REG-CP1A-2425-ASM-SET 4-MATH
Suggested solutions

Conventional Questions

1. (a) $\angle BAD = \angle BCE$ *(given)*

$\angle CBE = \angle BDA$ *(alt. ∠s, BC//AD)*

$\triangle ABD \sim \triangle CEB$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) Let $BE = x$ cm.

$$\frac{BD}{BE} = \frac{AD}{CB}$$ 1M

$$\frac{x+45}{x} = \frac{85}{34}$$

$$x = 30$$

We have $BD = 75$ cm.

$AB^2 + BD^2 = 40^2 + 75^2 = 7225\text{ cm}^2$ 1M

$AD^2 = 85^2 = 7225\text{ cm}^2 = AB^2 + BD^2$

Thus, $\triangle ABD$ is a right-angled triangle. 1A

2. (a) $AD = CB$ *(property of square)*

$\angle ADR = 90°$ *(property of square)*

$\angle CBP = 90°$ *(property of square)*

$= \angle ADR$

$\angle CPB = \angle RAB$ *(corr. ∠s, AR//PC)*

$\angle ARD = \angle RAB$ *(alt. ∠s, AB//DC)*

$= \angle CPB$

$\triangle ADR \cong \triangle CBP$ *(AAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\angle ARD + \angle DAR + 90° = 180°$

$$\angle ARD + \frac{\angle ARD}{4} = 90°$$

$$\angle ARD = 72°$$

Since $\triangle ADR \cong \triangle CBP$, we have $\angle CPB = \angle ARD = 72°$. 1M

Note that $QC = AR = PC$. $\triangle CPQ$ is an isosceles triangle. 1M

$\angle CQB = \angle CPB = 72°$ 1A

3. (a) (i) $AB = AD$ *(given)*

$\angle ABC = \angle ADC = 90°$ *(given)*

$AC = AC$ *(common side)*

$\triangle ABC \cong \triangle ADC$ *(RHS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) $\triangle ABC \cong \triangle ADC$ *(proved)*

$CD = BC$ *(corr. sides, $\cong$ $\triangle$s)*

$\angle ECD = \angle ECB$ *(corr. $\angle$s, $\cong$ $\triangle$s)*

$CE = CE$ *(common side)*

$\triangle BCE \cong \triangle DCE$ *(SAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\angle BFD + \angle FBC = 90° + 90° = 180°$

Therefore, $DE // BC$. 1M

We have $\angle DEC = \angle BCE$ and $\angle DEC = \angle BEC$.

Thus, we have $\angle BEC = \angle BCE$ and $BE = BC$. 1M

Then $DE = CD = CB = BE$ and $BCDE$ is a rhombus.

The claim is correct. 1A

4. (a) $\angle BAC = \angle ADC$ *(given)*

$\angle DAC = \angle ACB$ *(alt. $\angle$s, $AD // BC$)*

$\triangle ADC \sim \triangle CAB$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\dfrac{AC}{CB} = \dfrac{DC}{AB}$

$\dfrac{AC}{625} = \dfrac{168}{175}$ 1M

$AC = 600\text{ cm}$

$AB^2 + AC^2 = 175^2 + 600^2 = 390\,625\text{ cm}^2$

$BC^2 = 625^2 = 390\,625\text{ cm}^2 = AB^2 + AC^2$ 1M

We have $\angle BAC = 90°$.

Also, $\angle ADC = \angle BAC = 90°$.

The claim is agreed. 1A

5. (a) $\triangle ABE \sim \triangle CDE$ *(given)*

$\angle BAE = \angle DCE$ *(corr. $\angle$s, $\sim \triangle$s)*

$AB // DC$ *(alt. $\angle$s equal)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\angle BDC = \angle ABD$ 1M

$\angle BDC + \angle ACD = \angle BEC$

$\angle BDC + 2\angle BDC = 75°$

$\angle BDC = 25°$

$\angle ADC = \dfrac{180° - \angle ACD}{2}$ 1M

$= 65°$

$\angle ADB = 65° - 25°$

$= 40°$ 1A

6. (a) $DE = DE$ *(common side)*

$\angle DFE = 90°$ *(given)*

$\angle DCE = 90°$ *(property of rectangle)*

$= \angle DFE$

$AD = AE$ *(given)*

$\angle AED = \angle ADE$ *(base $\angle$s, isos. $\triangle$)*

$AD // BC$ *(property of rectangle)*

$\angle CED = \angle ADE$ *(alt. $\angle$s, $AD // BC$)*

$= \angle AED$

$\triangle CDE \cong \triangle FDE$ *(AAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) $AF = AE - FE = AE - CE = 5 - 1 = 4\text{ cm}$ 1M

