AD &= 96\text{ cm} \\ AB &= 65\text{ cm} \end{aligned}$$

Consider $\triangle BCF$.

$$\begin{aligned} BF^2 &= BC^2+CF^2 \\ BF &= \sqrt{96^2+(65-25)^2} \\ &= 104\text{ cm} \end{aligned}$$

Note that $\triangle ABG \cong \triangle EFD$, we have $BG = FD = 25$ cm.
Thus, $GF = BF - BG = 104 - 25 = 79$ cm.

39. B

Note that $\triangle ABE \sim \triangle DAE$.

$$\begin{aligned} \frac{AE}{DE} &= \frac{BE}{AE} \\ \frac{AE}{6.25\times\frac{16}{9+16}} &= \frac{6.25\times\frac{9}{9+16}}{AE} \\ AE^2 &= 9 \\ AE &= 3\text{ cm} \end{aligned}$$

Consider $\triangle ADE$.

$$\begin{aligned} AD^2 &= AE^2+DE^2 \\ AD &= \sqrt{3^2+4^2} \\ &= 5\text{ cm} \end{aligned}$$

Consider $\triangle ABE$.

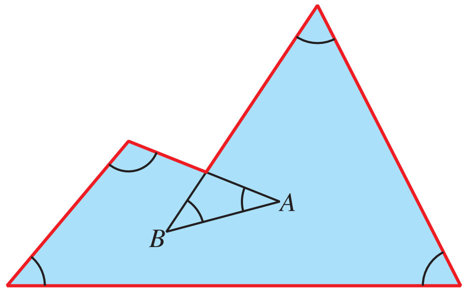
$$\begin{aligned} AB^2 &= AE^2+BE^2 \\ AB &= \sqrt{3^2+2.25^2} \\ &= 3.75\text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Required perimeter} &= 2(AB+AD) \\ &= 2(3.75+5) \\ &= 17.5\text{ cm} \end{aligned}$$

40. A

Consider the shaded pentagon as shown.

The reflex angle is equal to $\angle A + \angle B + 180°$.
By considering the sum of angles of the pentagon,



$$(\text{required sum}) + 180° = (5 - 2)180°$$
$$\text{required sum} = 360°$$

41. C

Note that $\triangle ADE \cong \triangle CDE$.

$\angle DCE = \angle DAE = 31°$

$\angle DEC = \angle ECF = 23°$

$$\angle CDE + \angle DCE + \angle DEF = 180°$$
$$\angle CDE + 31° + 23° = 180°$$
$$\angle CDE = 126°$$

$\angle ADE = \angle CDE = 126°$

$$\angle ADC + \angle ADE + \angle CDE = 360°$$
$$\angle ADC + 126° + 126° = 360°$$
$$\angle ADC = 108°$$

$$\angle BAD + \angle ADC = 180°$$
$$\angle BAD + 108° = 180°$$
$$\angle BAD = 72°$$

42. C

Note that $\triangle ABE \sim \triangle CBD$.

$$\frac{CD}{AE} = \frac{BC}{BA}$$
$$\frac{CD}{5} = \frac{5+15}{CD}$$
$$CD^2 = 100$$
$$CD = 10\text{ cm} \quad \text{or} \quad -10\text{ cm (rejected)}$$

43. C

Since $\triangle ABC$ is equilateral, we have $\angle BAC = 60°$.
Since $ACDE$ is a square, we have $\angle CAE = 90°$.
Consider the regular polygon $AEFGHIJKLM$.

$$10\angle EAM = (10 - 2)180°$$
$$\angle EAM = 144°$$

Consider the angles at point A.

$$\begin{aligned}\angle MAB + \angle BAC + \angle CAE + \angle EAM &= 360^\circ \\ \angle MAB + 60^\circ + 90^\circ + 144^\circ &= 360^\circ \\ \angle MAB &= 66^\circ\end{aligned}$$

Note that $AM = AE = AC = AB$, we have $\angle ABM = \angle BMA$.
Consider $\triangle ABM$.

$$\begin{aligned}\angle MAB + \angle ABM + \angle BMA &= 180^\circ \\ 66^\circ + 2\angle BMA &= 180^\circ \\ \angle BMA &= 57^\circ\end{aligned}$$

Consider the angles at point M.

$$\begin{aligned}\angle BMA + \angle AML + \angle BML &= 360^\circ \\ 57^\circ + 144^\circ + \angle BML &= 360^\circ \\ \angle BML &= 159^\circ\end{aligned}$$

44. B

$\angle AOB = 90^\circ - 52^\circ = 38^\circ$
$\angle OAB = \dfrac{180^\circ - 38^\circ}{2} = 71^\circ$
Required angle is S19°E.

