

1. B

Let O be the centre of the semi-circle.

Let M be the mid-point of AC .

$$OA = \frac{AB}{2} = \frac{17}{2} = 8.5 \text{ cm}$$

$$AM = \frac{AC}{2} = \frac{15}{2} = 7.5 \text{ cm}$$

$$OM = \sqrt{OA^2 - AM^2} = \sqrt{8.5^2 - 7.5^2} = 4 \text{ cm}$$

$$\sin \angle AOM = \frac{AM}{OA}$$

$$\angle AOM \approx 61.9^\circ$$

$$\angle AOM + \angle COM + \angle BOC = 180^\circ$$

$$2\angle AOM + \angle BOC = 180^\circ$$

$$\angle BOC \approx 56.1^\circ$$

Required area

$$\begin{aligned} &= \frac{(15)(4)}{2} + \pi(8.5^2) \times \frac{\angle BOC}{360^\circ} \\ &= 65.4 \text{ cm}^2 \end{aligned}$$

2. B

Let O be the centre of the circle and M be the mid-point of AB .

$$AM = \frac{AB}{2} = \frac{8}{2} = 4 \text{ cm}$$

$$\sin \angle AOM = \frac{AM}{OA}$$

$$\sin \angle AOM = \frac{4}{5}$$

$$\angle AOM \approx 53.1^\circ$$

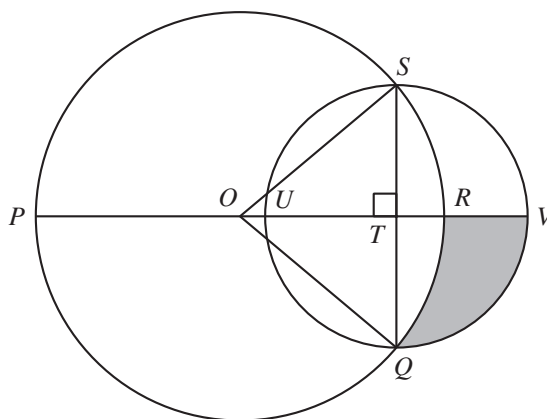
Required area

$$\begin{aligned} &= \pi 5^2 \times \frac{2\angle AOM}{360^\circ} - \frac{1}{2} \times 8 \times \sqrt{5^2 - 4^2} \\ &\approx 11 \text{ cm}^2 \end{aligned}$$

3. A

T is the centre of circle $QVSU$.

Let O be the centre of circle $PQRS$.



$$OS = OR = \frac{27 + 3}{2} = 15 \text{ cm}$$

$$ST = \sqrt{OS^2 - OT^2} = \sqrt{15^2 - (15 - 3)^2} = 9 \text{ cm}$$

$$\angle TOQ = \cos^{-1} \frac{12}{15} \approx 36.9^\circ$$

Required area

$$= \pi(9)^2 \times \frac{90^\circ}{360^\circ} - \left(\pi(15)^2 \times \frac{\angle ROQ}{360^\circ} - \frac{(12)(9)}{2} \right)$$

$$\approx 45 \text{ cm}^2$$

4. C

C is the centre of the circle $BFEG$.

Let O be the centre of circle $AGDF$.

Let r cm be the radius of the circle $BFEG$.

$$OC^2 + CF^2 = OF^2$$

$$\left(\frac{30 + r + 5}{2} - 5 \right)^2 + r^2 = \left(\frac{30 + r + 5}{2} \right)^2$$

$$r^2 - 5r - 150 = 0$$

$$r = 15 \quad \text{or} \quad -10 \text{ (rejected)}$$

$$\text{Radius of circle } AFDG = \frac{30 + 15 + 5}{2} = 25 \text{ cm}$$

$$\sin \angle COF = \frac{CF}{OF}$$

$$= \frac{15}{25}$$

$$\angle COF \approx 36.9^\circ$$

Required area

$$= 25^2 \pi \times \frac{360^\circ - 2\angle COF}{360^\circ} + \frac{(15 \times 2)(25 - 5)}{2} - \frac{\pi(15)^2}{2}$$

$$\approx 1508 \text{ cm}^2$$

5. D

Let O be the centre of the semi-circle and M be the mid-point of AC .

$$OA = \frac{AD}{2} = \frac{10}{2} = 5 \text{ cm}$$

$$AM = \frac{AC}{2} = \frac{4}{2} = 2 \text{ cm}$$

$$\sin \angle AOM = \frac{AM}{OA}$$

$$\sin \angle AOM = \frac{2}{5}$$

$$\angle AOM \approx 23.6^\circ$$

Required area

$$= \pi \times 5^2 \times \frac{2\angle AOM}{360^\circ} - \frac{1}{2} \times 4 \times \sqrt{5^2 - 2^2}$$

$$\approx 1.12 \text{ cm}^2$$

6. C

Let M be the mid-point of CD .

$$DM = 6 \sin 30^\circ = 3 \text{ cm}$$

$$\sin \angle DBM = \frac{DM}{BD}$$

$$\angle DBM \approx 48.6^\circ$$

$$\angle CBD = 2\angle DBM \approx 97.2^\circ$$

Required area

$$= \left[6^2 \pi \times \frac{60^\circ}{360^\circ} - \frac{1}{2} (6^2) \sin 60^\circ \right] + \left[4^2 \pi \times \frac{\angle CBD}{360^\circ} - \frac{1}{2} (4)^2 \sin \angle CBD \right]$$

$$\approx 8.89 \text{ cm}^2$$

7. C

Let θ be the angle subtended by the chord at the centre.

$$\sin \frac{\theta}{2} = \frac{24 \div 2}{13}$$

$$\theta \approx 135^\circ$$

$$\text{Required perimeter} = 2\pi(13) \times \frac{\theta}{360^\circ} + 24$$

$$\approx 55 \text{ cm}^2$$

8. D

$$\text{Radius} = \sqrt{7^2 + \left(\frac{48}{2}\right)^2}$$

$$= 25 \text{ cm}$$

Let θ be the angle of the sector corresponding to the shaded segment.