$DF = \sqrt{AD^2 - AF^2} = \sqrt{5^2 - 4^2} = 3\text{ cm}$ 1M

Area of $\triangle ADF = \frac{1}{2}(4)(3) = 6\text{ cm}^2$ 1A

7. (a) (i) $EA = EF$ *(given)*

$ED = ED$ *(common side)*

$\angle EAD = 90°$ *(property of rectangle)*

$\angle EFD = 90°$ *(given)*

$= \angle EAD$

$\triangle EAD \cong \triangle EFD$ *(RHS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) $\angle EBF = \angle FCD = 90°$ *(property of rectangle)*

$\angle BEF = 180° - 90° - \angle BFE$ *(∠ sum of △)*

$= 90° - \angle BFE$

$\angle EFD = 90°$

$\angle CFD = 180° - 90° - \angle BFE$ *(adj. ∠s on st. line)*

$= 90° - \angle BFE$

$= \angle BEF$

$\triangle EBF \sim \triangle FCD$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $FD = AD = 30\text{ cm}$

$CF = \sqrt{30^2 - 24^2} = 18\text{ cm}$

$BF = 30 - 18 = 12\text{ cm}$

Since $\triangle EBF \sim \triangle FCD$, we have

$\frac{EF}{FD} = \frac{BF}{CD}$ 1M

$\frac{EF}{30} = \frac{12}{24}$

$EF = 15\text{ cm}$ 1A

(ii) $DE = \sqrt{15^2 + 30^2} = 15\sqrt{5}\text{ cm}$

Let the shortest distance from G to DE be h cm.

$$\frac{(DE)(h)}{2} = \frac{(15)(30)}{2}$$ 1M

$$h \approx 13.4$$
$$> 13$$

Thus, such point G does not exist. 1A

8. (a) $\angle EFD = \angle AFB$ *(vert. opp. ∠s)*

$\angle EDF = 90° - \angle CBE$ *(given)*

$\angle ABC = 90°$ *(given)*

$\angle ABF = 90° - \angle CBE$

$= \angle EDF$

$\triangle DEF \sim \triangle BAF$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $\angle DEF = \angle BAF = 90°$ 1M

$\triangle BDE$ is a right-angled triangle. 1A

(ii) Let $DF = x$ cm. Then $AF = (25 - x)$ cm.

$$\frac{DF}{BF} = \frac{EF}{AF}$$

$$\frac{x}{25} = \frac{6}{25 - x}$$ 1M

$$25x - x^2 = 150$$

$$-x^2 + 25x - 150 = 0$$

$$x = 10 \quad \text{or} \quad 15 \text{ (rejected)}$$

$DE = \sqrt{10^2 - 6^2} = 8$ cm 1M

$BD = \sqrt{(25+6)^2 + 8^2} = \sqrt{1025}$ cm

Required perimeter $= (25 + 6) + 8 + \sqrt{1025}$

$= (39 + 5\sqrt{41})$ cm 1A

≈ 71.0 cm

9. (a) Slope of $L = \frac{5 - 0}{15 - 12} = \frac{5}{3}$ 1M

Slope of $L' = -\frac{3}{5}$

Required equation is

$$\frac{y-0}{x-12}=-\frac{3}{5}$$ 1M

$$3x+5y-36=0$$ 1A

(b) (i) $3x+5k-36=0$

$$x=\frac{36-5k}{3}$$

The coordinates of G are $\left(\frac{36-5k}{3},\ k\right)$. 1M

The equation of C is

$$x^2+y^2-2\left(\frac{36-5k}{3}\right)x-2ky+F=0$$

$$3x^2+3y^2-2(36-5k)x-6ky+3F=0$$

where F is a constant.

C passes through $Q\ (12,\ 0)$.

$$3(12)^2+0-2(36-5k)(12)-0+3F=0$$ 1M

$$3F=432-120k$$

The equation of C is $3x^2+3y^2-2(36-5k)x-6ky+432-120k=0$. 1

(ii) $3(4)^2+3(8)^2-2(36-5k)(4)-6k(8)+432-120k=0$ 1M

$$k=3$$

The coordinates of G are $(7,\ 3)$.

PG is a diameter of the required circle.

$PG=\sqrt{(15-7)^2+(5-3)^2}=\sqrt{68}$ 1M

$$\text{Required area}=\pi\left(\frac{\sqrt{68}}{2}\right)^2$$

$$=17\pi$$ 1A

10. (a) Let the coordinates of G be $(h,\ 26)$.

Note that G lies on the perpendicular bisector of AB.

$$h=\frac{5+13}{2}$$ 1M

$$=9$$

The equation of C is

$$(x-9)^2+(y-26)^2=(5-9)^2+(23-26)^2$$ 1M

$$(x-9)^2+(y-26)^2=25$$ 1A

(b) $\sqrt{(k-9)^2+(38-26)^2}=15$ 1M

$$k^2-18k=0$$

$$k=18 \quad \text{or} \quad 0 \text{ (rejected)}$$ 1A

(c) (i) T, P and G are collinear. 1A

(ii) Radius of C is 5.

