DAYCP-PROB-2425-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. A	3. C	4. B	5. D
6. C	7. C	8. D	9. C	10. D
11. A	12. B	13. B	14. B	15. A
16. C	17. D	18. B	19. B	20. D
21. B	22. D	23. C	24. C	25. C
26. C	27. A	28. D	29. C	30. B
31. C	32. C	33. D	34. B	35. A
36. C	37. C	38. C	39. B	40. C
41. C	42. A	43. B	44. B	45. D
46. C	47. B	48. D	49. D	50. D

1. A

$$\text{Required probability} = \frac{3+2+2+1}{8\times 7}$$
$$= \frac{1}{7}$$

2. A

$$\frac{n}{4+6+n} = \frac{1}{3}$$
$$3n = 10 + n$$
$$n = 5$$

3. C

Probability of drawing a yellow ball is $\frac{n}{n+10}$.
Probability of drawing a black ball is $\frac{10}{n+10}$.

$$31 = 35\left(\frac{n}{n+10}\right) + 21\left(\frac{10}{n+10}\right)$$
$$31n + 310 = 35n + 210$$
$$n = 25$$

4. B

$$\text{Required probability} = \frac{C_2^4}{C_2^7}$$
$$= \frac{2}{7}$$

5. D

$$\text{Required probability} = 1 - \left(\frac{5}{5^2}\right)$$
$$= \frac{4}{5}$$

6. C

Required probability

$$= 1 - \left(\frac{4}{7}\right)^2$$
$$= \frac{33}{49}$$

7. C

$$\text{Required probability} = \frac{2+4+2}{6 \times 5}$$
$$= \frac{4}{15}$$

8. D

$$\text{Required probability} = \frac{5+3+1}{C_2^6}$$
$$= \frac{3}{5}$$

9. C

$$\text{Required probability} = \frac{C_2^4}{C_2^5}$$
$$= \frac{3}{5}$$

10. D

$$56 = 144\left(\frac{1+2}{36}\right) + x\left(1 - \frac{3}{36}\right)$$
$$x = 48$$

11. A

$$\text{Required probability} = \frac{1+2+3+4+5}{6^2}$$
$$= \frac{5}{12}$$

12. B

$$\text{Required probability} = \frac{3+1}{C_2^8}$$
$$= \frac{1}{7}$$

13. B

$$\text{Expected gain} = -4 + 12\left[1 - \left(\frac{3}{6}\right)^2\right]$$
$$= \$5$$

14. B

$$\text{Required probability} = \frac{C_2^3 + C_2^2}{C_2^5}$$
$$= \frac{2}{5}$$

15. A

$$\frac{C_2^3}{C_2^5} = \frac{k-2}{20}$$
$$k = 8$$

16. C

Square of numbers in box A: 4, 16, 36, 64

Square of numbers in box B: 1, 9, 25, 49, 81

$$\text{Required probability} = \frac{2+2+2+2}{4 \times 5}$$
$$= 0.4$$

17. D

$$\text{Required probability} = 1 \times \frac{3}{5}$$
$$= \frac{3}{5}$$

18. B

$$\text{Expected gain} = 20\left(\frac{4}{5^2}\right) + 5\left(\frac{2+1}{5^2}\right) + 10\left(1 - \frac{4}{5^2} - \frac{2+1}{5^2}\right)$$
$$= \$11$$

19. B

The possible prime numbers are 71, 73 and 79.

$$\text{Required probability} = \frac{3}{10}$$

20. D

$$\text{Required probability} = 1 - \frac{C_2^2}{C_2^4}$$
$$= \frac{5}{6}$$

21. B

$$\text{Required probability} = \frac{C_4^{15}C_2^{11} + C_5^{15}C_1^{11} + C_6^{15}}{C_6^{26}}$$
$$= \frac{113}{230}$$

22. D

$$\text{Required probability} = \frac{5!2!}{6!}$$
$$= \frac{1}{3}$$

23. C

$$\text{Required probability} = \frac{C_1^4 C_3^5}{C_4^9}$$
$$= \frac{20}{63}$$

24. C

$$\text{Required probability} = 1 - \left(1 - \frac{2}{8}\right)\left(1 - \frac{3}{8}\right)\left(1 - \frac{4}{8}\right)$$
$$= \frac{49}{64}$$

25. C

$$\text{Required probability} = C_4^5(0.8)^4(0.2) + (0.8)^5$$
$$= 0.737\,28$$

26. C

$$\text{Required probability} = \frac{3}{5} \times \frac{7}{8} + \frac{2}{5} \times \frac{6}{8}$$
$$= \frac{33}{40}$$

27. A

$$\text{Required probability} = 0.6 \times 0.15 + 0.4 \times 0.2$$
$$= 0.17$$

28. D

$$\text{Required probability} = \frac{5}{6} \times \frac{4}{5} \times \frac{1}{4}$$
$$= \frac{1}{6}$$

29. C

$$\text{Required probability} = \frac{C_2^{12}}{C_2^{15}}$$
$$= \frac{22}{35}$$

30. B

Required probability $= 1 - (0.8)^5 - C_4^5(0.8)^4(0.2)$

$= 0.262\,72$

31. C

Let X be the number of black balls drawn from bag A.