$$\cos \frac{\theta}{2} = \frac{7}{25}$$

$$\theta \approx 147^\circ$$

$$\text{Required area} = 25^2 \pi \times \frac{\theta}{360^\circ} - \frac{(48)(7)}{2}$$

$$\approx 636 \text{ cm}^2$$

9. D

$$PT = \frac{PU + US}{2} = \frac{23 + 73}{2} = 48 \text{ cm}$$

Let r cm be the radius.

$$OP^2 = PT^2 + OT^2$$

$$r^2 = 48^2 + (r - 36)^2$$

$$72r = 3600$$

$$r = 50$$

Let V be a point on RT such that $QV \perp RT$.

$$\sin \angle QOR = \frac{QV}{OQ}$$

$$\sin \angle QOR = \frac{48 - 23}{50}$$

$$\angle QOR = 30^\circ$$

$$\text{Required area} = \pi(50)^2 \times \frac{30^\circ}{360^\circ}$$

$$= \frac{625\pi}{3} \text{ cm}^2$$

10. A

Radius of the circle is 13 cm.

Let θ be the angle subtended by $\widehat{AB}$.

$$\sin \frac{\theta}{2} = \frac{5}{13}$$

$$\theta \approx 45.2^\circ$$

Required length

$$= 2\pi(13) \times \frac{\theta}{360^\circ}$$

$$\approx 10 \text{ cm}$$

11. C

$$\angle ACB = \angle ABC = 70^\circ$$

$$\angle BCD = \angle ACB + \angle ACD = 70^\circ + 20^\circ = 90^\circ$$

Thus, BD is a diameter of the circle and $\angle BAD = 90^\circ$.

$$\angle ABD = \angle ACD = 20^\circ$$

Consider $\triangle ABD$.

$$\cos 20^\circ = \frac{12}{BD}$$

$$BD \approx 12.8 \text{ cm}$$

$$\angle DBC = \angle ABC - \angle ABD = 70^\circ - 20^\circ = 50^\circ$$

Consider $\triangle BCD$.

$$\sin 50^\circ = \frac{CD}{BD}$$

$$CD \approx 10 \text{ cm}$$

12. C

The centre of the circle lies on AC .

Since E is the mid-point of BD , we have $AC \perp BD$.

$$\angle DAC = \angle CBE = 22^\circ$$

$$\angle ADE + \angle DAE + \angle AED = 180^\circ$$

$$\angle ADE + 22^\circ + 90^\circ = 180^\circ$$

$$\angle ADE = 68^\circ$$

13. C

$$\angle AOB = 2\angle ADB = 2x$$

Consider $\triangle OBC$.

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ$$

$$\angle BOC + y + y = 180^\circ$$

$$\angle BOC = 180^\circ - 2y$$

$$\angle AOC = \angle AOB + \angle BOC$$

$$= 2x + (180^\circ - 2y)$$

$$= 180^\circ + 2x - 2y$$

14. B

Let O be the centre of semi-circle PQT .

$$RQ = OT = \frac{PQ}{2} = 40 \text{ cm}$$

$$\angle RPQ = \tan^{-1} \frac{QR}{PQ} \approx 26.6^\circ$$

$$\angle ROQ = 2\angle RPQ \approx 53.1^\circ$$

Required area

$$= \frac{(80)(40)}{2} - \pi(40)^2 \times \frac{\angle ROQ}{360^\circ} - \frac{1}{2}(40)(40 \sin \angle ROQ)$$

$$\approx 218 \text{ cm}^2$$

15. D

$$\angle COD = 2\angle CBD = 2 \times 32^\circ = 64^\circ$$

$$\angle BDO = \angle CBD = 32^\circ$$

$$\angle BEO = \angle ODE + \angle DOE = 32^\circ + 64^\circ = 96^\circ$$

16. A

Note that $\angle ACB = 90^\circ$ and AB is a diameter of the circle.

$$\text{Required area} = \frac{1}{2} \times \left(\frac{25}{2}\right)^2 \pi - \frac{(7)(24)}{2}$$

$$\approx 161 \text{ cm}^2$$

17. A

$$\angle ABC = 90^\circ$$

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$90^\circ + 32^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 58^\circ$$

$$\angle BDC = \angle BAC = 58^\circ$$

$$\angle BCD = \angle BDC = 58^\circ$$

$$\angle BCD = \angle ACB + \angle ACD$$

$$58^\circ = 32^\circ + \angle ACD$$

$$\angle ACD = 26^\circ$$

$$\angle AED = \angle ACD + \angle BDC$$

$$= 26^\circ + 58^\circ$$

$$= 84^\circ$$

18. B

Let O be the centre of the circle, and N be the mid-point of AB such that $ON \perp AB$.

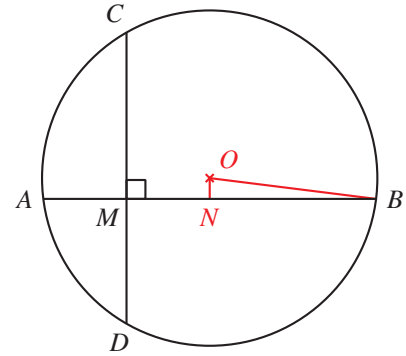
$$\triangle AMD \sim \triangle CMB$$

$$\frac{AM}{MD} = \frac{CM}{MB}$$

$$CM = 4$$

$$NB = \frac{2+6}{2} = 4 \text{ and } ON = 4 - \frac{4+3}{2} = 0.5$$

$$\text{Radius} = \sqrt{4^2 + 0.5^2} \approx 4.03$$



19. A

$$\angle OCA = \angle OAC = 38^\circ$$

$$\angle AOC + \angle OCA + \angle OAC = 180^\circ$$

$$\angle AOC + 38^\circ + 38^\circ = 180^\circ$$

$$\angle AOC = 104^\circ$$

$$\angle AOC + 2\angle ABC = 360^\circ$$

$$108^\circ + 2\angle ABC = 360^\circ$$

$$\angle ABC = 128^\circ$$

$$\angle BAC = \angle OCA = 38^\circ$$

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$38^\circ + 128^\circ + \angle ACB = 180^\circ$$

$$\angle ACB = 14^\circ$$

20. D

Note that $\triangle AED \sim \triangle BEC$.

