REG-CP1B-2425-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a) Required number = 11!

$= 39\,916\,800$ 1A

(b) Required number $= C_4^8 \times C_3^5 \times 3!4!$ 1M+1M

$= 100\,800$ 1A

2. (a) Required probability $= \dfrac{C_4^4}{C_4^{25}}$ 1M

$= \dfrac{1}{12\,650}$ 1A

(b) Required probability $= \dfrac{C_4^4 + C_2^4 C_1^9 C_1^{12} + C_2^9 C_2^{12}}{C_4^{25}}$ 1M

$= \dfrac{11}{46}$ 1A

3. (a) Required probability $= \dfrac{C_5^5 + C_5^6 + C_5^7}{C_5^{18}}$ 1M

$= \dfrac{1}{306}$ 1A

(b) Required probability $= 1 - \dfrac{1}{306}$ 1M

$= \dfrac{305}{306}$ 1A

4. (a) Required number $= C_5^8 C_2^5$ 1M

$= 560$ 1A

(b) Required number $= C_3^8 C_4^5 + C_2^8 C_5^5$ 1M

$= 308$ 1A

5. (a) Required probability $= \dfrac{C_4^{12}}{C_4^{20}}$ 1M

$= \dfrac{33}{323}$ 1A

(b) Required probability $= \dfrac{C_1^{12} C_3^8}{C_4^{20}}$ 1M

$= \dfrac{224}{1615}$ 1A

(c) Required probability $= \dfrac{C_1^8 C_3^{12} + C_2^8 C_2^{12}}{C_4^{20}}$ 1M

$= \dfrac{3608}{4845}$ 1A

6. (a) $y = 7$ 1A

$67 - \dfrac{1}{2}[(40 + x) + 51] = 18$

$x = 7$ 1A

(b) Mean $= \dfrac{34 + 35 + \ldots + 83}{20}$

$= 58$

Standard deviation $= \sigma \approx 13.5$

Required standard score $= \dfrac{62 - 58}{\sigma}$ 1M

≈ 0.296 1A

(c) Sum of the two deleted data $= 58 \times 2 = 116$

The only possible set of deleted data is $\{42, 74\}$. 1M

In this case, new standard deviation is 13.2, which is lower than the original standard deviation.

The new mean is also 58.

Thus, the new standard score increases. 1M

The claim is disagreed. 1A

7. (a) Let the mean score be μ marks.

$\dfrac{x - \mu}{4} - \dfrac{56 - \mu}{4} = 1.5 - (-1)$ 1M

$\dfrac{x - 56}{4} = 2.5$

$x = 66$ 1A

(b) The original mean score is 60 marks.

If David left the group, the mean score remains unchanged and the standard deviation of the score increases. 1A

New standard score of Joan $= \dfrac{56 - 60}{\sigma} > \dfrac{56 - 60}{4}$

The standard score of Joan increases. 1A

8. (a) It could happen that the 18th and the 19th students got 50 marks that make up the median 50 marks. 1M

The probability is therefore greater than 0.5. So, the claim is disagreed. 1A

(b) Standard score of Peter relative to boys $= \dfrac{48 - 52}{12} = -\dfrac{1}{3}$

Standard score of Mary relative to girls $= \dfrac{48 - 52}{10} = -\dfrac{2}{5} < -\dfrac{1}{3}$ 1M

The claim is agreed. 1A

9. (a) Let the mean and standard deviation of the distribution be μ marks and σ marks respectively.

$$\begin{cases} \dfrac{75-\mu}{\sigma} = 1.5 \\ \dfrac{45-\mu}{\sigma} = -1 \end{cases}$$ 1M

Solving, we have $\mu = 57$. 1A

(b) Since mean $= 57 < 60 =$ median of the distribution, half of the students' scores are not lower than 57. 1M

The claim is disagreed. 1A

10. (a) Let m marks be the mean of the test.

