1. (a) $f(x) = (8x^2 + ax + 8)(3x^2 + 7x + r) + bx + c$ 1M

$= 24x^4 + (56 + 3a)x^3 + \ldots$

$56 + 3a = 47$ 1M

$a = -3$ 1A

(b) (i) Let $g(x) = A(8x^2 + ax + 8) + bx + c$, where A is a constant. 1M

$f(x) - g(x) = [(8x^2 + ax + 8)(3x^2 + 7x + r) + bx + c] - [A(8x^2 + ax + 8) + bx + c]$

$= (8x^2 + ax + 8)(3x^2 + 7x + r - A)$

Thus, $f(x) - g(x)$ is divisible by $8x^2 + ax + 8$. 1

(ii) $f(x) - g(x) = 0$

$(8x^2 - 3x + 8)(3x^2 + 7x + r - A) = 0$

$8x^2 - 3x + 8 = 0$ or $3x^2 + 7x + r - A = 0$ 1M

For $8x^2 - 3x + 8 = 0$,

$\Delta = (-3)^2 - 4(8)(8) = -247 < 0$. The equation has no real roots. 1M

For $3x^2 + 7x + r - A = 0$, the equation has at most 2 real roots.

Thus, $f(x) - g(x) = 0$ has at most 2 real roots.

The claim is disagreed. 1A

2. (a) Let $f(x) = (x^2 + x - 1)(Ax + B)$, where A and B are constants. 1M

$$\begin{cases} f(-7) = -123 = (49 - 7 - 1)(-7A + B) \\ f(3) = 297 = (9 + 3 - 1)(3A + B) \end{cases}$$ 1M

Solving, we have $A = 3$ and $B = 18$.

Required quotient is $3x + 18$. 1A

(b) $f(x) = 0$

$(x^2 + x - 1)(3x + 18) = 0$

$3(x^2 + x - 1)(x + 6) = 0$

$x = -6$ or $x^2 + x - 1 = 0$ 1M

For $x^2 + x - 1 = 0$, $x \approx 0.618$ or $x \approx -1.62$. 1M

The equation $f(x) = 0$ has only one positive root.

The claim is not correct. 1A

3. (a) $f(x) = (x - 2)(x^2 - 3x + 4) + r$ 1A

$f(1) = 0$ 1M

$(1 - 2)(1 - 3 + 4) + r = 0$

$r = 2$ 1A

(b) $f(x) = (x - 2)(x^2 - 3x + 4) + 2$

$= x^3 - 5x^2 + 10x - 6$

$= (x - 1)(x^2 - 4x + 6)$ 1M

We have $g(x) = x^2 - 4x + 6$. 1A

Consider the equation $x^2 - 4x + 6 = 0$.

$$\Delta = 4^2 - 4(1)(6)$$ 1M

$$= -8$$

$$< 0$$

The equation has no real roots.

The claim is agreed. 1A

4. (a) $f(x) = (x+2)(ax^2 + bx + c)$

$$= ax^3 + (2a+b)x^2 + (2b+c)x + 2c$$

We have $a = -2$, $2a + b = 0$ and $2c = -6$. 1M

Solving, we have $a = -2$, $b = 4$ and $c = -3$. 3A

(b) $0 = (x+2)(-2x^2 + 4x - 3)$

$x = -2$ or $-2x^2 + 4x - 3 = 0$ 1M

For $-2x^2 + 4x - 3 = 0$, we have

$$\Delta = 4^2 - 4(-2)(-3)$$ 1M

$$= -8$$

$$< 0$$

The equation $-2x^2 + 4x - 3 = 0$ has no real roots and hence no irrational roots.

Note that -2 is not an irrational number.

The claim is disagreed. 1A

5. (a) Let $q(x) = Ax + B$, where A and B are constants.

$$g(x) = f(x)(Ax+B) + (2Ax + 2B)$$ 1M

$$= (21x^3 - 2x^2 - 8x - 2)(Ax + B) + 2Ax + 2B$$

Compare the coefficients of x^3 and x.

$$\begin{cases} -33 = 21B - 2A \\ 8 = -8B - 2A + 2A \end{cases}$$ 1M

Solving, we have $A = 6$ and $B = -1$.

