REG-FG-2425-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. C	3. C	4. A	5. C
6. A	7. B	8. D	9. B	10. B
11. D	12. B	13. D	14. A	15. B
16. A	17. B	18. C	19. B	20. B
21. B	22. D	23. D	24. A	25. C
26. B	27. C	28. B	29. C	30. A

1. B

$$2f(3)-5=2[2(3)^2-7(3)+5]-5$$
$$=-1$$

2. C

$$9g(-1)=9[3^{-1}-2(-1)]$$
$$=21$$

3. C

$$f(a)-f(-a)=-2[(a)^2-(-a)^2]-5[(a)-(-a)]+[4-4]$$
$$=-5(2a)$$
$$=-10a$$

4. A

$$f(3)-f(-2)=[(3)^2-(-2)^2]-4[(3)-(-2)]+[k-k]$$
$$=-15$$

5. C

$$g(a+1)-g(a-1)=[(a+1)^2-(a-1)^2]-3[(a+1)-(a-1)]$$
$$=(4a)-3(2)$$
$$=4a-6$$

6. A

$$f(2)-f(-2)=2[(2)^2-(-2)^2]-5[(2)-(-2)]+[k-k]$$
$$=-20$$

7. B

$$f(1-k) = (1-k)^2 - (1-k) + 1$$
$$= k^2 - k + 1$$
$$= f(k)$$

8. D

$$f(2m-1) = 3(2m-1)^2 - 2(2m-1) + 1$$
$$= 12m^2 + (-12-4)m + (3+2+1)$$
$$= 12m^2 - 16m + 6$$

9. B

$$f(1+\beta) - f(1-\beta) = 3[(1+\beta)^2 - (1-\beta)^2] - [(1+\beta) - (1-\beta)] - [2-2]$$
$$= 3(4\beta) - (2\beta)$$
$$= 10\beta$$

10. B

$$k^2 + 5k - 4 = 2k$$
$$k^2 + 3k - 4 = 0$$
$$k = -4 \quad \text{or} \quad 1$$

11. D

$$g(-1) = g(7)$$
$$(-1)^2 - 2k(-1) + 1 = (7)^2 - 2k(7) + 1$$
$$2 + 2k = 50 - 14k$$
$$k = 3$$

12. B

$$(-3)^2 - 7(-3) + 2k = 12$$
$$30 + 2k = 12$$
$$k = -9$$

13. D

$$2 = 1^2 - b(1) + 3$$
$$b = 2$$
$$f(-1) = (-1)^2 - 2(-1) + 3$$
$$= 6$$

14. A

$$\begin{cases} f(0) = 1 = h(-3) + k \\ f(8) = 1 = (8 + h)(5) + k \end{cases}$$

Solving, we have $h = -5$ and $k = -14$.

15. B

The graph has no x-intercepts.

The equation $4x^2 - 6x + k = 0$ has no real roots.

$$\Delta = 6^2 - 4(4)(k) < 0$$

$$36 - 16k < 0$$

$$k > \frac{9}{4}$$

16. A

Coordinates of the vertex are $(2,\ -7)$.

Coefficient of x^2 is 2 (positive).

The graph opens upwards.

The answer is A.

17. B

Coordinates of the vertex are $(6,\ 16)$.

$$0 = -(x - 6)^2 + 16$$

$$0 = -x^2 + 12x - 20$$

$$x = 2 \quad \text{or} \quad 10$$

The coordinates of C and D are $(10,\ 0)$ and $(2,\ 0)$ respectively.

Required area $= (10 - 2)(16 - 0)$

$= 128$

18. C

Coordinates of V are $(5,\ -9)$.

$$0 = (x - 5)^2 - 9$$

$$x - 5 = \pm\sqrt{9}$$

$$x = 2 \quad \text{or} \quad 8$$

Coordinates of A and B are $(2,\ 0)$ and $(8,\ 0)$ respectively.

Required area $= \dfrac{(8 - 2)(9)}{2}$

$= 27$

19. B

The x-intercepts are -1 and 5.

The equation of the axis of symmetry is

$x = \dfrac{(-1)+5}{2}$

$x = 2$

When $x = 2$, $y = -(2+1)(2-5) = 9$.

The coordinates of the vertex are $(2,\ 9)$.

20. B

$b = y\text{-intercept} = -10$

Let the value of another x-intercept be β.

1 and β are roots of $-2x^2 + ax - 10 = 0$.

$$1 \times \beta = \frac{-10}{-2}$$

$$\beta = 5$$

Axis of symmetry is $x = \dfrac{1+5}{2} = 3$.

21. B

The x-intercepts are -3 and 4.

The equation of the graph is in the form $y = a(x+3)(x-4)$, where a is a constant.

$6 = a(0+3)(0-4)$

$a = -\dfrac{1}{2}$

Required equation is $y = -\dfrac{1}{2}(x+3)(x-4)$.

22. D

$f(x) = a(x-1)^2 - 2$, where a is a constant.

