REG-CP1B-2425-ASM-SET 1-MATH
Suggested solutions

Conventional Questions

1. The equation $kx^2 - (3k+8)x + 3k + 1 = 0$ has at most 1 real root.

$$\Delta = (3k+8)^2 - 4(k)(3k+1) \le 0 \quad \text{1M}$$
$$-3k^2 + 44k + 64 \le 0$$
$$k \le -\frac{4}{3} \quad \text{or} \quad k \ge 16 \quad \text{1A}$$

Note that the graph of $y = f(x)$ does not lie below the x-axis.
The coefficient of x^2 is positive. We have $k > 0$. 1A
Thus, we have $k \ge 16$. 1M

2. (a) Consider $(3-k)x^2 + (k-3)x + (1-k) = 0$.

$$\Delta = (k-3)^2 - 4(3-k)(1-k) \ge 0 \quad \text{1M}$$
$$-3k^2 + 10k - 3 \ge 0$$
$$\frac{1}{3} \le k \le 3 \quad \text{1M}$$

Note that $3 - k \ne 0$. We have $\frac{1}{3} \le k < 3$. 1A

(b) If $k > 4$, then the graph does not intersect the x-axis.
Since $3 - k < -1 < 0$, the graph of $y = f(x)$ opens downwards. 1M
We have $f(x) < 0$ for all real values of x.
The claim is agreed. 1A

3. $\log_4 y = \frac{6}{2}x + 6$ 1M+1A

$y = 4^{3x+6}$ 1A

4. We have $\alpha + \beta = p$ and $\alpha\beta = p - 2$.

$$\log_4(\alpha+\beta) = \log_2 \alpha + \log_2 \beta$$
$$\frac{\log(\alpha+\beta)}{2\log 2} = \frac{\log \alpha\beta}{\log 2} \quad \text{1M}$$
$$\log p = 2\log(p-2)$$
$$p = (p-2)^2 \quad \text{1M}$$
$$0 = p^2 - 5p + 4$$
$$p = 4 \quad \text{or} \quad 1 \text{ (rejected)} \quad \text{1A}$$

5. (a) Slope of the graph $= \dfrac{12-6}{3-0} = 2$

$$\log_2 y - 6 = 2(x-0) \qquad \text{1M}$$

$$\log_2 y = 2x + 6$$

$$y = 2^{2x+6} \qquad \text{1M}$$

$$y = 2^6 \cdot 2^{2x}$$

$$= 64 \cdot 4^x$$

Thus, $a = 64$ and $b = 4$. 1A+1A

(b) $64(4^t) - 64(4^{t-1}) = 786\,432$ 1M

$$4^t(1-4^{-1}) = 12\,288$$

$$4^t = 16\,384$$

$$t \log 4 = \log 16\,384 \qquad \text{1M}$$

$$t = 7 \qquad \text{1A}$$

6. $\dfrac{\text{area of } A_{k+1}}{\text{area of } A_k} = (\sqrt{3})^2 = 3.$ 1A

$$\frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)^2 \sin 60^\circ \left(1 + 3 + 3^2 + \ldots + 3^{n-1}\right) > 10^6 \qquad \text{1M}$$

$$\frac{1}{\sqrt{3}} \times \frac{3^n - 1}{3 - 1} > 10^6$$

$$3^n > 2 \times 10^6 \times \sqrt{3} + 1$$

$$n \log 3 > \log\left(2 \times 10^6 \times \sqrt{3} + 1\right) \qquad \text{1M}$$

$$n > 13.7$$

The least value of n is 14. 1A

7. (a) Let a and r be the first term and common ratio of the sequence respectively.

$$\frac{ar^5}{ar^2} = \frac{3072}{48} \qquad \text{1M}$$

$$r^3 = 64$$

$$r = 4$$

$$a = 3$$

We have $A(n) = 3(4^{n-1})$. 1A

(b) $B(n) = \log[3(4^{n-1})]$

$$= \log 3 + (n-1)\log 4 \qquad \text{1M}$$

$$B(1) + B(2) + B(3) + \ldots + B(k) > 2023$$

$$k \log 3 + [1 + 2 + 3 + \ldots + (k-1)] \log 4 > 2023$$

$$k \log 3 + \frac{k(k-1)}{2} \cdot \log 4 > 2023 \quad \text{1M}$$

$$k^2 \log 2 + (\log 3 - \log 2)k - 2023 > 0$$

$$k > 81.7 \quad \text{or} \quad k < -82.3 \text{ (rejected)} \quad \text{1M}$$

The least value of k is 82. 1A

8. (a) $\dfrac{7}{\alpha} = \dfrac{\beta}{7}$ 1M

$$\alpha = \frac{49}{\beta}$$

$\log_7 \alpha = \log_7 49 - \log_7 \beta$ 1M

$= 2 - \log_7 \beta$ 1A

(b) $\log_7 \beta - \log_\beta \alpha = \log_\alpha \beta - \log_7 \beta$ 1M

$2\log_7 \beta - \dfrac{\log_7 \alpha}{\log_7 \beta} = \dfrac{\log_7 \beta}{\log_7 \alpha}$ 1M

Let $u = \log_7 \beta$.