Let $AE = 2k$ cm, where k is a non-zero constant.

$$\frac{DE}{CE} = \frac{AE}{BE}$$

$$\frac{DE}{3k} = \frac{2k}{4k}$$

$$DE = 1.5k$$

$$\text{Required area} = 6 \times \frac{BE}{DE}$$

$$= 6 \times \frac{4}{1.5}$$

$$= 16 \text{ cm}^2$$

21. B

$$\angle BOC = 2\angle BAC = 64^\circ$$

$$\angle OCE + \angle BOC = \angle BEC$$

$$\angle OCE + 64^\circ = 79^\circ$$

$$\angle OCE = 15^\circ$$

$$\angle AOC + \angle OAC + \angle OCA = 180^\circ$$

$$\angle AOC + 15^\circ + 15^\circ = 180^\circ$$

$$\angle AOC = 150^\circ$$

We have $\angle AOD : \angle COD = \widehat{AD} : \widehat{DC} = 1 : 2$.

$$\angle AOD + \angle COD + \angle AOC = 360^\circ$$

$$\frac{\angle COD}{2} + \angle COD + 150^\circ = 360^\circ$$

$$\angle COD = 140^\circ$$

$$\angle CAD = \frac{\angle COD}{2} = 70^\circ$$

22. D

Let $\angle BOC = \theta$.

$$\angle OAB = \angle OBA = \angle BOC = \theta$$

$$\angle AOB + \angle OAC + \angle OBA = 180^\circ$$

$$\angle AOB + \theta + \theta = 180^\circ$$

$$\angle AOB = 180^\circ - 2\theta$$

$$\angle AOB + \angle BOC = \angle AOC$$

$$(180^\circ - 2\theta) + \theta = 130^\circ$$

$$\theta = 50^\circ$$

$$\widehat{AB} : \widehat{BC} = \angle AOB : \angle BOC$$

$$= 80^\circ : 50^\circ$$

$$= 8 : 5$$

23. C

$$\angle ABC = 180^\circ - 68^\circ = 112^\circ$$

$$\angle ABD = 90^\circ$$

$$\angle CBD = 112^\circ - 90^\circ = 22^\circ$$

$$\angle BDC = \angle CBD = 22^\circ$$

$$\angle ADB = 68^\circ - 22^\circ = 46^\circ$$

24. B

$$\angle BAC = \angle ACD = 46^\circ$$

$$\angle ABD = \angle ACD = 46^\circ$$

$$\angle ADB = \angle ABD = 46^\circ$$

$$\begin{aligned}\angle ABD + \angle ADB + \angle BAD &= 180^\circ \\ 46^\circ + 46^\circ + (46^\circ + \angle CAD) &= 180^\circ \\ \angle CAD &= 42^\circ\end{aligned}$$

25. B

$$\begin{aligned}\angle AOB &= \angle COD = \angle DOE \\ \angle AOB + \angle BOC + \angle COD + \angle DOE &= 180^\circ \\ 3\angle AOB + 54^\circ &= 180^\circ \\ \angle AOB &= 42^\circ \\ \angle OAD &= \angle ODA \\ \angle AOD + \angle OAD + \angle ODA &= 180^\circ \\ (42^\circ + 54^\circ + 42^\circ) + 2\angle ODA &= 180^\circ \\ \angle ODA &= 21^\circ \\ \angle DOF + \angle ODF + \angle OFD &= 180^\circ \\ (54^\circ + 42^\circ) + 21^\circ + \angle OFG &= 180^\circ \\ \angle OFG &= 63^\circ\end{aligned}$$

26. A

$$\begin{aligned}\angle COD &= \angle BOC = \angle AOB = 70^\circ \\ \angle AOD &= 360^\circ - 70^\circ \times 3 = 150^\circ \\ \angle ACD &= \frac{\angle AOD}{2} = \frac{150^\circ}{2} = 75^\circ\end{aligned}$$

27. C

Note that regular octagon is cyclic.

Let O be the centre of the circumcircle.

$$\begin{aligned}\angle GOH &= \frac{360^\circ}{8} \\ &= 45^\circ\end{aligned}$$

We have $r = \frac{45^\circ}{2} = 22.5^\circ$ and $p : q : r = \widehat{BAH} : \widehat{BAH} : \widehat{GH} = 2 : 2 : 1$.

$$\begin{aligned}p + q + r &= 2r + 2r + r \\ &= 5r \\ &= 5(22.5^\circ) \\ &= 112.5^\circ\end{aligned}$$

28. C

Let $\angle RPQ = x$.

$$\angle QOR = 2\angle RPQ = 2x$$

$$\angle POQ = \frac{\angle QOR}{2} = x$$

$$\angle OQP = \angle QOR = 2x$$

$$\angle OPQ = \angle OQP = 2x$$

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

$$x + 2x + 2x = 180^\circ$$

$$x = 36^\circ$$

29. B

Let $\angle AOD = \theta$.