45. B

Note that $\triangle BDC \sim \triangle BED$.

$$\frac{CD}{DE} = \frac{BC}{BD}$$

$$\frac{50}{\left(\frac{100}{3}\right)} = \frac{BC}{BD}$$

$$\frac{BC}{BD} = \frac{3}{2}$$

Note that $\triangle ABC \sim \triangle ADB$.

$$\frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB}$$

$$\frac{AB}{AD} = \frac{3}{2} = \frac{AC}{AB}$$

We have $AB = \dfrac{3AD}{2}$.

$$\frac{3}{2} = \frac{AD + 50}{\left(\frac{3AD}{2}\right)}$$

$$AD = 40$$

46. C

Since $\triangle DEF$ is equilateral, we have $\angle EDF = 60°$.
Consider the regular pentagon $ABCDE$.

$$5\angle EDC = (5-2)180°$$
$$\angle EDC = 108°$$

Note that $DF = DE = DC$, we have $\angle FCD = \angle CFD$.

$$\angle FCD + \angle CFD + \angle FDC = 180°$$
$$2\angle FCD + (108° - 60°) = 180°$$
$$\angle FCD = 66°$$

47. B

I. ✗.
It has rotational symmetry.

II. ✓.
It is an isosceles triangle but is not an equilateral triangle.
It has reflectional symmetry but not rotational symmetry.

III. ✗.
The remaining angle is 26°.
It has neither reflectional symmetry nor rotational symmetry.

48. B

Note that $\angle ABC = \angle ECD$ and $AB // CE$.

$$\frac{\text{area of } \triangle ACE}{\text{area of } \triangle ABC} = \frac{CE}{AB}$$
$$\frac{\text{area of } \triangle ACE}{50} = \frac{4}{5}$$
$$\text{area of } \triangle ACE = 40\,\text{cm}^2$$

49. C

Note that $\triangle ABD \sim \triangle BCD$.
Let x cm^2 be the area of $\triangle BCD$.

$$\frac{x}{18} = \left(\frac{4}{3}\right)^2$$
$$x = 32$$

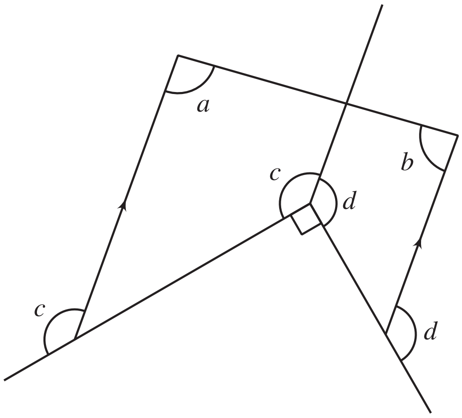
50. B

I. ✓.

Refer to the figure.

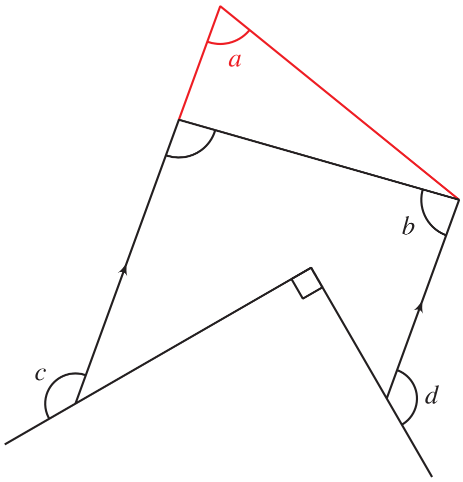
$c + d + 90° = 360°$

$c + d = 270°$



II. ✗.

Refer to the figure.



