REG-POLY-2425-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. B	3. C	4. D	5. D
6. D	7. D	8. A	9. D	10. B
11. C	12. C	13. C	14. A	15. C
16. B	17. C	18. D	19. A	20. B
21. B	22. B	23. A	24. C	25. D
26. A	27. B	28. A	29. A	30. D
31. D	32. B	33. B	34. C	35. C
36. D	37. A	38. A	39. C	40. D
41. B	42. C			

1. B

Remainder $= (-1)^{2023} + (-1)^{2022} + (-1)^{2021} + \ldots + (-1)$
$= [(-1) + 1] + [(-1) + 1] + \ldots + (-1)$
$= -1$

2. B

$f(1) = 2(1)^3 - 3(1)^2 - 8(1) + k = -12$
$k = -3$

We have $f(x) = 2x^3 - 3x^2 - 8x - 3$.

x	-1	$\frac{1}{2}$	3
$f(x)$	0	$-\frac{15}{2}$	0

Thus, $x + 1$ and $x - 3$ are factors of $f(x)$.

3. C

Note that $f(3) = (3 - 1)^3 - 6(3 - 1) + 4 = 0$.
Thus, $x - 3$ is a factor of $f(x)$.

4. D

Let $f(x) = kx^3 + (k+1)x^2 - 1$.

x	2	1	0	-1
$f(x)$	$12k + 3$	$2k$	-1	0

Thus, $x + 1$ is a factor of $f(x)$.

5. D

$$(-k)^4 - k^2(-k)^2 + 4(-k) + 12 = 0$$
$$-4k + 12 = 0$$
$$k = 3$$

6. D

$$(-3)^3 + 27(-3)^2 + k(-3) - 3 = 0$$
$$-3k + 213 = 0$$
$$k = 71$$

7. D

$$6\left(-\frac{k}{3}\right)^4 + 2k\left(-\frac{k}{3}\right)^3 + \left(-\frac{k}{3}\right)^2 + 6\left(-\frac{k}{3}\right) + 9 = 0$$
$$\frac{k^2}{9} - 2k + 9 = 0$$
$$k = 9$$

8. A

$$2(2)^2 + 2 + m = 0$$
$$m = -10$$
$$2x^2 + x - 10 = (x-2)(2x+5)$$

Thus, $2x^2 + x + m$ is also divisible by $2x + 5$.

9. D

$$9\left(\frac{c}{3}\right)^4 - c^2\left(\frac{c}{3}\right)^2 + 2\left(\frac{c}{3}\right) - 6 = 0$$
$$\frac{2c}{3} - 6 = 0$$
$$c = 9$$

10. B

Let $f(x) = (x+1)Q(x)$, where $Q(x)$ is a polynomial.

$$f(2x-1) = [(2x-1)+1]Q(2x-1)$$
$$= 2xQ(2x-1)$$

Thus, $2x$ must be a factor of $f(2x-1)$.

11. C

Let $f(x) = (x + 2)Q(x)$, where $Q(x)$ is a polynomial.

$$\begin{aligned} f(2x + 3) &= [(2x + 3) + 2]Q(2x + 3) \\ &= (2x + 5)Q(2x + 3) \end{aligned}$$

Thus, $2x + 5$ must be a factor of $f(2x + 3)$.

12. C

Let $f(x) = (3x - 7)Q(x)$, where $Q(x)$ is a polynomial.

$$\begin{aligned} f(x + 2) &= [3(x + 2) - 7]Q(x + 2) \\ &= (3x - 1)Q(x + 2) \end{aligned}$$

Thus, $3x - 1$ must be a factor of $f(x + 2)$.

13. C

Let $f(x) = (2x + 3)Q(x)$, where $Q(x)$ is a polynomial.

$$\begin{aligned} f(x - 5) &= [2(x - 5) + 3]Q(x - 5) \\ &= (2x - 7)Q(x - 5) \end{aligned}$$

Thus, $2x - 7$ must be a factor of $f(x - 5)$.

14. A

Let $f(x) = (x - 1)q(x)$, where $q(x)$ is a polynomial.

$$\begin{aligned} f(3x - 1) &= [(3x - 1) - 1]q(3x - 1) \\ &= (3x - 2)q(3x - 1) \end{aligned}$$

Thus, $3x - 2$ must be a factor of $f(3x - 1)$.