$$\angle ABD = \frac{\angle AOD}{2} = \frac{\theta}{2}$$

$$\frac{\angle BAC}{\angle ABD} = \frac{\widehat{BC}}{\widehat{AD}}$$

$$\frac{\angle BAC}{\left(\frac{\theta}{2}\right)} = \frac{1}{4}\angle BAC = \frac{\theta}{8}$$

$$\angle ABT + \angle TAB + \angle ATB = 180^\circ$$

$$\frac{\theta}{2} + \frac{\theta}{8} + 120^\circ = 180^\circ$$

$$\theta = 96^\circ$$

30. C

$$\frac{\angle BAD}{\angle BDC} = \frac{\widehat{BD}}{\widehat{BC}}$$

$$\frac{\angle BAD}{20^\circ} = \frac{2}{1}$$

$$\angle BAD = 40^\circ$$

$$\angle ADB = 90^\circ$$

$$\angle ABD + \angle ADB + \angle BAD = 180^\circ$$

$$\angle ABD = 50^\circ$$

31. C

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$2(44^\circ) + \angle AOB = 180^\circ$$

$$\angle AOB = 92^\circ$$

$$\angle BOD = 2\angle BCD$$

$$92^\circ + \angle AOD = 2(62^\circ)$$

$$\angle AOD = 32^\circ$$

32. D

$$\angle ACE = \angle AEC = 60^\circ$$

$$\angle ACD + \angle AED = 180^\circ$$

$$(60^\circ + 28^\circ) + (60^\circ + \angle CED) = 180^\circ$$

$$\angle CED = 32^\circ$$

Since $AB = CD$, we have $\angle ACB = \angle CED = 32^\circ$.

33. C

$$\angle ACD = 90^\circ$$

$$\angle BAC = \angle BCA$$

$$\angle BAD + \angle BCD = 180^\circ$$

$$(\angle BAC + 50^\circ) + (\angle BCA + 90^\circ) = 180^\circ$$

$$2\angle BCA + 140^\circ = 180^\circ$$

$$\angle BCA = 20^\circ$$

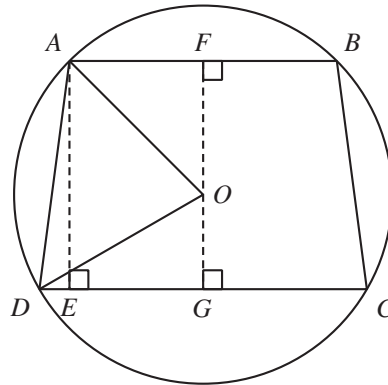
34. D

Note that $\angle ADC = \angle BCD = 80^\circ$ and $AB \parallel CD$.

Let E be a point on CD such that $AE \perp CD$.

Let F and G be the mid-points of AB and CD respectively.

Denote the centre of the circle by O .



Consider $\triangle ADE$.

$$\tan \angle ADE = \frac{AE}{DE}$$

$$\tan 80^\circ = \frac{AE}{\left(\frac{8-6}{2}\right)}$$

$$AE = \tan 80^\circ \text{ cm}$$

Let $OG = x$ cm. Consider $\triangle ODG$ and $\triangle OAF$.

$$\begin{aligned} r^2 &= 3^2 + (AE - x)^2 = 4^2 + x^2 \\ -2(AE)x &= 7 - AE^2 \\ x &\approx 2.22 \end{aligned}$$

$$\begin{aligned} \text{Required area} &= r^2\pi \\ &= (4^2 + x^2)\pi \\ &\approx 65.7 \text{ cm}^2 \end{aligned}$$

35. A

$$\begin{aligned} \angle ABC &= 90^\circ \\ \angle ABC + \angle BAC + \angle ACB &= 180^\circ \\ 90^\circ + \angle BAC + 42^\circ &= 180^\circ \\ \angle BAC &= 48^\circ \end{aligned}$$

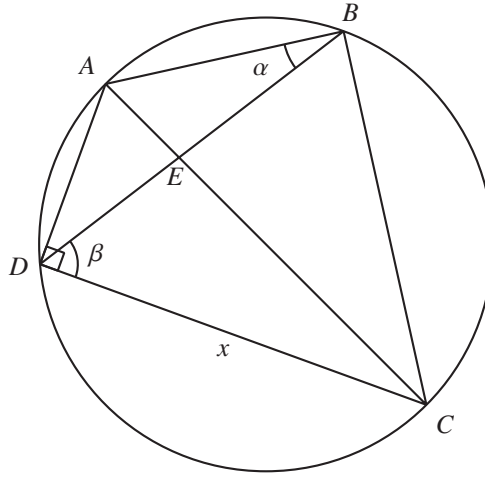
$$\begin{aligned} \frac{\angle CAD}{\angle BAC} &= \frac{\widehat{CD}}{\widehat{BC}} \\ \frac{\angle CAD}{48^\circ} &= \frac{2}{3} \\ \angle CAD &= 32^\circ \end{aligned}$$

$$\begin{aligned} \angle BCD + \angle BAD &= 180^\circ \\ \angle BCD + (48^\circ + 32^\circ) &= 180^\circ \\ \angle BCD &= 100^\circ \end{aligned}$$

$$\begin{aligned} \frac{\angle ABE}{\angle ACB} &= \frac{\widehat{AE}}{\widehat{AB}} \\ \frac{\angle ABE}{42^\circ} &= \frac{6}{7} \\ \angle ABE &= 36^\circ \\ \angle CBF &= \angle ABC - \angle ABE = 90^\circ - 36^\circ = 54^\circ \\ \angle BFC + \angle CBF + \angle BCF &= 180^\circ \\ \angle BFC + 54^\circ + 100^\circ &= 180^\circ \\ \angle BFC &= 26^\circ \end{aligned}$$

36. C

Draw circle $ABCD$.



Note that AC is a diameter of the circle and $\angle ABC = 90^\circ$.