Required probability

$= \mathrm{P}(X = 2) \times \dfrac{6}{12} + \mathrm{P}(X = 1) \times \dfrac{5}{12} + \mathrm{P}(X = 0) \times \dfrac{4}{12}$

$= \dfrac{C_2^5}{C_2^8} \times \dfrac{6}{12} + \dfrac{C_1^5 C_1^3}{C_2^8} \times \dfrac{5}{12} + \dfrac{C_2^3}{C_2^8} \times \dfrac{4}{12}$

$= \dfrac{7}{16}$

32. C

Required probability $= 1 - \left(\dfrac{4}{10}\right)^3$

$= \dfrac{117}{125}$

33. D

Required probability $= 1 - (0.3)(0.2)(0.6)$

$= 0.964$

34. B

$\left(\dfrac{6}{10}\right)^{n-1}\left(\dfrac{4}{10}\right) < 0.05$

$0.6^{n-1} < 0.125$

$(n-1)\log 0.6 < \log 0.125$

$n > 5.07$

The minimum value of n is 6.

35. A

Required probability $= \dfrac{C_4^5 + C_3^5 C_1^2 + C_3^5 C_1^8}{C_4^{15}}$

$= \dfrac{1}{13}$

36. C

Required probability $= \dfrac{4}{8} \times \dfrac{4}{9} + \dfrac{4}{8} \times \dfrac{3}{9}$

$= \dfrac{7}{18}$

37. C

$$\text{Required probability} = 1 - \left(\frac{1}{2} \times \frac{C_2^5}{C_2^8} + \frac{1}{2} \times \frac{C_2^4}{C_2^{10}}\right)$$
$$= \frac{317}{420}$$

38. C

$$\text{Required probability} = 1 - 0.8^5$$
$$= 0.672\,32$$

39. B

$$\text{Required probability} = 1 - \frac{1}{6} \times \frac{1}{3} \times \frac{11}{12} \times \frac{3}{4}$$
$$= \frac{277}{288}$$

40. C

$$\text{Required probability} = 1 - 0.7^5 - C_4^5(0.7)^4(0.3)$$
$$= 0.471\,78$$

41. C

$$\text{Required probability} = \frac{C_4^{12}}{C_4^{17}}$$
$$= \frac{99}{476}$$

42. A

$$\text{Required probability} = \frac{7!3!}{9!}$$
$$= \frac{1}{12}$$

43. B

$$\text{Required probability} = \left(\frac{15}{18}\right)^2$$
$$= \frac{25}{36}$$

44. B

$$\text{Required probability} = \frac{1}{8} + \frac{1}{8} - \frac{6!}{8!}$$
$$= \frac{13}{56}$$

45. D

$$\text{Required probability} = \frac{C_4^{10} + C_1^6 C_3^{10} + C_2^6 C_2^{10}}{C_4^{16}}$$
$$= \frac{321}{364}$$

46. C

$$\text{Required probability} = 1 - \frac{C_2^4 + C_2^6 + C_2^5}{C_2^{15}} - \frac{C_1^4 C_1^5 C_1^6}{C_3^{15}}$$
$$= \frac{86}{195}$$

47. B

$$\text{Required probability} = \frac{C_5^8 5!7!}{12!}$$
$$= \frac{7}{99}$$

48. D

Needs at least 3 draws $\Leftrightarrow$ no gold coins obtained in first 2 draws

$$\text{Required probability} = \frac{C_2^3}{C_2^7} = \frac{1}{7}$$

49. D

$$\text{Required probability} = 1 - (1 - 0.6)(1 - 0.55)$$
$$= 0.82$$

50. D

$$\text{Required probability} = \frac{C_8^{12} + C_1^6 C_7^{12} + C_2^6 C_6^{12} + C_3^6 C_5^{12} + C_4^6 C_4^{12}}{C_8^{18}}$$
$$= \frac{214}{221}$$

