REG-CP1A-2425-ASM-SET 3-MATH
Suggested solutions

Conventional Questions

1. (a) $f(x) = (x+p)(x^2+qx-23)-44$ 1M

$= x^3+(p+q)x^2+(qp-23)x+(-23p-44)$

Comparing coefficients,

$$\begin{cases} p+q=r \\ qp-23=-15 \\ -23p-44=2 \end{cases}$$ 1M

Solving, we have $p=-2$, $q=-4$ and $r=-6$. 1A

(b) $0 = x^3-6x^2-15x+2$

$= (x+2)(x^2-8x+1)$ 1M+1A

$x+2=0$ or $x^2-8x+1=0$

$x=-2$ or $x = \dfrac{8 \pm \sqrt{64-4}}{2(1)}$

$= 4 \pm \sqrt{15}$

Since $8 \pm \sqrt{15}$ are not rational,

Thus, the claim is agreed. 1A

2. (a) $f(x) = (3x-1)(x^2-2x-3)+r$ 1A

$f(2) = (6-1)(4-4-3)+r = 0$ 1M

$r = 15$ 1A

(b) $f(x) = (3x-1)(x^2-2x-3)+15$

$= 3x^3-7x^2-7x+18$

$= (x-2)(3x^2-x-9)$

Therefore, $g(x) = 3x^2-x-9$.

In the equation $g(x)=0$, $\Delta = (-1)^2-4(3)(-9) = 109 > 0$. 1M

Roots of $g(x)=0$ are real. The claim is agreed. 1A

3. (a) Compare the coefficient of x^2 and the constant term.

$$\begin{cases} -6 = (1)(b)+(a)(-1)+(-2)(2) \\ 6 = -2b \end{cases}$$ 1M

Solving, we have $a=-1$ and $b=-3$. 1A+1A

(b) $f(x) = g(x)$

$(x^2 - x - 2)(2x^2 - x - 3) - (2x^2 - x - 3) = 0$ 1M

$(2x^2 - x - 3)(x^2 - x - 3) = 0$

$(2x - 3)(x + 1)(x^2 - x - 3) = 0$

$x = -1$ or $\frac{3}{2}$ or $\frac{1 \pm \sqrt{1^2 - 4(1)(-3)}}{2}$ 1M

$x = -1$ or $\frac{3}{2}$ or $\frac{1 \pm \sqrt{13}}{2}$

The roots $\frac{1 \pm \sqrt{13}}{2}$ are not rational.

The claim is disagreed. 1A

4. (a) Let $f(x) = (x^2 - 1)Q(x) + 5x - 25$, where $Q(x)$ is a polynomial. 1M

$f(1) = 1 + a - 5 + b = 0 + 5 - 25$ 1M

$a + b = -16$

$f(-1) = -1 - a - 5 + b = 0 - 5 - 25$

$-a + b = -24$ 1M

Solving, we have $a = 4$ and $b = -20$ 1A

(b) $f(x) = x^{15} + 4x^{11} - 5x^4 - 20 = 0$

$x^{11}(x^4 + 4) - 5(x^4 + 4) = 0$

$(x^4 + 4)(x^{11} - 5) = 0$ 1M

$x^{11} = 5$ or $x^4 = -4$ (rejected)

$x = \sqrt[11]{5}$ which is irrational 1M

The claim is not correct. 1A

5. (a) $p(x) = (x + r)(x^2 - rx + 2r^2) - 4r^3$ 1A

$p(r) = (r + r)(r^2 - r^2 + 2r^2) - 4r^3$ 1M

$= 4r^3 - 4r^3$

$= 0$

$p(x)$ is divisible by $x - r$. 1

(b) $0 = (x + r)(x^2 - rx + 2r^2) - 4r^3$

$= x^3 + r^2x - 2r^3$

$= (x - r)(x^2 + rx + 2r^2)$ 1M

$x = r$ or $x^2 + rx + 2r^2 = 0$ 1A

When $x^2 + rx + 2r^2 = 0$, $\Delta = r^2 - 4(1)(2r^2) = -7r^2 < 0$. 1M

The roots of the equation $x^2 + rx + 2r^2 = 0$ are not real.

The claim is disagreed. 1A

6. (a) $f(x) = (x - 2)(x^2 - 3x + 4) + r$ 1A

So, $(x-1)g(x) \equiv (x-2)(x^2-3x+4)+r$.

Put $x = 1$,

$0 = (-1)(1-3+4)+r$ 1M

$r = 2$ 1A

(b) $f(x) = (x-2)(x^2-3x+4)+2$

$= x^3 - 5x^2 + 10x - 6$

$= (x-1)(x^2-4x+6)$ 1M

Therefore, $g(x) = x^2 - 4x + 6$. When $g(x) = 0$, 1A

$\Delta = 4^2 - 4(1)(6) = -8 < 0$ 1M

The roots of $g(x) = 0$ are not real, i.e., they are not rational numbers.