We have $\angle ACD = \angle ABD = \alpha$ and $\angle BAC = \angle BDC = \beta$.

$$\begin{aligned} BC &= AC \sin \beta \\ &= \left(\frac{x}{\cos \alpha} \right) \sin \beta \\ &= \frac{x \sin \beta}{\cos \alpha} \end{aligned}$$

37. B

$$\angle AOC = 2 \times 74^\circ = 148^\circ.$$

$$\angle COD = 148^\circ - 52^\circ = 96^\circ.$$

$$\angle ODC = \frac{180^\circ - 96^\circ}{2} = 42^\circ.$$

38. B

Let $\angle ABC = \theta$.

$$\angle ADE = \angle CDF = \angle ABC = \theta$$

$$\angle BCD = \angle CDF + \angle CFD = \theta + 28^\circ$$

$$\angle BAD = \angle ADE + \angle AED = \theta + 50^\circ$$

$$\angle BAD + \angle BCD = 180^\circ$$

$$(\theta + 50^\circ) + (\theta + 28^\circ) = 180^\circ$$

$$\theta = 51^\circ$$

$$\angle ACB = \angle ABC = 51^\circ$$

$$\angle ACB = \angle CAF + \angle AFC$$

$$51^\circ = \angle CAD + 28^\circ$$

$$\angle CAD = 23^\circ$$

39. A

$$\angle BCD = 90^\circ$$

$$\angle BCA = \angle BCD - \angle ACD = 90^\circ - 36^\circ = 54^\circ$$

$$\angle BCD = \angle CDE + \angle CED$$

$$90^\circ = 68^\circ + \angle CED$$

$$\angle CED = 22^\circ$$

$$\angle BCA = \angle CAE + \angle CEA$$

$$54^\circ = \angle CAE + 22^\circ$$

$$\angle CAE = 32^\circ$$

$$\angle CBD = \angle CAD = 32^\circ$$

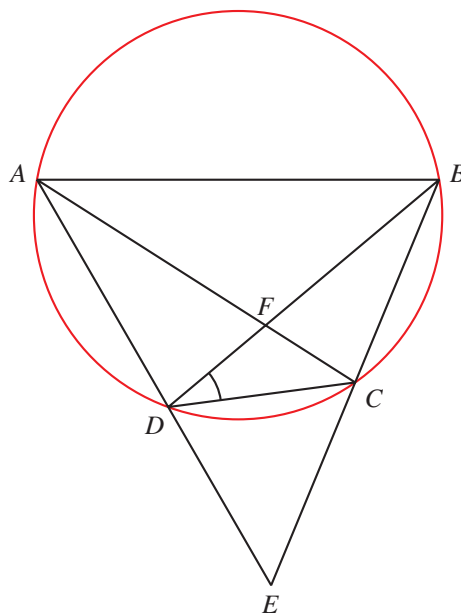
$$\angle BFC + \angle CBF + \angle BCF = 180^\circ$$

$$\angle BFC + 32^\circ + 54^\circ = 180^\circ$$

$$\angle BFC = 94^\circ$$

40. C

Draw circle $ABCD$.



Consider the cyclic quadrilateral $ABCD$.

We have $\angle BAD = \angle DCE$ and $\angle ABC = \angle CDE$.

Since $CE = DE$, we have $\angle DCE = \angle CDE = \angle ABC = \angle BAD$.

Thus, we have $EA = EB$.

$$AE - DE = BE - CE$$

$$AD = BC$$

$$\angle ACD = \angle BDC$$

Consider $\triangle CDF$.

$$\angle CDF + \angle FCD = 64^\circ$$

$$2\angle CDF = 64^\circ$$

$$\angle CDF = 32^\circ$$

41. C

$$\frac{\sin \angle TPQ}{5} = \frac{\sin 120^\circ}{7}$$

$$\angle TPQ \approx 38.2^\circ \quad \text{or} \quad 140^\circ \text{ (rejected)}$$

Let X be a point on the circle such that XQ is a diameter of the circle.

$$\angle PXQ = \angle TPQ \text{ and radius} = \frac{XQ}{2} = \frac{1}{2} \times \frac{PQ}{\sin \angle PXQ} \approx 5.66 \text{ cm}$$

42. B

$$\angle EAD = \angle EDA$$

$$\angle AED + \angle EAD + \angle EDA = 180^\circ$$

$$70^\circ + 2\angle EAD = 180^\circ$$

$$\angle EAD = 55^\circ$$

$$\angle ACD = \angle EAD = 55^\circ$$

$$\angle CDB = \angle CBG = 40^\circ$$

$$\angle CHD + \angle CDH + \angle DCH = 180^\circ$$

$$\angle CHD + 40^\circ + 55^\circ = 180^\circ$$

$$\angle CHD = 85^\circ$$

43. C

$$\angle OAT = 90^\circ$$

$$\text{Reflex } \angle AOC = 2\angle ABC = 2 \times 120^\circ = 240^\circ$$

Consider the polygon $AOCT$.

$$\angle OAT + \angle ATC + \angle TCO + (\text{reflex } \angle AOC) = 360^\circ$$