$$\frac{86-m}{8} = 1.5$$ 1M

$$m = 74$$

Standard score of Ringo $= \dfrac{68-74}{8}$

$= -0.75$ 1A

(b) (i) Required standard deviation $= 8(1+30\%) = 10.4$ marks 1A

(ii) Let z and x be the original standard score and the original score of a student respectively.

$$z = \frac{x-74}{8}$$

New standard score $= \dfrac{[x(1+30\%)+3]-[74(1+30\%)+3]}{10.4}$ 1M+1A

$= \dfrac{1.3(x-74)}{10.4}$

$= \dfrac{x-74}{8}$

$= z$

The claim is agreed. 1A

11. (a) $AB = BC \tan 70°$

≈ 54.9 cm 1A

(b) (i) $CD = AB \approx 54.9$ cm

$50^2 = 20^2 + CD^2 - 2(20)(CD)\cos\angle BCD$ 1M

$\angle BCD \approx 65.3°$ 1A

(ii) Let E be a point on AC such that $BE \perp AC$.

Let F be a point on CD such that $EF \perp AC$.

Required angle is $\angle BEF$. 1M

$BE = 20\sin 70° \approx 18.8$ cm

$EF = \dfrac{20}{\cos 20°} - 20\sin 70° \approx 2.49$ cm 1M

$CF = 20\tan 20° \approx 7.28$ cm

$BF^2 = 20^2 + CF^2 - 2(20)(CF)\cos\angle BCD$ 1M

$BF \approx 18.2\text{ cm}$

$BF^2 = BE^2 + EF^2 - 2(BE)(EF)\cos\angle BEF$

$\angle BEF \approx 72.4°$ 1A

Required angle is 72.4°.

12. (a) Let K be a point on EF such that $CK \perp EF$.

Consider $\triangle CFK$.

$$\cos\angle CFK = \frac{\left(\frac{60-30}{2}\right)}{50}$$

$$\angle CFK \approx 72.5°$$

$\angle DCF = 180° - \angle CFK \approx 107°$ 1A

Consider $\triangle DCF$.

$$DF^2 = 30^2 + 50^2 - 2(30)(50)\cos\angle DCF$$

$DF \approx 65.6\text{ cm}$ 1A

(b) Let J be a point on FH such that $CJ \perp FH$.

$AC = \sqrt{30^2 + 30^2} = 30\sqrt{2}\text{ cm}$

$HF = \sqrt{60^2 + 60^2} = 60\sqrt{2}\text{ cm}$

Consider $\triangle CFJ$.

$CJ = \sqrt{50^2 - \left(\frac{HF - AC}{2}\right)^2}$ 1M

$= 5\sqrt{82}\text{ cm}$

Height of the frustum is $5\sqrt{82}$ cm. 1A

(c) $BD = AC = 30\sqrt{2}\text{ cm}$

$BF = DF \approx 65.6\text{ cm}$

Consider $\triangle BDF$.

Let $s = \frac{BD + BF + DF}{2}$.

Area of $\triangle BDF = \sqrt{s(s - BD)(s - DF)(s - BF)}$ 1M

$\approx 1320\text{ cm}^2$

Let h cm be the required distance.

Consider the volume of the tetrahedron $CBDF$.

$\frac{1}{3}(\text{area of }\triangle BDF)(h) = \frac{1}{3}\left[\frac{(30)(30)}{2}\right](5\sqrt{82})$ 1M

$h \approx 15.5$ 1A

Required distance is 15.5 cm.

13. (a) $\angle BAC = 180° - 56° - 82° = 42°$

$$\frac{BC}{\sin 42°} = \frac{20}{\sin 56°}$$ 1M

$BC \approx 16.1\text{ cm}$ 1A

(b) (i) $10^2 = BC^2 + 20^2 - 2(BC)(20)\cos\angle ABC$ 1M

$\angle ABC \approx 29.8°$ 1A

(ii) Let E be a point on BD such that $CE \perp BD$.

Let F be a point on AB such that $EF \perp BD$.