Note that a becomes smaller and b becomes larger.

The result of $a + d$ becomes smaller, while the result of $b + c$ becomes larger.

The equation cannot hold in both cases.

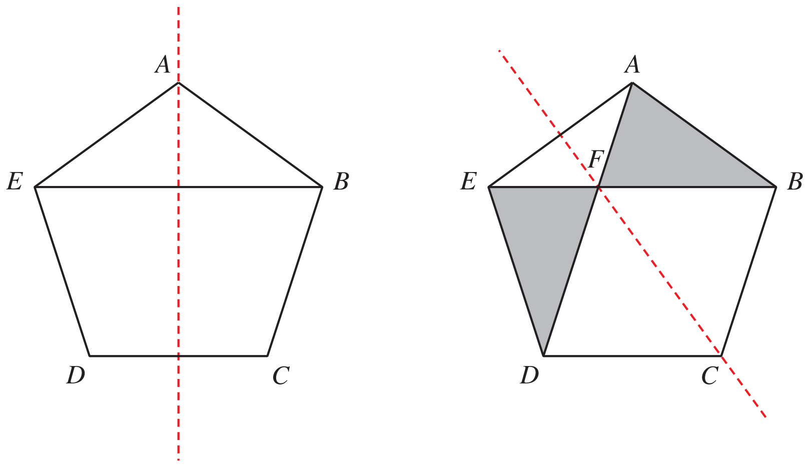
III. ✓.

Note that $a + b = 180°$ and $c + d = 270°$.

$$\begin{aligned} a + b + c + d &= 180° + 270° \\ &= 450° \end{aligned}$$

51. D

The regular pentagon is symmetric.



I. ✓.

Consider the regular pentagon $ABCDE$.

$$5\angle CDE = (5-2)180°$$
$$\angle CDE = 108°$$

Since $DC // EB$.

$$\angle BED + \angle CDE = 180°$$
$$\angle BED + 108° = 180°$$
$$\angle BED = 72°$$

II. ✓.

III. ✓.

52. A

I. ✓.

Exterior angle $= 180° - 135°$

$= 45°$

Consider the sum of exterior angles.

$$n(45°) = 360°$$
$$n = 8$$

II. ✓.

III. ✗.

The number of axes of reflectional symmetry is 8.

53. C

Consider the regular decagon $ABCDEFGHIJ$.

$$10\angle JIH = (10-2)180^\circ$$
$$\angle JIH = 144^\circ$$

Consider the regular pentagon $HKLMI$.

$$5\angle HIM = (5-2)180^\circ$$
$$\angle HIM = 108^\circ$$

Since $\triangle IMN$ is equilateral, $\angle MIN = 60^\circ$.

$$\angle JIH + \angle HIM + \angle MIN + \angle NIJ = 360^\circ$$
$$144^\circ + 108^\circ + 60^\circ + \angle NIJ = 360^\circ$$
$$\angle NIJ = 48^\circ$$

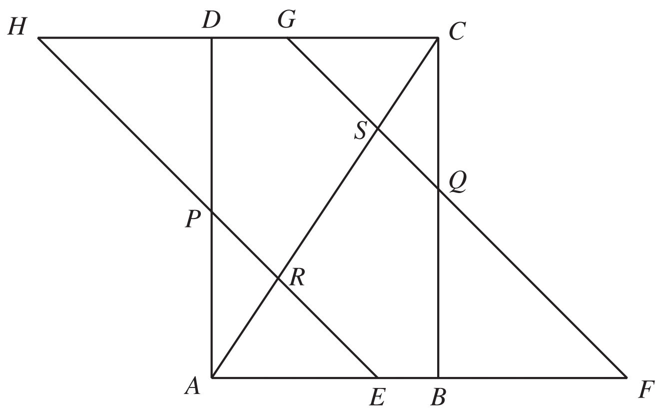
Note that $JI = IH = IM = IN$, we have $\angle IJN = \angle INJ$.

$$\angle IJN + \angle INJ + \angle NIJ = 180^\circ$$
$$2\angle INJ + 48^\circ = 180^\circ$$
$$\angle INJ = 66^\circ$$

54. C

I. ✗.

Take $\angle GCQ = 90^\circ$ such that $ABCD$ is a rectangle, where $CD \neq CB$.