$$90^\circ + 20^\circ + \angle TCO + 240^\circ = 360^\circ$$

$$\angle TCO = 10^\circ$$

$$\angle ODC = \angle OCD = 10^\circ$$

$$\angle COD + \angle ODC + \angle OCD = 180^\circ$$

$$\angle COD + 10^\circ + 10^\circ = 180^\circ$$

$$\angle COD = 160^\circ$$

$$\angle AOD = (\text{reflex } \angle AOC) - \angle COD$$

$$= 240^\circ - 160^\circ$$

$$= 80^\circ$$

44. C

$$\angle PAE = \angle PEA$$

$$\angle APE + \angle PAE + \angle PEA = 180^\circ$$

$$84^\circ + 2\angle PAE = 180^\circ$$

$$\angle PAE = 48^\circ$$

$$\angle ACE = \angle PAE = 48^\circ$$

$$\angle AGB + \angle GAB + \angle ABG = 180^\circ$$

$$\angle AGB + 27^\circ + 66^\circ = 180^\circ$$

$$\angle AGB = 87^\circ$$

$$\angle CGF = \angle AGB = 87^\circ$$

$$\angle CFD = \angle CGF + \angle FCG$$

$$= 87^\circ + 48^\circ$$

$$= 135^\circ$$

45. C

$$\angle ADC = 180^\circ - 100^\circ = 80^\circ$$

$$\angle ADB = 80^\circ \times \frac{2}{2+3} = 32^\circ$$

$$\angle BCA = \angle ADB = 32^\circ$$

$$\angle DAT = \angle ADC - \angle ATD = 50^\circ$$

$$\angle ACT = \angle DAT = 50^\circ$$

$$\angle BCT = 32^\circ + 50^\circ = 82^\circ$$

46. D

Let O be the centre of the circle. $AQOP$ is a square of length 9 cm.

$$\angle ACB = \angle ABC = 45^\circ, QC = BP = \frac{9}{\tan 45^\circ} = 9 \text{ cm},$$

$$CO = BO = \frac{9}{\sin 45^\circ} = 9\sqrt{2} \text{ cm}$$

$$\text{Perimeter} = (9 + 9) + (9 + 9) + (9\sqrt{2} + 9\sqrt{2}) \approx 61 \text{ cm}$$

47. C

$$\angle ACB = 90^\circ$$

Let $\angle BAC = \theta$.

We have $\angle CAD = \frac{\theta}{2}$ and $\angle ACD = \theta$.

$$\angle BCD + \angle BAD = 180^\circ$$

$$(90^\circ + \theta) + \left(\theta + \frac{\theta}{2}\right) = 180^\circ$$

$$\theta = 36^\circ$$

$$\angle AEC = \angle BAC = 36^\circ$$

$$\angle ACE + \angle CAE + \angle AEC = 180^\circ$$

$$(\angle DCE + 36^\circ) + (42^\circ + 18^\circ) + 36^\circ = 180^\circ$$

$$\angle DCE = 48^\circ$$

48. C

$$\angle ACP = \angle ADC = 102^\circ$$

$$\angle RAC = \angle APC + \angle ACP$$

$$= 26^\circ + 102^\circ$$

$$= 128^\circ$$

$$\angle RAD = \angle DAC = \frac{\angle RAC}{2} = 64^\circ$$

$$\angle ARD + \angle RAD = \angle ADC$$

$$\angle ARD + 64^\circ = 102^\circ$$

$$\angle ARD = 38^\circ$$

49. B

$$\angle RSV = \angle RQS = 44^\circ$$

$$\angle QSP = \angle QPU = 40^\circ$$

$$\angle PST + \angle QSP + \angle QSR + \angle RSV = 180^\circ$$

$$\angle PST + 40^\circ + 33^\circ + 44^\circ = 180^\circ$$

$$\angle PST = 63^\circ$$

Since $TP = TS$, we have $\angle TPS = \angle PST$.

$$\angle UTV + \angle TPS + \angle PST = 180^\circ$$

$$\angle UTV + 63^\circ + 63^\circ = 180^\circ$$

$$\angle UTV = 54^\circ$$

50. B

$$\angle ABC = \angle ACM = 40^\circ \quad (\text{circle } ABC + \text{tangent } MN)$$

$$\angle ADC = \angle DBC + \angle BCD$$

$$= 40^\circ + 40^\circ$$

$$= 80^\circ$$

$$\angle CDE = \angle ECM = 40^\circ \quad (\text{circle } CDE + \text{tangent } MN)$$

$$\angle ADE = \angle ADC - \angle CDE = 80^\circ - 40^\circ = 40^\circ$$

$$\angle ACD = \angle ADE = 40^\circ \quad (\text{circle } CDE + \text{tangent } AB)$$

$$\angle CAD + \angle ADC + \angle ACD = 180^\circ$$

$$\angle CAD + 80^\circ + 40^\circ = 180^\circ$$

$$\angle CAD = 60^\circ$$

51. A

Let $\angle OBC = \theta$.

$$\angle OCB = \angle OBC = \theta$$

$$\angle CAD = \angle ABC = \theta + 50^\circ$$

$$\angle BAC = \angle CAD = \theta + 50^\circ$$

$$\angle BOC = 2\angle BAC = 2(\theta + 50^\circ) = 2\theta + 100^\circ$$

$$\angle BOC + \angle OBC + \angle OCB = 180^\circ$$

$$(2\theta + 100^\circ) + \theta + \theta = 180^\circ$$

$$\theta = 20^\circ$$

52. B

Let O be the centre of the semi-circle $ABCD$.

We have $OC \perp EF$ and $OB \perp BE$.

Thus, $\angle BOC = 360^\circ - 90^\circ - 90^\circ - 90^\circ = 90^\circ$.

$$\angle AOC = \angle AFE + 90^\circ = 132^\circ$$

$$\angle AOB = 132^\circ - 90^\circ = 42^\circ$$

$$\angle ACB = \frac{\angle AOB}{2} = 21^\circ$$

53. C

$$\angle GFD = \angle GDF$$

$$\angle DGF + \angle GFD + \angle GDF = 180^\circ$$

$$66^\circ + 2\angle GDF = 180^\circ$$

$$\angle GDF = 57^\circ$$

$$\angle BDC = \angle ABC = 30^\circ$$

$$\angle BDF = \angle CDG - \angle BDC - \angle GDF = 130^\circ - 30^\circ - 57^\circ = 43^\circ$$

$$\angle BCF = \angle BDF = 43^\circ$$

$$\angle ECF = \angle EFG = 20^\circ$$

$$\angle BCE = \angle BCF + \angle ECF = 43^\circ + 20^\circ = 63^\circ$$

54. C

$$\angle ABC + \angle ADC = 180^\circ$$

$$\angle ABC + 46^\circ = 180^\circ$$

$$\angle ABC = 134^\circ$$

$$\angle ACE = \angle ABC = 134^\circ$$

Since $AB = BC$, we have $\angle ADB = \angle BDC$.