Conventional Questions

51. (a) Required probability $= \frac{6!2!}{7!}$ 1M

$= \frac{2}{7}$ 1A

(b) Required probability $= 1 - \frac{C_3^5}{C_3^7}$ 1M

$= \frac{5}{7}$

52. (a) Required probability $= \frac{C_2^3 C_2^4 C_2^5}{C_6^{12}}$ 1M+1M

$= \frac{15}{77}$ 1A

(b) Required probability $= \frac{C_1^3 C_5^9}{C_6^{12}}$ 1M

$= \frac{9}{22}$ 1A

53. (a) Required probability $= \frac{C_5^6 + C_5^8}{C_5^{18}}$ 1M+1M

$= \frac{31}{4284}$ 1A

(b) Required probability $= 1 - \frac{31}{4284}$ 1M

$= \frac{4253}{4284}$ 1A

54. (a) Required number $= 14!$

$= 87\,178\,291\,200$ 1A

(b) Required probability $= \frac{C_6^9 6!8!}{14!}$ 1M+1A

$= \frac{4}{143}$ 1A

55. Required probability $= \frac{C_1^2 C_3^{22}}{C_5^{25}}$ 1M+1M

$= \frac{4}{69}$ 1A

56. (a) Required probability $= \frac{C_6^9}{C_6^{20}}$ 1M

$= \frac{7}{3230}$ 1A

(b) Required probability $= \frac{C_6^9 + C_5^9 C_1^{11} + C_4^9 C_2^{11}}{C_6^{20}}$ 1M

$= \frac{70}{323}$ 1A

57. (a) Number of ways $= C_2^6 = 15$ 1A

(b) Required probability $= 1 - \dfrac{15}{C_4^{12}}$ 1M

$= \dfrac{32}{33}$ 1A

58. (a) Required probability $= \dfrac{9!4!}{12!}$ 1M

$= \dfrac{1}{55}$ 1A

(b) Required probability $= \dfrac{C_4^9 \times 4! \times 8!}{12!}$ 1M

$= \dfrac{14}{55}$ 1A

59. (a) Required number $= C_4^{28}$

$= 20\,475$ 1A

(b) Required probability $= 1 - \dfrac{C_4^{15} + C_4^{13}}{C_4^{28}}$ 1M+1M

$= \dfrac{283}{315}$ 1A

60. (a) Required number $= 9!$

$= 362\,880$ 1A

(b) Required probability $= \dfrac{C_4^6 \times 4! \times 5!}{362\,880}$ 1M+1M

$= \dfrac{5}{42}$ 1A

61. (a) Required probability $= \dfrac{C_3^5}{C_3^9}$ 1M

$= \dfrac{5}{42}$ 1A

(b) Required probability $= 1 - \dfrac{5}{42} - \dfrac{C_3^4}{C_3^9}$ 1M

$= \dfrac{5}{6}$ 1A

62. (a) Required number $= C_5^{20}$

$= 15\,504$ 1A

(b) Required probability $= \dfrac{C_3^{11}C_2^9 + C_4^{11}C_1^9 + C_5^{11}}{C_5^{20}}$ 1M

$= \dfrac{781}{1292}$ 1A

63. (a) Required probability $= \dfrac{C_2^3 C_1^7 C_1^6}{C_4^{16}}$ 1M

$= \dfrac{9}{130}$ 1A

(b) Required probability $= \dfrac{C_2^3 C_1^7 C_1^6 + C_1^3 C_2^6 C_1^7 + C_1^3 C_1^6 C_2^7}{C_4^{16}}$ 1M

$= \dfrac{9}{20}$ 1A

64. (a) Required probability $= \dfrac{C_2^6}{C_2^{10}}$ 1M

$= \dfrac{1}{3}$ 1A

(b) Required probability $= \dfrac{1}{3} \times 1 + \dfrac{C_1^4 C_1^6}{C_2^{10}} \times \dfrac{C_2^7}{C_2^8} + \dfrac{C_2^4}{C_2^{10}} \times \dfrac{C_2^2 + C_2^6}{C_2^8}$ 1M

$= \dfrac{17}{21}$ 1A

65. (a) Required probability $= \dfrac{C_5^5 C_1^8}{C_6^{13}}$ 1M

$= \dfrac{2}{429}$ 1A

(b) Required probability $= 1 - \dfrac{2}{429} - \dfrac{C_4^5 C_2^8}{C_6^{13}}$ 1M

$= \dfrac{392}{429}$ 1A

66. (a) Required probability $= \dfrac{5! \times 4!}{9!}$ 1M

$= \dfrac{1}{126}$ 1A

(b) Required probability $= \dfrac{C_4^6 \times 4! \times 5!}{9!}$ 1M

$= \dfrac{5}{42}$ 1A

67. (a) Required probability $= \dfrac{C_6^{18}}{C_8^{20}}$ 1M

$= \dfrac{14}{95}$ 1A

(b) Required probability $= \dfrac{C_8^{10} (C_1^2)^8}{C_8^{20}}$ 1M

$= \dfrac{384}{4199}$ 1A

(c) Required probability $= \dfrac{C_2^{10} C_4^8 (C_1^2)^4}{C_8^{20}}$ 1M

$= \dfrac{1680}{4199}$ 1A