15. C

$$-2 = 8\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 - 5$$

$$a = 8$$

$$\begin{aligned} g(1) &= 8(1)^3 + 8(1)^2 - 5 \\ &= 11 \end{aligned}$$

16. B

$$\begin{aligned} f(-1) = (-1)^{2014} + a(-1) + b &= 3 \\ 1 - a + b &= 3 \\ a - b &= -2 \end{aligned}$$

Thus, $2a - 2b + 1 = 2(a - b) + 1 = 2(-2) + 1 = -3$.

17. C

$$(-2)^3 + a(-2)^2 + 6a(-2) + 16 = 0$$
$$-8a + 8 = 0$$
$$a = 1$$
$$f(-1) = (-1)^3 + (-1)^2 + 6(-1) + 16$$
$$= 10$$

18. D

$$g\left(\frac{1}{2}\right) = \frac{k}{8} - \frac{5}{4} - k + 3 = 0$$
$$k = 2$$
$$g(-2) = 2(-8) - 5(4) - 4(-2) + 3 = -25$$

19. A

$$a(3)^3 - 2a(3)^2 - 27 = 0$$
$$9a - 27 = 0$$
$$a = 3$$

20. B

$$f(-1) = k(-1)^3 + 5(-1)^2 - 46(-1) - 12k = 25$$
$$k = 2$$
$$\text{Remainder} = f(3)$$
$$= 2(3)^3 + 5(3)^2 - 46(3) - 24$$
$$= -63$$

21. B

$$f(-2) = k(-2)^3 - 3k(-2)^2 - (-2) + 3 = -75$$
$$k = 4$$
$$\text{Remainder} = f(1)$$
$$= 4(1)^3 - 12(1)^2 - (1) + 3$$
$$= -6$$

22. B

$$P(-1) = (-1)^{2025} - 3(-1) + k = 4$$
$$-1 + 3 + k = 4$$
$$k = 2$$
$$\text{Remainder} = P(1)$$
$$= (1)^{2025} - 3(1) + 2$$
$$= 0$$

23. A

$$0 = 4\left(\frac{1}{2}\right)^2 + m\left(\frac{1}{2}\right) + n$$

$$n = -1 - \frac{m}{2}$$

$$\text{Required remainder} = 4\left(-\frac{1}{2}\right)^2 + m\left(-\frac{1}{2}\right) + n$$

$$= 1 - \frac{m}{2} + \left(-1 - \frac{m}{2}\right)$$

$$= -m$$

24. C

$$0 = k(3)^3 + 2k(3)^2 + 90$$

$$k = -2$$

$$\text{Remainder} = -2(-1)^3 - 4(-1)^2 + 90$$

$$= 88$$

25. D

$$0 = (-2)^3 + 2(-2)^2 + 3(-2) + k$$

$$k = 6$$

Required remainder

$$= (3)^3 - 6(3) - 1$$

$$= 8$$

26. A

$$(1)^8 + a(1)^7 + b = 2a$$

$$1 + a + b = 2a$$

$$b = a - 1$$

$$\text{Remainder} = (-1)^8 + a(-1)^7 + (a - 1)$$

$$= 0$$

27. B

$$a\left(-\frac{4}{3}\right)^4 - 27\left(-\frac{4}{3}\right)^2 + b = 0$$

$$\frac{256a}{81} + b - 48 = 0$$

$$b = -\frac{256a}{81} + 48$$

$$\text{Remainder} = a\left(\frac{4}{3}\right)^4 - 27\left(\frac{4}{3}\right)^2 + \left(-\frac{256a}{81} + 48\right)$$

$$= 0$$

28. A

$$8\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right)^2 + k = 0$$

$$k = 0$$

$$\text{Remainder} = 8\left(-\frac{1}{2}\right)^3 - 4\left(-\frac{1}{2}\right)^2$$