$$\angle ADB = \frac{\angle ADC}{2} = \frac{46^\circ}{2} = 23^\circ$$

$$\angle ACB = \angle ADB = 23^\circ$$

$$\angle BCE = \angle ACB + \angle ACE = 23^\circ + 134^\circ = 157^\circ$$

55. B

$$EA = ED$$

$$\angle EDA = \frac{180^\circ - 86^\circ}{2} = 47^\circ$$

$$\angle ACD = \angle EDA = 47^\circ$$

$$\angle FGH = 180^\circ - \angle FEH = 94^\circ$$

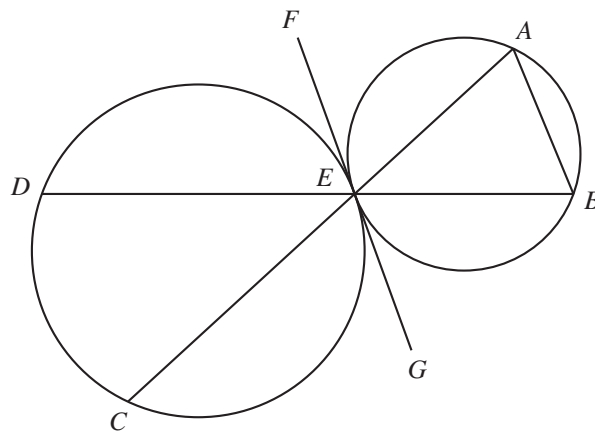
$$\begin{aligned}\angle GBC &= \frac{180^\circ - 94^\circ}{2} = 43^\circ \\ \angle BDC &= \angle GBC = 43^\circ \\ \angle AID &= \angle BDC + \angle ACD = 90^\circ\end{aligned}$$

56. B

$$\begin{aligned}\angle AEF &= 180^\circ - 125^\circ = 55^\circ \\ \angle EAF &= \angle EFB - \angle AEF \\ &= 120^\circ - 55^\circ \\ &= 65^\circ \\ \text{Since } BA &= BC, \text{ we have } \angle BCA = \angle BAC = 65^\circ. \\ \angle ABC &= 180^\circ - \angle BAC - \angle BCA \\ &= 180^\circ - 65^\circ - 65^\circ \\ &= 50^\circ \\ \angle DFB &= 180^\circ - \angle DFB - \angle FBD \\ \angle CDE &= 180^\circ - 120^\circ - 50^\circ \\ &= 10^\circ\end{aligned}$$

57. D

Draw the common tangent FG at point E .



$$\begin{aligned}BE^2 &= AB^2 + AE^2 - 2(AB)(AE) \cos \angle BAE \\ BE^2 &= 10^2 + 16^2 - 2(10)(16) \cos 60^\circ \\ BE &= 14 \text{ cm} \\ DE &= BD - BE = 35 - 14 = 21 \text{ cm} \\ \angle DCE &= \angle FED = \angle ABE \\ \angle DEC &= \angle BEA\end{aligned}$$

We have $\triangle ABE \sim \triangle CDE$.

$$\frac{AE}{CE} = \frac{BE}{DE}$$

$$\frac{16}{CE} = \frac{14}{21}$$

$$CE = 24 \text{ cm}$$

58. A

$$\angle CAB = \angle FBC = 39^\circ$$

$$\angle ACD = \angle AGC + \angle CAB = 29^\circ + 39^\circ = 68^\circ$$

Since $DA = DC$, we have $\angle CAD = \angle ACD = 68^\circ$.

$$\angle ADC = 180^\circ - 68^\circ - 68^\circ = 44^\circ$$

59. C

$$\angle BED = \angle CBD = 40^\circ$$

$$\angle DBE + \angle BDE + \angle BED = 180^\circ$$

$$\angle DBE + 88^\circ + 40^\circ = 180^\circ$$

$$\angle DBE = 52^\circ$$

Since $DE = EF$, we have $\angle EBF = \angle DBE = 52^\circ$.

$$\angle AFB = \angle BDE = 88^\circ$$

$$\angle BAF + \angle AFB = \angle CBF$$

$$\angle BAF + 88^\circ = 52^\circ + 52^\circ + 40^\circ$$

$$\angle BAF = 56^\circ$$

60. A

Let $\angle ACE = \theta$.

$$\angle BEC = \angle ACE = \theta$$

$$\angle ACB = \angle ADB = 28^\circ$$

$$\angle EBC = \angle ECQ = 74^\circ$$

$$\angle EBC + \angle BCE + \angle BEC = 180^\circ$$

$$74^\circ + (28^\circ + \theta) + \theta = 180^\circ$$

$$\theta = 39^\circ$$

$$\angle BCE = \angle ACB + \angle ACE = 28^\circ + 39^\circ = 67^\circ$$

61. C

$$\angle AOC + \angle OAC + \angle OCA = 180^\circ$$

$$\angle AOC + 2(27^\circ) = 180^\circ$$

$$\angle AOC = 126^\circ$$

$$\begin{aligned}\angle BOC &= \angle AOC \times \frac{2}{1+2} = 84^\circ \\ \angle BAC &= \frac{\angle BOC}{2} = 42^\circ \\ \angle BCD &= \angle BAC = 42^\circ\end{aligned}$$

