

| Solution  | Marks  |
|---|--|
| <b>ELITE-2425-MOCK-SET 11-MATH-CP 1</b>   |  |
| <b>Suggested solutions</b>  |  |
| 1. $m^8 \left( \frac{n^{-2}}{m^3} \right)^3 = m^8 \times \frac{n^{-6}}{m^9}$<br>$= \frac{1}{m^{9-8}n^6}$<br>$= \frac{1}{mn^6}$  | 1M<br><br>1M<br><br>1A                       |
| 2. $2x^2 - 8x(y - z)^2 = 2x[x^2 - 4(y - z)^2]$<br>$= 2x[x + 2(y - z)][x - 2(y - z)]$<br>$= 2x(x + 2y - 2z)(x - 2y + 2z)$  | 1M<br><br><br>1M+1A                          |
| 3. $\frac{2 - m}{2} = \frac{5n + 3m}{n}$<br>$2n - mn = 10n + 6m$<br>$-mn - 6m = 8n$<br>$m = \frac{-8n}{n + 6}$  | 1M<br><br>1M<br><br>1A                       |
| 4. Let the number of cartons of orange juice and pineapple juice be $x$ and $y$ respectively.<br>$\begin{cases} x + y = 50 \\ \frac{x}{5} - \frac{y}{5} = 4 \end{cases}$<br>Solving,<br>$\frac{50 - y}{5} - \frac{y}{5} = 4$<br>$50 - 2y = 20$<br>$y = 15$<br>There are 15 cartons of pineapple juice on the bench. | 1M<br>1M<br><br><br>1M<br><br><br>1A         |
| 5. (a) $\frac{3x - 2}{5} > x - 6$<br>$-\frac{2x}{5} > -\frac{28}{5}$<br>$x < 14$<br>$3x - 12 \geq 0$<br>$x \geq 4$<br>Thus, $4 \leq x < 14$ .<br>(b) 10   | 1A<br><br><br><br><br><br><br>1A<br>1M<br>1A |

| Solution           |  | Marks                      |         |         |         |         |         |            |      |      |      |      |      |                    |   |   |    |    |   |                |
|--------------------|--|----------------------------|---------|---------|---------|---------|---------|------------|------|------|------|------|------|--------------------|---|---|----|----|---|----------------|
| 6.                 | (a) The coordinates of A and C are (2, 0) and (8, −8) respectively.<br>(b) $AB = \sqrt{(8 - 2)^2 + 8^2} = 10$<br>$AC = \sqrt{(8 - 2)^2 + (-8)^2} = 10 = AB$<br>The lengths of AB and AC are equal.   | 1A+1A<br>1M<br>1A          |         |         |         |         |         |            |      |      |      |      |      |                    |   |   |    |    |   |                |
| 7.                 | (a) 60.5 marks<br>(b) <table border="1"><tr><td>Score</td><td>31 – 40</td><td>41 – 50</td><td>51 – 60</td><td>61 – 70</td><td>71 – 80</td></tr><tr><td>Class mark</td><td>35.5</td><td>45.5</td><td>55.5</td><td>65.5</td><td>75.5</td></tr><tr><td>Number of students</td><td>5</td><td>5</td><td>10</td><td>15</td><td>5</td></tr></table><br>Mean mark = $\frac{35.5 \times 5 + 45.5 \times 5 + \dots + 75.5 \times 5}{40}$<br>= 58 | Score                      | 31 – 40 | 41 – 50 | 51 – 60 | 61 – 70 | 71 – 80 | Class mark | 35.5 | 45.5 | 55.5 | 65.5 | 75.5 | Number of students | 5 | 5 | 10 | 15 | 5 | 1A<br>2A<br>1A |
| Score              | 31 – 40  | 41 – 50                    | 51 – 60 | 61 – 70 | 71 – 80 |         |         |            |      |      |      |      |      |                    |   |   |    |    |   |                |
| Class mark         | 35.5   | 45.5                       | 55.5    | 65.5    | 75.5    |         |         |            |      |      |      |      |      |                    |   |   |    |    |   |                |
| Number of students | 5  | 5                          | 10      | 15      | 5       |         |         |            |      |      |      |      |      |                    |   |   |    |    |   |                |
| 8.                 | (a) Let $W = kL^3$ , where $k$ is a non-zero constant.<br>$75 = k \times 5^3$<br>$k = \frac{3}{5}$<br>Thus, $W = \frac{3L^3}{5}$ .<br>(b) Required percentage = $\frac{\frac{3}{5}[L(1 + 60\%)]^3 - \frac{3}{5}L^3}{\frac{3}{5}L^3} \times 100\%$<br>= +309.6%   | 1A<br>1M<br>1A<br>1M<br>1A |         |         |         |         |         |            |      |      |      |      |      |                    |   |   |    |    |   |                |
| 9.                 | (a) Maximum absolute error = $16 \times 1\% = 0.16$ mm.<br>Denote the length of a quality component by $L$ mm.<br>Then $15.84 \leq L < 16.16$ .<br>(b) Since $16.20 \text{ mm} > 16.16 \text{ mm}$ ,<br>it is not a quality component.   | 1M<br>1M+1A<br>1M<br>1A    |         |         |         |         |         |            |      |      |      |      |      |                    |   |   |    |    |   |                |