We have quotient $= 6x - 1$ and remainder $= 12x - 2$. 1A

(b)
$$g(x) = 0$$

$$(6x-1)(21x^3 - 2x^2 - 8x - 2) + 2(6x - 1) = 0$$

$$(6x-1)(21x^3 - 2x^2 - 8x) = 0$$ 1M

$$x(6x-1)(3x-2)(7x+4) = 0$$ 1M

$$x = 0 \quad \text{or} \quad \frac{1}{6} \quad \text{or} \quad \frac{2}{3} \quad \text{or} \quad -\frac{4}{7}$$ 1M

All the roots are rational numbers.

The equation has no irrational roots.

The claim is not correct. 1A

6. (a) Let $g(x) = (2x^2 - 9x + 14)(Ax + B) + 3x - 1$, where A and B are constants. 1M

$$\begin{cases} 33 = [2(2)^2 - 9(2) + 14](2A + B) + 3(2) - 1 \\ 21 = [2(-1)^2 - 9(-1) + 14](-A + B) + 3(-1) - 1 \end{cases}$$ 1M

Solving, we have $A = 2$ and $B = 3$.

Required quotient is $2x + 3$. 1A

(b) $$(2x^2 - 9x + 14)(2x + 3) + 3x - 1 = 2(x + 2)(3x - 1)$$

$$(2x^2 - 9x + 14)(2x + 3) + (3x - 1)[1 - 2(x + 2)] = 0$$ 1M

$$(2x + 3)[(2x^2 - 9x + 14) - (3x - 1)] = 0$$ 1M

$$(2x + 3)(2x^2 - 12x + 15) = 0$$

$2x + 3 = 0$ or $2x^2 - 12x + 15 = 0$

$x = -\frac{3}{2}$ $\qquad$ $x = \frac{12 \pm \sqrt{12^2 - 4(2)(15)}}{2(2)}$ 1M

$= \frac{6 \pm \sqrt{6}}{2}$

$\frac{6 \pm \sqrt{6}}{2}$ are not rational numbers.

The claim is disagreed. 1A

7. (a) $f(x) = 2(x - 2)(x^2 - 4x + 1) + ax + b$ 1M

$= 2x^3 - 12x^2 + (18 + a)x + (b - 4)$

We have $2b = -12$, $18 + a = 7a$ and $c = b - 4$. 1M

Solving, we have $a = 3$, $b = -6$ and $c = -10$. 1A

(b) $0 = 2(x - 2)(x^2 - 4x + 1) + 3x - 6$

$= (x - 2)[2(x^2 - 4x + 1) + 3]$ 1M

$= (x - 2)(2x^2 - 8x + 5)$

$x = 2$ or $\frac{8 \pm \sqrt{8^2 - 4(2)(5)}}{2(2)}$

$= 2$ or $\frac{4 \pm \sqrt{6}}{2}$ 1A

There is 1 rational root. 1A

8. (a) $f(-1) = 0 = -4(-1)^3 + (a + 2)(-1)^2 + 2(-1) - 3b$ 1M

$a - 3b = -4$

$f(2) = 9 = -4(2)^3 + (a + 2)(2)^2 + 2(2) - 3b$ 1M

$4a - 3b = 29$

Solving, we have $a = 11$ and $b = 5$. 1A

(b) $f(x) = -4x^3 + 13x^2 + 2x - 15$

$= (-x^2 + 2x + 3)(4x - 5)$ 1M

So, $g(x) = 4x - 5$.

$$kx(4x-5) = (-x^2+2x+3)(4x-5) \qquad \text{1M}$$

$$(4x-5)(x^2+(k-2)x-3) = 0 \qquad \text{1M}$$

$$x = \frac{5}{4} \quad \text{or} \quad x^2+(k-2)x-3=0$$

$\Delta = (k-2)^2 - 4(1)(-3) = (k-2)^2 + 12 > 0$ for all real values of k.

$x^2 + (k-2)x - 3 = 0$ has two distinct real roots.

The claim is agreed. 1A

9. (a) $f(x) = (x^2-1)(x+h) + 4x + k$ 1A

$$f(-1) = (1-1)(-1+h) - 4 + k = 0 \qquad \text{1M}$$

$$k = 4 \qquad \text{1A}$$

(b)

$$(x^2-1)(x+h) + 4x + 4 = 0$$

$$(x+1)(x-1)(x+h) + 4(x+1) = 0$$

$$(x+1)[(x-1)(x+h)+4] = 0$$

$$(x+1)[x^2+(h-1)x+(4-h)] = 0$$

$x = -1 \quad \text{or} \quad x^2+(h-1)x+(4-h)=0$ 1M

Suppose the equation $x^2+(h-1)x+(4-h)=0$ has repeated roots.

$$\Delta = (h-1)^2 - 4(4-h) = 0 \qquad \text{1M}$$

$$h^2 + 2h - 15 = 0$$

$$h = -5 \quad \text{or} \quad 3$$

Suppose $x = -1$ is a root of $x^2+(h-1)x+(4-h)=0$.

$$(-1)^2 + (h-1)(-1) + (4-h) = 0 \qquad \text{1M}$$

$$h = 3$$

When $h = -5$, the equation $f(x) = 0$ has two distinct roots.