Note that $\angle GCS \neq 45^\circ$ and $\angle GCS \neq \angle QCS$.

Thus, $\triangle CGS$ and $\triangle CQS$ are not congruent triangles.

II. ✓.

Let K be the intersection of FG produced and AD produced.

$$\begin{aligned}
\angle ABC &= \angle ADC && \textit{(opp. } \angle \textit{s, //gram)} \\
\angle FBQ &= 180^\circ - \angle ABC && \textit{(adj. } \angle \textit{s on st. line)} \\
\angle PDH &= 180^\circ - \angle ADC && \textit{(adj. } \angle \textit{s on st. line)} \\
&= \angle FBQ \\
\angle BFQ &= \angle CGQ && \textit{(alt. } \angle \textit{s, } HC // AF) \\
CG &= CQ && \textit{(given)} \\
\angle CGQ &= \angle\angle CQG && \textit{(base. } \angle \textit{s, isos. } \triangle) \\
\angle CQG &= \angle AKF && \textit{(alt. } \angle \textit{s, } AK // BC) \\
\angle DPH &= \angle AKF && \textit{(alt. } \angle \textit{s, } EH // FK) \\
&= \angle BFQ \\
\triangle BFQ &\sim \triangle DPH && \textit{(AA)}
\end{aligned}$$

III. ✓.

We have $\triangle BFQ \sim \triangle DPH$.

$$\begin{aligned}
\angle AEP &= \angle BFQ && \textit{(corr. } \angle \textit{s, } EH // FG) \\
\angle BFQ &= \angle DPH && \textit{(corr. } \angle \textit{s, } \cong \triangle \textit{s)} \\
\angle APE &= \angle DPH && \textit{(vert. opp. } \angle \textit{s)} \\
&= \angle AEP \\
AE &= AP && \textit{(sides opp. equal } \angle \textit{s)}
\end{aligned}$$

55. B

Note that $\triangle AEF \sim \triangle ADC$.

Let $x\ \text{cm}^2$ be the area of $\triangle AEF$.

Then the area of $\triangle ADC$ is $(x + 21)\ \text{cm}^2$.

$$\begin{aligned}
\frac{x}{x + 21} &= \left(\frac{AF}{AC}\right)^2 \\
&= \left(\frac{4}{3 + 5}\right)^2 \\
x &= 7
\end{aligned}$$

Consider $\triangle ABF$ and $\triangle AFE$.

$$\begin{aligned}
\frac{\text{area of } \triangle ABF}{\text{area of } \triangle AFE} &= \frac{BF}{FE} \\
\frac{\text{area of } \triangle ABF}{7} &= \frac{8}{7} \\
\text{area of } \triangle ABF &= 8\ \text{cm}^2
\end{aligned}$$

Since $\triangle AEF \sim \triangle ADC$.

$$\frac{AD}{AE} = \frac{AC}{AF}$$

$$\frac{AD}{\left(\frac{3AF}{4}\right)} = \frac{3+5}{4}$$

$$AD = \frac{3AF}{2}$$

$$\frac{AF}{FD} = \frac{1}{\frac{3}{2}-1}$$

$$= 2$$

Consider $\triangle ABF$ and $\triangle BDF$.

$$\frac{\text{area of } \triangle ABF}{\text{area of } \triangle BDF} = \frac{AF}{FD}$$