| Solution       |   | Marks   |  |  |               |   |   |               |                                    |   |   |
|----------------|---|---|--|--|---------------|---|---|---------------|------------------------------------|---|---|
| 10.            | <p>(a) <math>\angle ADE = 90^\circ</math> (given)</p> <p><math>\angle ABC = 90^\circ</math> (given)</p> <p><math>\quad = \angle ADE</math></p> <p><math>\angle EAD = \angle CAB</math> (common <math>\angle</math>)</p> <p><math>\triangle ADE \sim \triangle ABC</math> (AA)</p> <table border="1"> <tr> <th colspan="3">Marking Scheme</th></tr> <tr> <td><b>Case 1</b></td><td>Any correct proof with correct reasons.</td><td>2</td></tr> <tr> <td><b>Case 2</b></td><td>Any correct proof without reasons.</td><td>1</td></tr> </table> <p>(b) <math>EB = AE = 20</math> cm.</p> <p><math>DE = \sqrt{20^2 - 16^2} = 12</math> cm</p> <p>Since <math>\triangle ADE \sim \triangle ABC</math>,</p> $\frac{BC}{AB} = \frac{DE}{AD}$ $\frac{BC}{20 + 20} = \frac{12}{16}$ <p><math>BC = 30</math> cm</p> <p>Since <math>\angle CBE = 90^\circ</math>, <math>CE</math> is a diameter of the required circle.</p> <p>Let the radius of the circle be <math>r</math>.</p> $(2r)^2 - 20^2 = 30^2$ $r^2 = 325$ <p>Required area is <math>r^2\pi = 325\pi</math> cm<sup>2</sup>.</p> | Marking Scheme  |  |  | <b>Case 1</b> | Any correct proof with correct reasons. | 2 | <b>Case 2</b> | Any correct proof without reasons. | 1 | <p>1A</p> <p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p> |
| Marking Scheme |   |   |  |  |               |   |   |               |                                    |   |   |
| <b>Case 1</b>  | Any correct proof with correct reasons.   | 2   |  |  |               |   |   |               |                                    |   |   |
| <b>Case 2</b>  | Any correct proof without reasons.  | 1   |  |  |               |   |   |               |                                    |   |   |
| 11.            | <p>(a) <math>(50 + b) - 21 = 32</math></p> <p><math>b = 3</math></p> <p><math>37 - (20 + a) = 9</math></p> <p><math>a = 8</math></p> <p>Mean of the distribution = <math>\frac{21 + 23 + \dots + 53}{18} = 33.5</math></p> <p>Standard deviation of the distribution <math>\approx 8.73</math></p> <p>(b) Original median of the distribution = 33</p> <p>Age of the new teacher = 33</p> <p>Change in standard deviation <math>\approx 8.49 - 8.73</math></p> <p><math>\approx -0.232</math></p>   | <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> |  |  |               |   |   |               |                                    |   |   |