When $h = 3$, the equation $f(x) = 0$ has repeated root $x = -1$ only.

Thus, the possible value of h is -5. 1A

10. (a) $p(x) = (x^2-2x+2)(10x^2+ax-19) + (bx+41)$ 1M

Since $p(x)$ is divisible by $(x+1)(x-1)$, we have $p(1) = p(-1) = 0$.

$$\begin{cases} 0 = (1-2+2)(10+a-19)+b+41 \\ 0 = (1+2+2)(10-a-19)-b+41 \end{cases} \qquad \text{1M}$$

Solving, we have $a = 7$ and $b = -39$. 1A+1A

(b) $0 = p(x)$

$0 = (x^2 - 2x + 2)(10x^2 + 7x - 19) - 39x + 41$

$= 10x^4 - 13x^3 - 13x^2 + 13x + 3$

$= (x^2 - 1)(10x^2 - 13x - 3)$ 1M

$= (x + 1)(x - 1)(2x - 3)(5x + 1)$

$x = \pm 1 \quad \text{or} \quad \frac{3}{2} \quad \text{or} \quad -\frac{1}{5}$ 1M

The equation $p(x) = 0$ has 4 rational roots. 1A

11. (a) $f(x) = (x - 1)[x^2 + (h - 1)x - h] + k$ We have $f(-h) = 5$. 1M

$(-h - 1)(h^2 - (h - 1)h - h) + k = 5$ 1M

$k = 5$ 1A

We have $f(2) = 0$.

$(2 - 1)(2^2 + (h - 1)2 - h) + 5 = 0$

$h = -7$ 1A

Thus, $h = -7$ and $k = 5$.

(b) $f(x) = 0$

$(x - 1)(x^2 - 8x + 7) + 5 = 0$

$x^3 - 9x^2 + 15x - 2 = 0$ 1A

$(x - 2)(x^2 - 7x + 1) = 0$ 1M

$x = 2 \quad \text{or} \quad x^2 - 7x + 1 = 0$

Consider the equation $x^2 - 7x + 1 = 0$.

$$x = \frac{7 \pm \sqrt{7^2 - 4(1)(1)}}{2}$$

$$x = \frac{7 \pm 3\sqrt{5}}{2}$$

The roots are irrational numbers.

The claim is disagreed. 1A

12. (a) $f(x) = (x^2 + cx + c)(4x - 8) + 17x + 29$ 1M

$$f\left(-\frac{3}{2}\right) = 0$$

$\left(\frac{9}{4} - \frac{3c}{2} + c\right)(-6 - 8) - \frac{51}{2} + 29 = 0$ 1M

$c = 4$ 1A

(b) $f(x) = 0$

$$(x^2 + 4x + 4)(4x - 8) + 17x + 29 = 0$$

$$4x^3 + 8x^2 + x - 3 = 0$$

$$(2x + 3)(2x^2 + x - 1) = 0 \quad \text{1M}$$

$$(2x + 3)(x + 1)(2x - 1) = 0$$

$$x = -\frac{3}{2} \quad \text{or} \quad -1 \quad \text{or} \quad \frac{1}{2} \quad \text{1M}$$

$-\frac{3}{2}$, -1 and $\frac{1}{2}$ are rational numbers.

There are 3 rational roots. 1A

13. (a) $p(x) = (x + r)(x^2 - rx + 2r^2) - 4r^3$ 1A

$p(r) = (r + r)(r^2 - r^2 + 2r^2) - 4r^3$ 1M

$= 4r^3 - 4r^3$

$= 0$

$p(x)$ is divisible by $x - r$. 1

(b) $0 = (x + r)(x^2 - rx + 2r^2) - 4r^3$

$= x^3 + r^2x - 2r^3$

$= (x - r)(x^2 + rx + 2r^2)$ 1M

$x = r$ or $x^2 + rx + 2r^2 = 0$ 1A

When $x^2 + rx + 2r^2 = 0$, $\Delta = r^2 - 4(1)(2r^2) = -7r^2 < 0$. 1M

The roots of the equation $x^2 + rx + 2r^2 = 0$ are not real.