$4 = a(0-1)^2 - 2$

$a = 6$

$f(x) = 6(x-1)^2 - 2$

23. D

A. ✗. $0 = (x+2)^2 - 9$

$0 = x^2 + 4x - 5$

$x = -5 \quad \text{or} \quad 1$

The x-intercepts are -5 and 1.

B. ✗. $y\text{-intercept} = (0+2)^2 - 9 = -5$

C. ✗. The coordinates of the vertex of the graph are $(-2,\ -9)$.

D. ✓.

24. A

I. ✓. Coefficient of x^2 is 1, which is positive.

II. ✓. When $y = 0$, $x = h$ or k.
Since $hk < 0$, h and k are of opposite signs and cannot be equal.

III. ✗. When $x = 0$, $y = hk < 0$.
We have y-intercept $= hk < 0$.

25. C

$$y = -2x^2 + 6x + 1$$
$$= -2\left[x^2 - 2(x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2\right] + \frac{11}{2}$$
$$= -2\left(x - \frac{3}{2}\right)^2 + \frac{11}{2}$$

Required equation is

$$x = \frac{3}{2}$$
$$2x - 3 = 0$$

26. B

$$y = -x^2 + 4x + 5$$
$$= -[x^2 - 2(x)(2) + 2^2] + 9$$
$$= -(x - 2)^2 + 9$$

Required equation is $x = 2$.

27. C

$$y = x^2 - 6x + 11$$
$$= [x^2 - 2(x)(3) + 3^2] + 2$$
$$= (x - 3)^2 + 2$$

Required coordinates are $(3,\ 2)$.

28. B

Coefficient of $x^2 = -1 < 0 \quad \Rightarrow \quad$ the graph open downwards

x-coordinate of vertex $= -\dfrac{-4}{2(-1)} = -2$

29. C

$$y = -2x^2 + 16x - 6$$
$$= -2[x^2 - 2(x)(4) + 4^2] + 26$$
$$= -2(x - 4)^2 + 26$$

Greatest value is 26.

30. A

$y = x^2 + 6x - 1$

$= [x^2 - 2(x)(3) + 3^2] - 10$

$= (x + 3)^2 - 10$

Required coordinates are $(-3, -10)$.

Conventional Questions

31. (a) $-12 = 0^2 + a(0) + b$

$b = -12$ 1A

$-7 = (-5)^2 + a(-5) - 12$

$a = 4$ 1A

(b) $-12 = x^2 + 4x - 12$ 1M

$0 = x^2 + 4x$

$x = 0 \quad \text{or} \quad -4$ 1A

The coordinates of C are $(-4,\ -12)$.

$x^2 + 4x - 12 = 0$ 1M

$x = -6 \quad \text{or} \quad 2$

The coordinates of A and B are $(2,\ 0)$ and $(-6,\ 0)$ respectively. 1A

Required area $= \dfrac{(8+4)(12)}{2}$ 1M

$= 72$ 1A

32. (a) (i) $k = 2x^2 - 8x + 3$

$0 = 2x^2 - 8x + (3 - k)$

α and β are roots of the equation $2x^2 - 8x + (3 - k) = 0$.

$\alpha + \beta = -\dfrac{-8}{2}$ 1M

$= 4$ 1A

(ii) $\alpha\beta = \dfrac{3 - k}{2}$ 1A

(b) $3BP = 7PA$

$3\beta = 7(-\alpha)7\alpha + 3\beta \quad = 0$ 1M

Solve $\begin{cases} 7\alpha + 3\beta = 0 \\ \alpha + \beta = 4 \end{cases}$, we have $\alpha = -3$ and $\beta = 7$. 1A+1A

$\alpha\beta = \dfrac{3 - k}{2}$

$(-3)(7) = \dfrac{3 - k}{2}$

$k = 45$ 1A

33. (a) $P(n) = -2n^2 + 120n + 100$

$= -2[n^2 - 2(n)(30) + 30^2 - 30^2] + 100$ 1M

$= -2(n - 30)^2 + 1900$

Maximum daily profit is \$1900, which is smaller than \$2000. 1A

The claim is disagreed. 1A

(b) $Q(n) = P(n) + 150$

$= -2(n - 30)^2 + 2050$ 1M

Maximum daily profit is \$2050. 1A

It is possible.

34. (a) $f(x) = 60x - 2x^2$

$= -2[x^2 - 2(x)(15) + 15^2 - 15^2]$ 1M

$= -2(x - 15)^2 + 450$

Required coordinates are $(15,\ 450)$. 1A

(b) (i) $A = x\left(\dfrac{120 - 4x}{2}\right)$ 1A

$= 60x - 2x^2$ 1A

(ii) Area of each small rectangle

$= \dfrac{60x - 2x^2}{3}$

$= -\dfrac{2}{3}(x - 15)^2 + 150$ 1M

Maximum area is $150\,\text{cm}^2$, which is greater than $140\,\text{cm}^2$.

The claim is agreed. 1A

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