Required angle is $\angle CEF$. 1M

$BF = BC \approx 16.1\text{ cm}$

$CE = FE = BC \sin\frac{82°}{2} \approx 10.6\text{ cm}$

$CF^2 = BC^2 + BF^2 - 2(BC)(BF)\cos\angle ABC$ 1M

$CF \approx 8.29\text{ cm}$

$CF^2 = EF^2 + CE^2 - 2(EF)(CE)\cos\angle CEF$ 1M

$\angle CEF \approx 46.1° > 45°$

The claim is agreed. 1A

14. (a) (i) Let $s = \frac{13+14+15}{2} = 21$.

Required area $= \sqrt{21(21-13)(21-14)(21-15)}$ 1M

$= 84\text{ cm}^2$ 1A

(ii) $\frac{14(AE)}{2} = 84$

$AE = 12\text{ cm}$ 1A

(b) Required angle is $\angle AEF$. 1M

$13^2 = 14^2 + 15^2 - 2(14)(15)\cos\angle ADB$

$\angle ADB \approx 53.1°$

$\angle CBD = \angle ADB \approx 53.1°$

$BE = \sqrt{AB^2 - AE^2} = 5\text{ cm}$ 1M

$EF = 5\tan\angle CBD \approx 6.67\text{ cm}$

$\cos\angle AEF = \frac{EF}{AE}$ 1M

$\angle AEF \approx 56.3°$ 1A

15. (a) $$\frac{\sin\angle BAD}{85} = \frac{\sin 60°}{102}$$ 1M

$\angle BAD \approx 46.2°$ or $134°$ (rejected)

$\angle BDA = 180° - \angle BAD - 60° \approx 73.8°$

$AB^2 = 102^2 + 85^2 - 2(102)(85)\cos\angle BDA$

$AB \approx 113\text{ cm}$ 1A

$\angle BDC = 140° - \angle BDA \approx 66.2°$

$BC = 85 \tan \angle BDC \approx 193 \text{ cm}$ 1A

$CD = \dfrac{85}{\cos \angle BDC} \approx 211 \text{ cm}$ 1A

(b) (i) Let G be a point on BC such that $AG \perp BC$.

Let H be a point on CD such that $GH \perp BC$.

Required angle is $\angle AGH$. 1M

$AC = \sqrt{CD^2 - AD^2} \approx 184 \text{ cm}$

$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos \angle ACB$

$\angle ACB \approx 34.8°$

$AG = AC \sin \angle ACB \approx 105 \text{ cm}$ 1M

$CG = AC \cos \angle ACB \approx 151 \text{ cm}$

$\dfrac{GH}{85} = \dfrac{CG}{BC}$

$GH \approx 66.7 \text{ cm}$

$CH = \sqrt{GH^2 + CG^2} \approx 165 \text{ cm}$

$\angle ACD = \tan^{-1} \dfrac{102}{AC} \approx 29.0°$

$AH^2 = AC^2 + CH^2 - 2(AC)(CH)\cos \angle ACD$

$AH \approx 89.3 \text{ cm}$

$AH^2 = AG^2 + GH^2 - 2(AG)(GH)\cos \angle AGH$ 1M

$\angle AGH \approx 57.5°$ 1A

(ii) Note that B is at a direction of N15°E from F.

Consider the area of the shadow.

$\dfrac{BD(BF \sin 15°)}{2} + \dfrac{(BC)(BF \cos 15°)}{2} = 2 \times 100^2$ 1M

$BF \approx 192 \text{ cm}$

Let K be the projection of A on the horizontal ground.

Note that F, B and K are collinear.

$AK = AG \sin \angle AGH \approx 88.8 \text{ cm}$ 1M

$BK = \sqrt{AB^2 - AK^2} \approx 70.1 \text{ cm}$

$\tan \phi = \dfrac{AK}{BF + BK}$ 1M

$\phi \approx 18.7° < 20°$

The claim is not correct. 1A

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