The claim is disagreed. 1A

14. (a) Let $h(x) = g(x)(Ax + B) + Cx^2 + Dx + E$, where A, B, C, D and E are constants. 1M

Compare the coefficients of x^4.

$12 = 6A$ 1M

$A = 2$

Compare the coefficients of x^3.

$8 = 6B + A$

$B = 1$

Required quotient is $2x + 1$. 1A

(b) $h(x) = g(x)(2x + 1) + R(x)$

$g(x) = 6x^3 + x^2 - 18x + 8$

$= (2x - 1)(3x^2 + 2x - 8)$

$= (2x - 1)(3x - 4)(x + 2)$ 1M

Note that $R(-2) = 0$ and $R\left(\frac{1}{2}\right) = 0$.

We have $R(x) = r(x+2)(2x-1)$ for some constant r. 1M

$$h(x) = 0$$

$$(2x-1)(3x-4)(x+2)(2x+1) + r(x+2)(2x-1) = 0$$

$$(2x-1)(x+2)[(3x-4)(2x+1)+r] = 0 \quad \text{1M}$$

We have $x = \frac{1}{2}$ or $x = -2$ or $(3x-4)(2x+1)+r = 0$.
-2 is a negative root of $h(x) = 0$.
The claim is disagreed. 1A

15. (a) $f(-1) = h = -h - 1 + 9 + k$ 1M

$$2h - k = 8$$

$$0 = f\left(\frac{1}{2}\right) = \frac{h}{8} - \frac{1}{4} - \frac{9}{2} + k \quad \text{1M}$$

$$h + 8k = 38$$

Solving, we have $h = 6$ and $k = 4$. 1A+1A

(b)

$$f(x) = 6$$

$$6x^3 - x^2 - 9x - 2 = 0$$

$$(x+1)(6x^2 - 7x - 2) = 0 \quad \text{1M}$$

$$x = -1 \quad \text{or} \quad \frac{7 \pm \sqrt{7^2 - 4(6)(-2)}}{2(6)} \quad \text{1M}$$

$$= -1 \quad \text{or} \quad \frac{7 \pm \sqrt{97}}{12}$$

There are some irrational roots in $f(x) = 6$.
The claim is disagreed. 1A

16. (a) $f(12) = 12^3 - 96(12) - 576 = 0$ 1M
$f(x) = (x-12)(x^2 + 12x + 48)$ 1A

(b) (i) Volume of $A = \frac{1}{3}\pi(9)^2(18)$

$$= 486\pi \text{ cm}^3$$

Let h_A cm be the depth of water in container A after 44 seconds.

$$\left(\frac{h_A}{18}\right)^3 = \frac{35 \times 2\pi}{486\pi}$$

$$h_A^3 = 840$$

$$h_A \approx 9.44 \quad \text{1A}$$

Let h_B cm be the depth of water in container B after 44 seconds.

$$\pi 4^2 h_B = 44 \times \pi$$

$$h_B = 2.75 \quad \text{1A}$$

$$< h_A$$

The water level in container A is higher. 1A

(ii) Let h cm be the equal water level in both container.

Consider the time taken to reach equal water level in both containers.

$$\frac{486\pi\left(\frac{h}{18}\right)^3 - 35\times 2\pi}{\pi} = \frac{\pi 4^2(h) - 44\pi}{2\pi} \quad \text{1M}$$

$$\frac{h^3}{12} - 70 = 8h - 22$$

$$h^3 - 96h - 576 = 0$$

$$(h-12)(h^2+12h+48) = 0 \quad \text{1M}$$

$$h = 12 \quad \text{or} \quad h^2+12h+48 = 0$$

Consider the equation $h^2 + 12h + 48 = 0$.

$$\Delta = 12^2 - 4(1)(48)$$

$$= -48$$

$$< 0$$

The equation has no real roots.