62. B

$$\begin{aligned}\angle BCA &= \angle BAC = \frac{180^\circ - 30^\circ}{2} = 75^\circ \\ \angle CDE &= \angle ACB = 75^\circ \\ \angle DCE &= 180^\circ - 2 \times 75^\circ = 30^\circ \\ \angle ECA &= 180^\circ - 75^\circ - 30^\circ = 75^\circ \\ \angle EFA &= 180^\circ - 75^\circ = 105^\circ\end{aligned}$$

63. B

$$\begin{aligned}\angle OTD &= 90^\circ \\ \angle BOC &= \angle COT \\ \angle DOT + \angle OTD + \angle ODT &= 180^\circ \\ 2\angle COT + 90^\circ + 30^\circ &= 180^\circ \\ \angle COT &= 30^\circ \\ \angle OTC + \angle OCT + \angle COT &= 180^\circ \\ 2\angle OTC + 30^\circ &= 180^\circ \\ \angle OTC &= 75^\circ \\ \angle OTA &= \angle ATC - \angle OTC = 110^\circ - 75^\circ = 35^\circ \\ \angle OAT &= \angle OTA = 35^\circ\end{aligned}$$

64. C

$$\begin{aligned}\angle QRT &= \angle QUT = 108^\circ \\ \angle PQR &= \angle RPV = 48^\circ \\ \angle QPR + \angle PQR &= \angle QRT \\ \angle QPR + 48^\circ &= 108^\circ \\ \angle QPR &= 60^\circ\end{aligned}$$

65. A

$$\begin{aligned}\angle AGE &= 2 \times 66^\circ = 132^\circ \\ \angle CEA &= 132^\circ \times \frac{6}{5+6} = 72^\circ \\ \angle CAT &= \angle CEA = 72^\circ \text{ and } \angle ATC = 180^\circ - 2 \times 72^\circ = 36^\circ\end{aligned}$$

66. B

$$\angle BCD + \angle BAD = 180^\circ$$

$$108^\circ + \angle BAD = 180^\circ$$

$$\angle BAD = 72^\circ$$

$$\frac{\widehat{BCD}}{\widehat{DE}} = \frac{\angle BAD}{\angle DAE}$$

$$\frac{4+5}{6} = \frac{72^\circ}{\angle DAE}$$

$$\angle DAE = 48^\circ$$

$$\frac{\widehat{ABCD}}{\widehat{BCD}} = \frac{\angle AED}{\angle BAD}$$

$$\frac{3+4+5}{4+5} = \frac{\angle AED}{72^\circ}$$

$$\angle AED = 96^\circ$$

$$\angle ADE + \angle AED + \angle DAE = 180^\circ$$

$$\angle ADE + 96^\circ + 48^\circ = 180^\circ$$

$$\angle ADE = 36^\circ$$

$$\angle TAE = \angle ADE = 36^\circ$$

67. A

$$\angle BCA = 90^\circ \text{ and } \angle CBA = 180^\circ - 90^\circ - 42^\circ = 48^\circ$$

$$\angle OBC = \frac{48^\circ}{2} = 24^\circ$$

$$\angle BOC = 180^\circ - 90^\circ - 24^\circ = 66^\circ$$

68. D

$$\angle PRQ = \angle PQV = 38^\circ$$

$$\angle PQR + \angle QPR + \angle PRQ = 180^\circ$$

$$\angle PQR + 11^\circ + 38^\circ = 180^\circ$$

$$\angle PQR = 131^\circ$$

$$\angle PSR + \angle PQR = 180^\circ$$

$$\angle PSR + 131^\circ = 180^\circ$$

$$\angle PSR = 49^\circ$$

Since $UP = US$, we have $\angle USP = \angle UPS$.

$$\angle USP + \angle UPS + \angle PUS = 180^\circ$$

$$2\angle USP + 26^\circ = 180^\circ$$

$$\angle USP = 77^\circ$$

$$\angle USP + \angle PSR + \angle RSW = 180^\circ$$

$$77^\circ + 49^\circ + \angle RSW = 180^\circ$$

$$\angle RSW = 54^\circ$$

69. C

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ$$

$$\angle BOC = 180^\circ - 2 \times 55^\circ$$

$$= 70^\circ$$

$$\angle OCT = 90^\circ$$

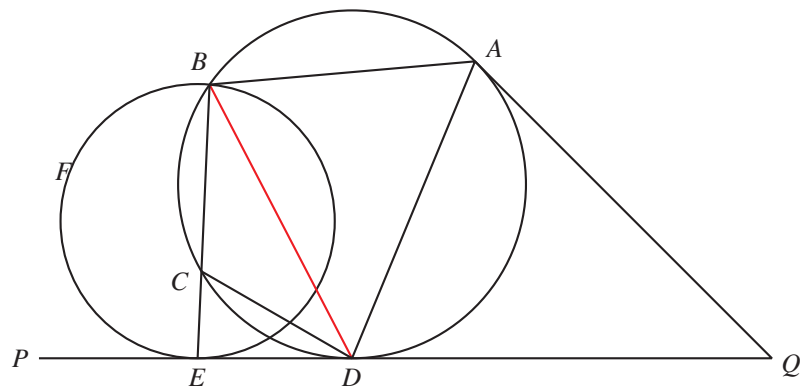
$$\angle OTC + \angle OCT + \angle COT = 180^\circ$$

$$\angle OTC = 180^\circ - 70^\circ - 90^\circ$$

$$\angle ATC = 20^\circ$$

70. **B**

Since $AQ = DQ$, we have $\angle ADQ = \frac{180^\circ - 50^\circ}{2} = 65^\circ$.



$$\angle ABD = \angle ADQ = 65^\circ$$

$$\angle CBD = 100^\circ - 65^\circ = 35^\circ$$

$$\angle CDE = \angle CBD = 35^\circ$$

$$\angle CDB = \angle CBD = 35^\circ$$

$$\angle PEB = \angle EBD + \angle BDE = 35^\circ + (35^\circ + 35^\circ) = 105^\circ$$