$$\frac{8}{\text{area of } \triangle BDF} = 2$$

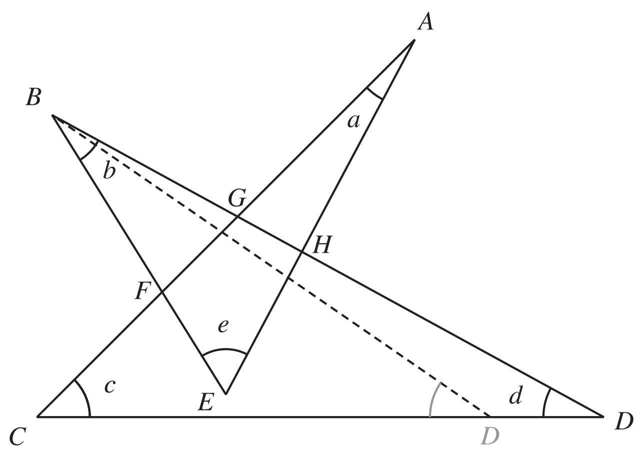
$$\text{area of } \triangle BDF = 4\,\text{cm}^2$$

Required area $= 21 + 7 + 8 + 4$

$= 40\,\text{cm}^2$

56. A

Consider the following figure.



Move the vertex D to the right.

As a result, angle b becomes larger while angle d becomes smaller.

I. ✗.

The value of $c + d$ becomes smaller after the adjustment.

II. ✗.

The value of $a + b - e$ becomes larger after the adjustment.

III. ✓.

$$
\begin{aligned}
a+b+c+d+e &= (a+e)+b+(c+d) \\
&= \angle BFG + b + \angle BGF \\
&= 180^\circ
\end{aligned}
$$

57. B

Note that $\triangle APQ \sim \triangle DQC$.

$$
\begin{aligned}
\frac{PQ}{QC} &= \frac{AQ}{DC} \\
&= \frac{2}{2+1} \\
&= \frac{2}{3}
\end{aligned}
$$

Consider $\triangle CPQ$.

$$
\begin{aligned}
\tan\angle PCQ &= \frac{PQ}{QC} \\
\angle PCQ &\approx 34^\circ
\end{aligned}
$$

Note that B, C, Q, P are concyclic.

We have $\angle PCQ = \angle PBQ$.

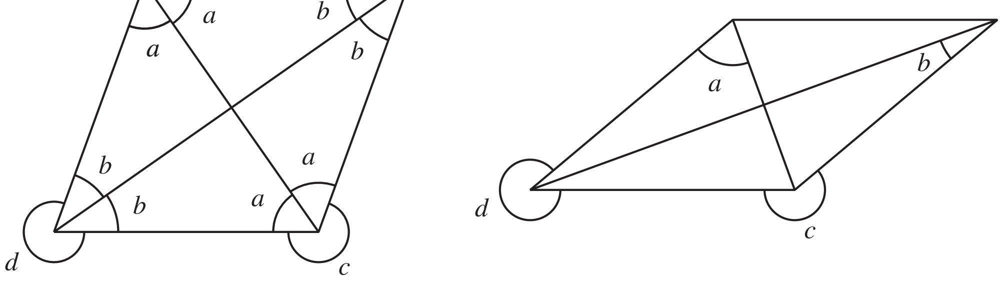
$$
\begin{aligned}
\tan\angle PBQ &= \frac{AQ}{AB} \\
\tan\angle PBQ &= \frac{2}{3} \\
\angle PBQ &\approx 34^\circ
\end{aligned}
$$

Thus, $\angle PCQ \approx 34^\circ$.

58. D

I. ✗.

Note that $a \neq b$.



II. ✓.

Note that rhombus is symmetrical. We have the angles in the figure above.

Consider the angles at the bottom right vertex, we have $2a + c = 360^\circ$.

III. ✓.

Note that $2a + 2b = 180°$.

$$\begin{aligned}(d + 2b) + (c + 2a) &= 360° + 360° \\ c + d + (2a + 2b) &= 720° \\ c + d &= 540°\end{aligned}$$

59. D

Since $\triangle ABC \cong \triangle DEC$, we have $CE = BC = 5$ cm and $AC = CD = 12$ cm.
Consider $\triangle CDE$.

$$\begin{aligned}DE^2 &= CE^2 + CD^2 \\ DE &= \sqrt{5^2 + 12^2} \\ &= 13\text{ cm}\end{aligned}$$

Consider $\triangle ACD$.