Required depth of water is 12 cm. 1A

17. (a) Let $f(x) = (x^2-4)(Ax+B) + kx + 24$, where A and B are constants. 1M

$$f(2) = 0$$

$$(4-4)(2A+B) + 2k + 24 = 0 \quad \text{1M}$$

$$k = -12 \quad \text{1A}$$

(b)

$$f(0) = 8$$

$$(0-4)(0+B) + 0 + 24 = 8 \quad \text{1M}$$

$$B = 4 \quad \text{1A}$$

$$f(-1) = 0$$

$$(1-4)(-A+4) + 12 + 24 = 0$$

$$A = -8 \quad \text{1A}$$

$$f(x) = 0$$

$$(x^2-4)(-8x+4) - 12x + 24 = 0$$

$$(x+2)(x-2)(-8x+4) - 12(x-2) = 0$$

$$(x-2)[(x+2)(-8x+4) - 12] = 0$$

$$(x-2)(-8x^2 - 12x - 4) = 0$$

$$-4(x-2)(x+1)(2x+1) = 0 \quad \text{1M}$$

$$x = 2 \quad \text{or} \quad -1 \quad \text{or} \quad -\frac{1}{2}$$

$-\frac{1}{2}$ is not an integer.

The claim is not correct. 1A

18. (a) Consider the constant term of $f(x)$.

$$0 - (1)(-1) + b = 15$$

$$b = 14 \qquad \text{1A}$$

We have $f\left(\frac{3}{2}\right) = 0$.

$$\frac{27}{2} - \left(\frac{3a}{2} + 1\right)(6 - 1) + 14 = 0 \qquad \text{1M}$$

$$a = 3 \qquad \text{1A}$$

(b) $f(x) = 4x^3 - (3x + 1)(4x - 1) + 14$

$= 4x^3 - 12x^2 - x + 15$ 1M

$= (2x - 3)(2x^2 - 3x - 5)$ 1M

$$f(x) + g(x) = 0$$

$$(2x - 3)(2x^2 - 3x - 5) + (2x^2 - 3x - 5)(x + 2) + 3x - 1 = 0 \qquad \text{1M}$$

$$(2x^2 - 3x - 5)(3x - 1) + 3x - 1 = 0$$

$$(3x - 1)(2x^2 - 3x - 4) = 0 \qquad \text{1M}$$

$$x = \frac{1}{3} \quad \text{or} \quad 2x^2 - 3x - 4 = 0$$

Consider the equation $2x^2 - 3x - 4 = 0$.

$$x = \frac{3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{41}}{4}$$

The roots are not rational.

The equation $f(x) + g(x) = 0$ has 1 rational root. 1A

19. (a)

$$f(-1) = g(-1)$$

$$a(-1)^3 - b(-1)^2 - (-1) + 2 = (-1)^3 + a(-1)^2 - b \qquad \text{1M}$$

$$-a - b + 3 = -1 + a - b$$

$$a = 2 \qquad \text{1A}$$

$$f(1) = 0$$

$$2(1)^3 - b(1)^2 - 1 + 2 = 0 \qquad \text{1M}$$

$$b = 3 \qquad \text{1A}$$

(b) $f(x) - 2g(x) = -1$

$7x^2 + x - 9 = 0$ 1M

$$x = \frac{-1 \pm \sqrt{1^2 - 4(7)(-9)}}{2(7)}$$

$$= \frac{-1 \pm \sqrt{253}}{14}$$

The roots are irrational.

The claim is agreed. 1A

20. (a) Let $f(x) = (x^2 - 4)(Ax + B) + kx - 12$, where A and B are constants. 1M

$$f(2) = 0$$

$$(4-4)(2A+B) + 2k - 12 = 0 \quad \text{1M}$$

$$k = 6 \quad \text{1A}$$

(b)

$$f(0) = 20$$

$$(0-4)(0+B) + 0 - 12 = 20 \quad \text{1M}$$

$$B = -8 \quad \text{1A}$$

$$f(-3) = -40$$

$$(9-4)(-3A-8) + 6(-3) - 12 = -40$$

$$A = -2 \quad \text{1A}$$

$$f(x) = 0$$

$$(x^2-4)(-2x-8) + 6x - 12 = 0$$

$$(x+2)(x-2)(-2x-8) + 6(x-2) = 0$$

$$(x-2)[(x+2)(-2x-8)+6] = 0$$

$$2(x-2)(-x^2-6x-5) = 0$$

$$-2(x-2)(x+1)(x+5) = 0 \quad \text{1M}$$

$$x = 2 \quad \text{or} \quad -1 \quad \text{or} \quad -5$$

All the roots are integers.
The claim is correct. 1A

21. (a) Let $f(x) = (x^2 - 4)Q(x) + kx + 64$, where $Q(x)$ is a polynomial. 1M

$$f(2) = 0$$

$$(4-4)Q(2) + 2k + 64 = 0$$

$$k = -32 \quad \text{1A}$$

We have $f(-2) = 0 + 64 + 64 = 128$.