Required ratio $= GP : PT$ 1M

$= 5 : (15 - 5)$

$= 1 : 2$ 1A

11. (a) Slope of $L_1 = \dfrac{6-0}{4-0} = \dfrac{3}{2}$

Slope of $L_2 = -\dfrac{2}{3}$ 1M

Equation of L_2 is

$y - 6 = -\dfrac{2}{3}(x-4)$ 1M

$y = -\dfrac{2x}{3} + \dfrac{26}{3}$ 1A

(b) (i) Γ is the perpendicular bisector of MN. 1M

Γ is parallel to L_1. 1A

(ii) The coordinates of N are $(13,\ 0)$. 1M

Required equation is

$\sqrt{(x-13)^2 + (y-0)^2} = \sqrt{(x-4)^2 + (y-6)^2}$ 1M

$x^2 + y^2 - 26x + 169 = x^2 + y^2 - 8x - 12y + 52$

$-18x + 12y + 117 = 0$

$-6x + 4y + 39 = 0$ 1A

12. (a) $(x-7)^2 + (y+2)^2 = (4-7)^2 + (2+2)^2$ 1M

$(x-7)^2 + (y+2)^2 = 25$ 1A

(b) $GF = \sqrt{(7-2)^2 + (-2-10)^2} = 13$

Radius of $C = 5 < FG$ 1M

Thus, F lies outside C. 1A

(c) (i) Γ is the perpendicular bisector of GF. 1A

(ii) The coordinates of mid-point of FG are $\left(\dfrac{9}{2},\ 4\right)$.

Required distance $= \sqrt{\left(\dfrac{9}{2} - 7\right)^2 + (4+2)^2} - 5$ 1M

$= \dfrac{3}{2}$ 1A

13. (a) (i) Γ is the perpendicular bisector of EF. 1A

(ii) Let $(x,\ y)$ be the coordinates of P.

$$\sqrt{(x-8)^2+(y+20)^2}=\sqrt{(x-2)^2+(y-4)^2} \qquad \text{1M}$$

$$x^2+y^2-16x+40y+464=x^2+y^2-4x-8y+20$$

$$x-4y-37=0 \qquad \text{1A}$$

(b) (i) Let $(a,\ b)$ be the coordinates of H.

$$\begin{cases} a-4b-37=0 \\ \dfrac{9-4}{5-2}\times\dfrac{b-4}{a-2}=-1 \end{cases} \qquad \text{1M}$$

$$\frac{5}{3}\times\frac{b-4}{(4b+37)-2}=-1 \qquad \text{1M}$$

$$b=-5$$

The coordinates of H are $(17,\ -5)$. 1A

(ii) Centre of C is the mid-point of GH.

The coordinates of centre are $(11,\ 2)$. 1A

Radius $=\dfrac{GH}{2}=\dfrac{\sqrt{(17-5)^2+(9+5)^2}}{2}=\sqrt{85}$

Distance of E from the centre of C

$=\sqrt{(11-8)^2+(2+20)^2}$ 1M

$=\sqrt{493}$

$>\sqrt{85}$

E lies outside C.

The claim is agreed. 1A

14. (a) Let the coordinates of S be $(x,\ y)$.

$$\sqrt{(x-18)^2+(y+70)^2}=\sqrt{(x+60)^2+(y+96)^2} \qquad \text{1M}$$

$$x^2+y^2-36x+140y+5224=x^2+y^2+120x+192y+12\,816$$

$$-156x-52y-7592=0$$

$$3x+y+146=0 \qquad \text{1}$$

We have S lies on $3x+y+146=0$.

(b) (i) $130=\sqrt{(x-18)^2+(y+70)^2}$

$16\,900=(x-18)^2+(-3x-146+70)^2$ 1M

$0=10x^2+420x-10\,800$

$x=-60$ or 18 (rejected)

The coordinates of S are $(-60,\ 34)$. 1A

(ii) The coordinates of T are $(0,\ -70)$. 1A

Note that Q, S and U are collinear and $QS:SU=130:60=13:6$.

Let the coordinates of U be $(a,\ b)$.

$$\frac{a+60}{-60-18}=\frac{6}{13} \quad \text{and} \quad \frac{b-34}{34+70}=\frac{6}{13}$$ 1M

$$a=-96 \qquad\qquad b=82$$

Slope of $RT=\dfrac{-70+96}{0+60}=\dfrac{13}{30}$ 1M

Slope of $TU=\dfrac{82+70}{-96-0}=-\dfrac{19}{12}$

Slope of $RU=\dfrac{82+96}{-96+60}=-\dfrac{89}{18}$

Since no two of the slopes have a product of -1, there is no right angle in $\triangle RTU$.

The claim is disagreed. 1A

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