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| 12. (a)  | $\frac{OM}{ON} = \frac{AM}{ND}$ $\frac{13.5 - ON}{ON} = \frac{5}{4}$ $ON = 6 \text{ cm}$ $\text{Required volume} = \frac{1}{3}\pi(4)^2(6)$ $= 32\pi \text{ cm}^3$   | 1M<br>1M<br>1A             |
| (b)      | <p>Since the volume of milk is greater than the volume of the lower cone, there are <math>36\pi - 32\pi = 4\pi \text{ cm}^3</math> of milk in the upper cone.</p> <p>Let the height of milk in the upper cone be <math>h \text{ cm}</math>.</p> $\left(\frac{h}{6}\right)^3 = \frac{4\pi}{32\pi}$ $h = 3$ <p>Curved surface area of the lower cone</p> $= \pi(4)(\sqrt{4^2 + 6^2})$ $= 8\sqrt{13}\pi$ <p>Total wet curved surface area</p> $= 8\sqrt{13}\pi \times \left(\frac{3}{6}\right)^2 + 8\sqrt{13}\pi$ $= 10\sqrt{13}\pi \text{ cm}^2$ $> 36\pi \text{ cm}^2$ <p>The claim is agreed.</p> | 1M<br>1M<br>1A<br>1M<br>1A |
| 13. (a)  | $(x - 6)^2 + (y + 5)^2 = 6^2 + 5^2$ $(x - 6)^2 + (y + 5)^2 = 61$  | 1M<br>1A                   |
| (b)      | <p>(i) <math>H = (12, 0)</math> and <math>K = (0, -10)</math></p> <p>(ii) <math>O, P</math> and <math>Q</math> are collinear.</p> <p>(iii) Required area = <math>12 \times 10</math></p> $= 120$  | 1A+1A<br>1A<br>1M<br>1A    |

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| 14. (a) $f(-1) = h = -h - 1 + 9 + k$  |  | 1M    |
| $2h - k = 8$  |  |       |
| $0 = f\left(\frac{1}{2}\right) = \frac{h}{8} - \frac{1}{4} - \frac{9}{2} + k$ |  | 1M    |
| $h + 8k = 38$   |  |       |
| Solving, we have $h = 6$ and $k = 4$ .  |  | 1A+1A |
| (b) $f(x) = 6$  |  |       |
| $6x^3 - x^2 - 9x - 2 = 0$   |  |       |
| $(x + 1)(6x^2 - 7x - 2) = 0$  |  | 1M    |
| $x = -1$ or $\frac{7 \pm \sqrt{7^2 - 4(6)(-2)}}{2(6)}$                        |  | 1M    |
| $= -1$ or $\frac{7 \pm \sqrt{97}}{12}$  |  |       |
| There are some irrational roots in $f(x) = 6$ .                               |  |       |
| The claim is disagreed.   |  | 1A    |
| 15. (a) $6 = ka^0$  |  |       |
| $k = 6$   |  | 1A    |
| $54 = 6 \times a^{-2}$  |  |       |
| $a^2 = \frac{1}{9}$   |  |       |
| $a = \frac{1}{3}$   |  | 1A    |
| (b) $f(x_1) = \frac{3}{f(x_2)}$   |  |       |
| $6 \times \frac{1}{3^{x_1}} = \frac{3}{6 \times \frac{1}{3^{x_2}}}$           |  | 1M    |
| $3^{x_1+x_2} = 12$  |  |       |
| $(x_1 + x_2) \log 3 = \log 12$  |  | 1M    |
| $x_1 + x_2 \approx 2.26$  |  | 1A    |
| 16. (a) Let the common difference be $d$ .                                    |  |       |
| $2014 + 15d = 1729$   |  | 1M    |
| $d = -19$   |  | 1A    |
| (b) $S(n) = \frac{n}{2}[2(2014) + (n-1)(-19)] < 0$                            |  | 1M    |
| $-19n^2 + 4047n < 0$  |  |       |
| $n < 0$ (rejected) or $n > 213$   |  | 1A    |
| The least value of $n$ is 214.  |  | 1A    |