$$(-2)(-6+5)^2 - 2p + q = 128 \quad \text{1M}$$

$$-2p + q = 130$$

We have $f(2) = 0$.

$$2(6+5)^2 + 2p + q = 0$$

$$2p + q = -242$$

Solving, we have $p = -93$ and $q = -56$. 1A

(b) $f(x) = 0$

$$x(3x+5)^2 - 93x - 56 = 0$$

$$9x^3 + 30x^2 - 68x - 56 = 0 \qquad \text{1M}$$

$$(x-2)(9x^2 + 48x + 28) = 0 \qquad \text{1M}$$

$$(x-2)(3x+2)(3x+14) = 0$$

$$x = 2 \quad \text{or} \quad -\frac{2}{3} \quad \text{or} \quad -\frac{14}{3}$$

The equation $f(x) = 0$ has 3 rational roots. 1A

22. (a) Let $p(x) = (x^3 - 6x^2 - 3x + 16)(Ax + B) + 11x - 28$, where A and B are non-zero constants. 1A

$$\begin{cases} 15 = (1 - 6 - 3 + 16)(A + B) + 11 - 28 \\ 0 = (216 - 216 - 18 + 16)(6A + B) + 66 - 28 \end{cases} \qquad \text{1M}$$

Solving, we have $A = 3$ and $B = 1$. 1A

Thus, $p(x) = (x^3 - 6x^2 - 3x + 16)(3x + 1) + 11x - 28$.

(b) $p(-2) = (-8 - 24 + 6 + 17)(-6 + 1) - 22 - 28$

$= 0$

Thus, $x + 2$ is a factor of $p(x)$. 1

(c) $0 = (x^3 - 6x^2 - 3x + 16)(3x + 1) + 11x - 28$

$0 = 3x^4 - 17x^3 - 15x^2 + 56x - 12$

$0 = (x+2)(x-6)(3x^2 - 5x + 1)$ 1M

$x = -2 \quad \text{or} \quad 6 \quad \text{or} \quad 3x^2 - 5x + 1 = 0$

For $3x^2 - 5x + 1 = 0$,

$$x = \frac{5 \pm \sqrt{5^2 - 4(3)(1)}}{2(3)} \qquad \text{1M}$$

$$= \frac{5 \pm 13}{6}$$

Note that $\frac{5 + \sqrt{13}}{6}$ is irrational.

The claim is disagreed. 1A

(d) $6, -2, 0, \frac{5}{3}$ 1A

23. (a) Let $f(x) = (x^2 + 6x - 7)(Ax + B)$, where A and B are constants. 1M

$$\begin{cases} f(-1) = 96 = (1 - 6 - 7)(-A + B) \\ f(2) = 9 = (4 + 12 - 7)(2A + B) \end{cases} \qquad \text{1M}$$

Solving, we have $A = 3$ and $B = -5$.

The quotient is $3x - 5$. 1A

(b) $(x^2 + 6x - 7)(3x - 5) = 2x + 14$

$(x + 7)(x - 1)(3x - 5) - 2(x + 7) = 0$ 1M

$(x + 7)[(x - 1)(3x - 5) - 2] = 0$

$(x + 7)(3x^2 - 8x + 3) = 0$ 1A

$$x = -7 \quad \text{or} \quad \frac{8 \pm \sqrt{8^2 - 4(3)(3)}}{2(3)}$$

$$= -7 \quad \text{or} \quad \frac{4 \pm \sqrt{7}}{3}$$

Since $\frac{4 \pm \sqrt{7}}{3}$ is irrational, the claim is agreed. 1A

24. (a) Let $f(x) = (3 - 4x - x^2)(Ax + B) + (35x - 28)$, where A and B are constants. 1M

$$f(x) = (3 - 4x - x^2)(Ax + B) + (35x - 28)$$
$$= -Ax^3 + (-4A - B)x^2 + (3A - 4B + 35)x + (3B - 28)$$

We have $3A - 4B + 35 = -1$ and $3B - 28 = -10$. 1M

Solving, we have $A = -4$ and $B = 6$.

We have $m = -A = 4$ and $n = -4A - B = 10$. 1A

(b) $f(-2) = 4(-2)^3 + 10(-2)^2 - (-2) - 10$

$= 0$

Thus, $x + 2$ is a factor of $f(x)$. 1

(c) $f(x) = 0$

$(x + 2)(4x^2 + 2x - 5) = 0$ 1M

$x = -2 \quad \text{or} \quad 4x^2 + 2x - 5 = 0$

$x = -2 \quad \text{or} \quad 0.896 \quad \text{or} \quad -1.40$ 1M

All the roots of the equation $f(x) = 0$ are real numbers.