$$= -2$$

29. A

$$-2(-2)^3 + a(-2)^2 + 6a(-2) - 32 = 0$$

$$-8a - 16 = 0$$

$$a = -2$$

$$\text{Remainder} = -2(2)^3 - 2(2)^2 - 12(2) - 32$$

$$= -80$$

30. D

$$a\left(\frac{b}{a}\right)^2 - b\left(\frac{b}{a}\right) + c = 0$$

$$\frac{b^2}{a} - \frac{b^2}{a} + c = 0$$

$$c = 0$$

$$\text{Remainder} = a\left(-\frac{b}{a}\right)^2 - b\left(-\frac{b}{a}\right)$$

$$= \frac{b^2}{a} + \frac{b^2}{a}$$

$$= \frac{2b^2}{a}$$

31. D

$$f(5) = 0 = 2(5)^2 - 13(5) + k$$

$$k = 15$$

$$\text{Required remainder} = f\left(-\frac{1}{2}\right)$$

$$= 2\left(-\frac{1}{2}\right)^2 - 13\left(-\frac{1}{2}\right) + 15$$

$$= 22$$

32. B

$$-2 = 2\left(-\frac{1}{2}\right)^3 - 5\left(-\frac{1}{2}\right)^2 - a^2\left(-\frac{1}{2}\right) + a$$

$$0 = \frac{a^2}{2} + a + \frac{1}{2}$$

$$a = -1$$

$$\text{Required remainder} = 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) - 1$$

$$= -7$$

33. B

$$(k^2 + k)(k + 1) = 0$$

$$k(k + 1)^2 = 0$$

$$k = -1 \quad \text{or} \quad 0 \text{ (rejected)}$$

$$\text{Remainder} = f(-2)$$

$$= [(-2)^2 - 1](-2 + 1)$$

$$= -3$$

34. C

$$a\left(-\frac{1}{3}\right)^8 + 81\left(-\frac{1}{3}\right)^3 + b\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + c = 0$$

$$\frac{a}{6561} + \frac{b}{9} + c - 2 = 0$$

$$\frac{a}{6561} + \frac{b}{9} + c = 2$$

$$\text{Remainder} = g\left(\frac{1}{3}\right)$$

$$= a\left(\frac{1}{3}\right)^8 + 81\left(\frac{1}{3}\right)^3 + b\left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) + c$$

$$= \frac{a}{6561} + \frac{b}{9} + c + 2$$

$$= (2) + 2$$

$$= 4$$

35. C

$$6(-1)^3 - 7(-1)^2 - 11(-1) + k = 0$$

$$k = 2$$

$$\text{Remainder} = f(3)$$

$$= 6(3)^3 - 7(3)^2 - 11(3) + 2$$

$$= 68$$

36. D

$(-k-1)^2 = 0$

$k = -1$

Remainder $= f(-1)$

$= [(-1)-1]^2$

$= 4$

37. A

$(4p)^2 + p(4p) + q = 0$

$20p^2 + q = 0$

$q = -20p^2$

Remainder $= f(-4p)$

$= (-4p)^2 + p(-4p) + (-20p^2)$

$= -8p^2$

38. A

$-2(-1)^9 + a(-1)^2 + b = 0$

$a + b = -2$

Remainder $= f(1)$

$= -2(1)^9 + a(1)^2 + b$

$= -2 + a + b$

$= -2 + (-2)$

$= -4$

39. C

$-(1)^5 + 3a(1)^4 + b = 0$

$3a + b = 1$

Remainder $= f(-1)$

$= -(-1)^5 + 3a(-1)^4 + b$

$= 1 + 3a + b$

$= 1 + (1)$

$= 2$

40. D

$h(1)^4 - k(1)^7 + 2 = 0$

$h - k + 2 = 0$

$k = h + 2$

Remainder $g(-1)$

$= h(-1)^4 - (h+2)(-1)^7 + 2$

$= h - (h+2)(-1) + 2$

$= 2h + 4$

41. B

$h(-1)^5 - k(-1)^9 + 6 = 0$

$-h + k + 6 = 0$

$k = h - 6$

Remainder $= g(1)$

$= h(1)^5 - (h-6)(1)^9 + 6$

$= h - (h-6) + 6$

$= 12$

42. C

$12 - h(-2)^2 + k(-2)^3 = 0$

$12 - 4h - 8k = 0$

$h = 3 - 2k$

Remainder $= g(2)$

$= 12 - (3-2k)(2)^2 + k(2)^3$

$= 16k$