$$\begin{aligned}AD^2 &= AC^2 + CD^2 \\ AD &= \sqrt{12^2 + 12^2} \\ &= 12\sqrt{2}\text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Required perimeter} &= (12 - 5) + 13 + 12\sqrt{2} \\ &\approx 37.0\text{ cm}\end{aligned}$$

60. D

$AB = BC = 3$ cm
Note that $\triangle FAE \sim \triangle FBC$.

$$\begin{aligned}\frac{AE}{BC} &= \frac{FA}{FB} \\ \frac{AE}{3} &= \frac{2}{2+3} \\ AE &= 1.2\text{ cm}\end{aligned}$$

$ED = AD - AE = 3 - 1.2 = 1.8$ cm

61. C

Note that $\triangle ADG \cong \triangle ABE$.
We have $\angle BAE = \angle DAG = 36°$ and $AG = AE$.
Since $ABCD$ is a square, we have $\angle BAD = 90°$.

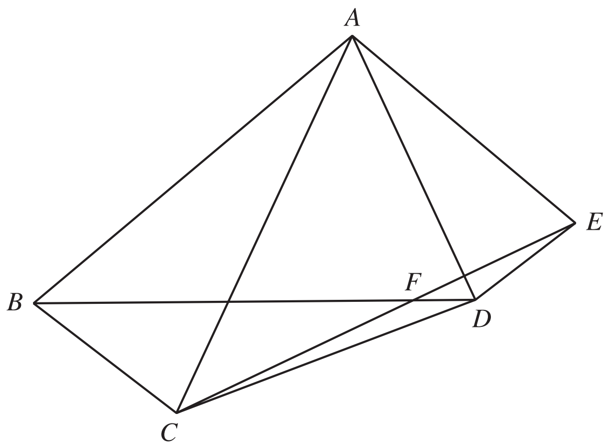
$$\begin{aligned}\angle DAF + \angle FAE + \angle EAB &= 90° \\ 9° + \angle FAE + 36° &= 90° \\ \angle FAE &= 45°\end{aligned}$$

Note that $\triangle AFG \cong \triangle AFE$, we have $\angle AEF = \angle AGF$.
Consider the angles at point E.

$$\begin{aligned}\angle BEA + \angle AEF + \angle CEF &= 180^\circ\\(180^\circ - 90^\circ - 36^\circ) + (180^\circ - 90^\circ - 36^\circ) + \angle CEF &= 180^\circ\\\angle CEF &= 72^\circ\end{aligned}$$

62. B

I. ✗.
Refer to the figure. We have $AB = AC$, $AD = AE$ and $\angle BAC = \angle DAE$.



Note that $\triangle ABC$ and $\triangle ADE$ are not congruent.

II. ✓.

$$\begin{aligned}AB &= AC && \textit{(given)}\\AD &= AE && \textit{(given)}\\\angle BAC &= \angle DAE && \textit{(given)}\\\angle BAD &= \angle BAC + \angle CAD\\\angle CAE &= \angle DAE + \angle CAD\\&= \angle BAC + \angle CAD\\&= \angle BAD\\\triangle ABD &\cong \triangle ACE && \textit{(SAS)}\end{aligned}$$

III. ✗.
Refer to the figure above.
Note that $\triangle BFC$ and $\triangle EFD$ are not congruent.

63. A

I. ✓.

$$\text{Interior angle} = \frac{(16-2)180°}{16}$$
$$= 157.5°$$

II. ✓.

III. ✗.

The number of axes of reflectional symmetry is 16.

64. C

$$(n-2)180° = 12 \times \frac{360°}{n}$$
$$n^2 - 2n - 24 = 0$$
$$n = 6 \quad \text{or} \quad -4 \text{ (rejected)}$$

A. ✗.

B. ✗.

$$\text{Each interior angle} = \frac{(6-2)180°}{6} = 120°$$

C. ✓.

Number of diagonals = $C_2^6 - 6 = 9$

D. ✗.

Number of folds of rotational symmetry is 6.

65. D

$$\angle BAC + 34° + 72° = 180°$$
$$\angle BAC = 74°$$
$$\angle ABC = \frac{180° - 74°}{2} = 53°$$
$$53° - 34° = 19°$$

The bearing of C from B is S19°W.

END OF PAPER