The claim is agreed. 1A

25. D

$$a\left(\frac{b}{a}\right)^2 - b\left(\frac{b}{a}\right) + c = 0$$

$$\frac{b^2}{a} - \frac{b^2}{a} + c = 0$$

$$c = 0$$

$$\text{Remainder} = a\left(-\frac{b}{a}\right)^2 - b\left(-\frac{b}{a}\right)$$
$$= \frac{b^2}{a} + \frac{b^2}{a}$$
$$= \frac{2b^2}{a}$$

26. D

$$(-3)^3 + 27(-3)^2 + k(-3) - 3 = 0$$
$$-3k + 213 = 0$$
$$k = 71$$

27. D

Let $f(x) = x^4 - 9x^2 - 24x - 16$.
Note that $f(-1) = f(4) = 0$.
$(x + 1)$ and $(x - 4)$ are factors of $f(x)$.
Thus, $x^4 - 9x^2 - 24x - 16 = (x^2 + 3x + 4)(x + 1)(x - 4)$.

28. A

$$f(2) = 0$$
$$8 - 8 + 2 + k = 0$$
$$k = -2$$

Remainder $= f(-1)$
$= -1 - 2 - 1 - 2$
$= -6$

29. D

$$f(5) = 0 = 2(5)^2 - 13(5) + k$$
$$k = 15$$

Required remainder $= f\left(-\frac{1}{2}\right)$
$= 2\left(-\frac{1}{2}\right)^2 - 13\left(-\frac{1}{2}\right) + 15$
$= 22$

30. A

$$p(-k) = -k^3 + k^3 - 4k - 16 = 0$$
$$k = -4$$

Remainder $= (-2)^3 - 4(-2)^2 + 4(-2) - 16$
$= -48$

31. C

$f\left(\frac{1}{p}\right) = 0$

$\frac{48}{p^4} - 3 = 0$

$p^4 = 16$

Remainder $= f(-p)$

$= 48(-p)^4 - 3$

$= 48(16) - 3$

$= 765$

32. C

$$a\left(-\frac{1}{3}\right)^8 + 81\left(-\frac{1}{3}\right)^3 + b\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + c = 0$$

$$\frac{a}{6561} + \frac{b}{9} + c - 2 = 0$$

$$\frac{a}{6561} + \frac{b}{9} + c = 2$$

Remainder $= g\left(\frac{1}{3}\right)$

$= a\left(\frac{1}{3}\right)^8 + 81\left(\frac{1}{3}\right)^3 + b\left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) + c$

$= \frac{a}{6561} + \frac{b}{9} + c + 2$

$= (2) + 2$

$= 4$

33. C

We have $g(x) = (x^2 - 5x + 2)(2x + 3) + ax + 3$.

$$g(1) = 0$$

$$(1 - 5 + 2)(2 + 3) + a + 3 = 0$$

$$a = 7$$

Remainder $= g\left(-\frac{1}{2}\right)$

$= \left(\frac{1}{4} + \frac{5}{2} + 2\right)(-1 + 3) - \frac{7}{2} + 3$

$= 9$

34. D

$$(-2)^3 - 6(-2)^2 - 3(-2) + k = 5$$

$$k = 31$$

35. $\boxed{\text{A}}$

$$(1)^8 + a(1)^7 + b = 2a$$
$$1 + a + b = 2a$$
$$b = a - 1$$

$$\text{Remainder} = (-1)^8 + a(-1)^7 + (a - 1)$$
$$= 0$$

36. $\boxed{\text{D}}$

Let $f(x) = (x^2 - 4)Q(x) + Ax + B$, where $Q(x)$ is a polynomial, A and B are constants.

$$\begin{cases} f(-2) = 0 = -2A + B \\ f(2) = 4 = 2A + B \end{cases}$$

Solving, we have $A = 1$ and $B = 2$.

Required remainder is $x + 2$.

37. $\boxed{\text{C}}$

Note that $f(3) = (3 - 1)^3 - 6(3 - 1) + 4 = 0$.

Thus, $x - 3$ is a factor of $f(x)$.