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| <p>17. (a) Required probability = <math>\frac{C_6^{18}}{C_8^{20}}</math><br/> <math>= \frac{14}{95}</math></p> <p>(b) Required probability = <math>\frac{C_8^{10}(C_1^2)^8}{C_8^{20}}</math><br/> <math>= \frac{384}{4199}</math></p> <p>(c) Required probability = <math>\frac{C_2^{10}C_4^8(C_1^2)^4}{C_8^{20}}</math><br/> <math>= \frac{1680}{4199}</math></p>   | <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>          |
| <p>18. (a) When <math>y = 0</math>, <math>3x^2 - 6mx + 4m^2 = 0</math>.</p> $\Delta = (6m)^2 - 4(3)(4m^2)$ $= -12m^2$ $< 0$ <p>The graph has no <math>x</math>-intercepts.</p> <p>(b) <math>y = f(x)</math></p> $= 3(x^2 - 2mx + m^2) + m^2$ $= 3(x - m)^2 + m^2$ <p>The coordinates of the vertex are <math>(m, m^2)</math>.</p> <p>(c) Coordinates of <math>A</math> and <math>B</math> are <math>(m, m^2)</math> and <math>(m, 0)</math> respectively.<br/> Since <math>\angle OBA = 90^\circ</math>, the circumcentre is the mid-point of <math>OA</math>.<br/> The coordinates of circumcentre are <math>\left(\frac{m}{2}, \frac{m^2}{2}\right)</math>.<br/> When <math>m = 2</math>, the coordinates of circumcentre are <math>(1, 2)</math> which does not lie on <math>y = x</math>.<br/> The claim is incorrect.</p> | <p>1M</p> <p>1</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> |

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| <p>19. (a) In <math>\triangle ABC</math>,</p> $BC^2 = 5^2 + 10^2 - 2(5)(10) \cos 80^\circ$ $BC \approx 10.4 \text{ cm}$ $5^2 = 10^2 + BC^2 - 2(10)(BC) \cos \angle ABD$ $\angle ABD \approx 28.3^\circ$ <p>In <math>\triangle ABD</math>,</p> $BD = 10 \cos \angle ABD$ $\approx 8.80 \text{ cm}$ $DC = BC - BD \approx 1.57 \text{ cm}$ <p>(b) (i) In <math>\triangle ABC</math>, when <math>\theta = 45^\circ</math>,</p> $BC^2 = 10^2 + 5^2 - 2(10)(5) \cos 45^\circ$ $BC \approx 7.37 \text{ cm}$ <p>The angle between the faces <math>ABD</math> and <math>ADC</math> is <math>\angle BDC</math>.</p> <p>In <math>\triangle BDC</math>,</p> $BC^2 = BD^2 + CD^2 - 2(BD)(CD) \cos \angle BDC$ $\angle BDC \approx 22.1^\circ$ $< 25^\circ$ <p>The claim is agreed.</p> <p>(ii) In <math>\triangle ABC</math>, when <math>\theta = 40^\circ</math>,</p> $BC^2 = 10^2 + 5^2 - 2(10)(5) \cos 40^\circ$ $BC \approx 6.96 \text{ cm}$ <p>So, <math>BC + CD \approx 8.53 \text{ cm} &lt; BD</math>.</p> <p>This violates the triangle inequality, implying that this situation is impossible.</p> | <p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1</p> |