$$2u - \frac{2-u}{u} = \frac{u}{2-u} \quad \text{1M}$$

$$2u^2(2-u) - (2-u)^2 = u^2$$

$$-2u^3 + 2u^2 + 4u - 4 = 0$$

$$-2(u-1)(u^2-2) = 0$$

$$u = 1 \text{ (rejected)} \quad \text{or} \quad -\sqrt{2} \text{ (rejected)} \quad \text{or} \quad \sqrt{2}$$

Required common difference

$= \log_7 \beta - \log_\beta \alpha$

$= \log_7 \beta - \dfrac{\log_7 \alpha}{\log_7 \beta}$

$= \sqrt{2} - \dfrac{2-\sqrt{2}}{\sqrt{2}}$ 1M

$= 1$ 1A

9. (a) We have $\alpha + \beta = \dfrac{c+3}{6}$ and $\alpha\beta = \dfrac{4c-2}{6} = \dfrac{2c-1}{3}$.

$\dfrac{7}{\alpha+6} = \dfrac{\beta+8}{7}$ 1M

$$49 = \alpha\beta + 8\alpha + 6\beta + 48$$

$49 = \dfrac{2c-1}{3} + 6\left(\dfrac{c+3}{6}\right) + 2\alpha + 48$ 1M

$\alpha = -\dfrac{5c+5}{6}$ 1

(b) $6\left(-\frac{5c+5}{6}\right)^2-(c+3)\left(-\frac{5c+5}{6}\right)+4c-2=0$ 1M

$$5c^2+\frac{47c}{3}+\frac{14}{3}=0$$

$$c=-\frac{14}{5} \quad \text{or} \quad -\frac{1}{3}$$

Note that when $c=-\frac{1}{3}$, the sum to infinity of the geometric sequence does not exist.

Thus, $c=-\frac{14}{5}$. 1A

$$\frac{\frac{15}{2}\left(\frac{14}{15}\right)^{n-1}\left[1-\left(\frac{14}{15}\right)^{n+3}\right]}{1-\frac{14}{15}}>37$$ 1M

$$-\left(\frac{14}{15}\right)^{2n+2}+\left(\frac{14}{15}\right)^{n-1}-\frac{74}{225}>0$$

$$-\frac{196}{225}\left(\frac{14}{15}\right)^{2n}+\frac{15}{14}\left(\frac{14}{15}\right)^{n}-\frac{74}{225}>0$$

$$0.590<\left(\frac{14}{15}\right)^{n}<0.640$$

$$\log 0.590<n\log\frac{14}{15}<\log 0.640$$ 1M

$$6.46<n<7.66$$

Thus, $n=7$. 1A

10. (a) $(10,\ 9)$ 1A

(b) $P_2P_3=8.1$

$P_3P_4=7.29$ 1A

Coordinates of P_4 are $(1.9,\ 1.71)$. 1A

(c) Total distance $=10+10\times0.9+10\times0.9^2+\ldots+10\times0.9^9$

$=\frac{10(1-0.9^{10})}{1-0.9}$ 1M

≈ 65.1 1A

(d) Let the coordinates of Q be $(a,\ b)$.

$a=10-10\times0.9^2+10\times0.9^4-\ldots$ 1M

$=\frac{10}{1-(-0.9^2)}$ 1M

$=\frac{1000}{181}$

$b=10\times0.9-10\times0.9^3+10\times0.9^5-\ldots$ 1M

$=\frac{9}{1-(-0.9^2)}$

$=\frac{900}{181}$

The coordinates of Q are $\left(\frac{1000}{181}, \frac{900}{181}\right)$. 1A

11. (a) $T_1 = \frac{1}{2}(3)(3)\sin 60°$ 1M

$= \frac{9\sqrt{3}}{4}$ 1A

(b) (i) $A_2B_2^2 = \left(3 \times \frac{2}{3}\right)^2 + \left(3 \times \frac{1}{3}\right)^2 - 2\left(3 \times \frac{2}{3}\right)\left(3 \times \frac{1}{3}\right)\cos 60°$ 1M

$A_2B_2 = \sqrt{3}$ 1A

(ii) Since $\triangle A_2B_2C_2$ is an equilateral triangle,

$T_2 = \frac{1}{2}(\sqrt{3})^2 \sin 60°$ 1M

$= \frac{3\sqrt{3}}{4}$ 1A

(c) (i) Common ratio $= \frac{3\sqrt{3}}{4} \div \frac{9\sqrt{3}}{4}$

$= \frac{1}{3}$ 1A

(ii) $T_n = \frac{9\sqrt{3}}{4} \times \left(\frac{1}{3}\right)^{n-1}$ 1A

(iii) $T_1 + T_2 + \ldots + T_n = \frac{\frac{9\sqrt{3}}{4}\left(1 - \left(\frac{1}{3}\right)^n\right)}{1 - \frac{1}{3}}$ 1M

$= \frac{27\sqrt{3}}{8}\left[1 - \left(\frac{1}{3}\right)^n\right]$ 1A

(iv) Required sum $= \frac{\frac{9\sqrt{3}}{4}}{1 - \frac{1}{3}}$ 1M

$= \frac{27\sqrt{3}}{8}$ 1A

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