38. $\boxed{\text{A}}$

$$a(3)^3 - 2a(3)^2 - 27 = 0$$
$$9a - 27 = 0$$
$$a = 3$$

39. $\boxed{\text{A}}$

$$3^3 + a(3)^2 - 3b = 6$$
$$3a - b = -7$$
$$3a - b + 7 = (-7) + 7 = 0$$

40. $\boxed{\text{A}}$

$$2(2)^2 + 2 + m = 0$$
$$m = -10$$
$$2x^2 + x - 10 = (x - 2)(2x + 5)$$

Thus, $2x^2 + x + m$ is also divisible by $2x + 5$.

41. $\boxed{\text{B}}$

$$a\left(-\frac{4}{3}\right)^4 - 27\left(-\frac{4}{3}\right)^2 + b = 0$$

$$\frac{256a}{81} + b - 48 = 0$$

$$b = -\frac{256a}{81} + 48$$

$$\text{Remainder} = a\left(\frac{4}{3}\right)^4 - 27\left(\frac{4}{3}\right)^2 + \left(-\frac{256a}{81} + 48\right)$$

$$= 0$$

42. D

Let R be the required remainder.

$$\begin{cases} 5 = a(-2)^2 + 3(-2) + b \\ R = a(2)^2 + 3(2) + b \end{cases}$$

Subtract one equation from another.

$$R - 5 = 6 - (-6)$$

$$R = 17$$

43. B

$$\text{Remainder} = (-1)^{2023} + (-1)^{2022} + (-1)^{2021} + \ldots + (-1)$$

$$= [(-1) + 1] + [(-1) + 1] + \ldots + (-1)$$

$$= -1$$

44. D

$$(-1)^{2n} + k(-1) - 2k = 0$$

$$1 - 3k = 0$$

$$k = \frac{1}{3}$$

45. A

$$8\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right)^2 + k = 0$$

$$k = 0$$

$$\text{Remainder} = 8\left(-\frac{1}{2}\right)^3 - 4\left(-\frac{1}{2}\right)^2$$

$$= -2$$

46. C

$$g\left(\frac{1}{2}\right) = 5$$
$$\frac{1}{16} + \frac{a}{2} + b = 5$$
$$b = \frac{79}{16} - \frac{a}{2}$$

Remainder $= g\left(-\frac{1}{2}\right)$

$$= \frac{1}{16} - \frac{a}{2} + \left(\frac{79}{16} - \frac{a}{2}\right)$$
$$= 5 - a$$

47. D

$$9\left(\frac{c}{3}\right)^4 - c^2\left(\frac{c}{3}\right)^2 + 2\left(\frac{c}{3}\right) - 6 = 0$$
$$\frac{2c}{3} - 6 = 0$$
$$c = 9$$

48. B

$$f(-1) = (-1)^{2014} + a(-1) + b = 3$$
$$1 - a + b = 3$$
$$a - b = -2$$

Thus, $2a - 2b + 1 = 2(a - b) + 1 = 2(-2) + 1 = -3$.

49. A

$$0 = 4\left(\frac{1}{2}\right)^2 + m\left(\frac{1}{2}\right) + n$$
$$n = -1 - \frac{m}{2}$$

Required remainder $= 4\left(-\frac{1}{2}\right)^2 + m\left(-\frac{1}{2}\right) + n$

$$= 1 - \frac{m}{2} + \left(-1 - \frac{m}{2}\right)$$
$$= -m$$

50. B

$$P(-1) = (-1)^{2025} - 3(-1) + k = 4$$
$$-1 + 3 + k = 4$$
$$k = 2$$

Remainder $= P(1)$

$$= (1)^{2025} - 3(1) + 2$$
$$= 0$$

51. $\boxed{C}$

$0 = k(3)^3 + 2k(3)^2 + 90$

$k = -2$

Remainder $= -2(-1)^3 - 4(-1)^2 + 90$

$= 88$

52. $\boxed{C}$

$f(-k) = 0$

$-k^3 + k^3 + 4k - 8 = 0$

$k = 2$

Remainder $= f(1)$

$= 1 + 2 - 4 - 8$

$= -9$

53. $\boxed{B}$

$(k^2 + k)(k + 1) = 0$

$k(k + 1)^2 = 0$

$k = -1$ or 0 (rejected)

Remainder $= f(-2)$

$= [(-2)^2 - 1](-2 + 1)$

$= -3$

54. $\boxed{C}$

Let $g(x) = (2x + 3)q(x)$, where $q(x)$ is a polynomial.

$g(4x - 3) = [2(4x - 3) + 3]q(4x - 3)$

$= (8x - 3)q(4x - 3)$

Thus, $8x - 3$ is a factor of $g(4x - 3)$.

END OF PAPER