The claim is disagreed. 1A

7. (a) $2(2)^3 + 19(2)^2 + 16(2) + k = 112$ 1M

$k = -12$

$f\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right)^3 + 19\left(-\frac{3}{2}\right)^2 + 16\left(-\frac{3}{2}\right) - 12$ 1M

$= 0$

Thus, $2x+3$ is a factor of $f(x)$. 1A

(b) $2x^3 + 19x^2 + 16x - 12 = 0$

$(2x+3)(x^2+8x-4) = 0$ 1M

$x = -\frac{3}{2}$ or $\frac{-8 \pm \sqrt{80}}{2}$

Since $\frac{-8 \pm \sqrt{80}}{2} = -4 \pm 2\sqrt{5}$ are irrational numbers.

The claim is disagreed. 1A

8. (a) Let $f(x) = (x^2-1)Q(x) + Ax + B$, where $Q(x)$ is a polynomial, A and B are constants. 1A

$\begin{cases} 3 = f(1) = A + B \\ -7 = f(-1) = -A + B \end{cases}$ 1M

Solving, we have $A = 5$ and $B = -2$.

Required remainder is $5x - 2$. 1A

(b) Let $f(x) = (x^2-1)(Cx+D) + 5x - 2$, where C and D are constants. 1A

Coefficient of $x = -C + 5 = 0$

$C = 5$ 1A

$f(0) = (-1)(D) - 2 = 0$

$D = -2$ 1A

Therefore, $f(x) = (x^2 - 1)(5x - 2) + (5x - 2)$. When $f(x) = 0$,

$$(x^2 - 1)(5x - 2) + (5x - 2) = 0$$
$$(5x - 2)(x^2) = 0$$
$$x = \frac{2}{5} \quad \text{or} \quad 0$$

The other root is $\frac{2}{5}$. 1A

9. (a) Let $f(x) = (x^2 + 3x + 2)(Ax + B) + ax + 32$, where A and B are constants. 1A

$f(-2) = 0 = (4 - 6 + 2)(-2A + B) - 2a + 32$ 1M

$a = 16$ 1A

(b) We have $f(1) = 0$ and $f(3) = -40$.

$$\begin{cases} 0 = (1 + 3 + 2)(A + B) + 16 + 32 \\ -40 = (9 + 9 + 2)(3A + B) + 48 + 32 \end{cases}$$ 1M

Solving, we have $A = 1$ and $B = -9$. 1A

$0 = f(x)$

$0 = (x + 1)(x + 2)(x - 9) + 16(x + 2)$

$0 = (x + 2)[(x + 1)(x - 9) + 16]$ 1M

$0 = (x + 2)(x^2 - 8x + 7)$

$0 = (x + 2)(x - 1)(x - 7)$

$x = -2 \quad \text{or} \quad 1 \quad \text{or} \quad 7$ 1A

All the roots of the equation are rational numbers.
The claim is correct. 1A

10. (a) $p(x) = (x^2 - 2x + 2)(10x^2 + ax - 19) + (bx + 41)$ 1M
Since $p(x)$ is divisible by $(x + 1)(x - 1)$, we have $p(1) = p(-1) = 0$.

$$\begin{cases} 0 = (1 - 2 + 2)(10 + a - 19) + b + 41 \\ 0 = (1 + 2 + 2)(10 - a - 19) - b + 41 \end{cases}$$ 1M

Solving, we have $a = 7$ and $b = -39$. 1A+1A

(b) $0 = p(x)$

$0 = (x^2 - 2x + 2)(10x^2 + 7x - 19) - 39x + 41$

$= 10x^4 - 13x^3 - 13x^2 + 13x + 3$

$= (x^2 - 1)(10x^2 - 13x - 3)$ 1M

$= (x + 1)(x - 1)(2x - 3)(5x + 1)$

$x = \pm 1 \quad \text{or} \quad \frac{3}{2} \quad \text{or} \quad -\frac{1}{5}$ 1M

The equation $p(x) = 0$ has 4 rational roots. 1A

11. (a) $f(-2) = 0$

$4(-2)^3 + b(-2)^2 + c(-2) + 18 = 0$ 1M

$2b - c = 7$

$f(1) = f\left(-\frac{3}{2}\right)$

$4 + b + c + 18 = 4\left(-\frac{3}{2}\right)^3 + b\left(-\frac{3}{2}\right)^2 + c\left(-\frac{3}{2}\right) + 18$ 1M

$b - 2c = 14$

Solving the simultaneous equations $\begin{cases} 2b - c = 7 \\ b - 2c = 14 \end{cases}$, 1M

we have $b = 0$ and $c = -7$. 1A

(b) $f(x) = 4x^3 - 7x + 18$

$= (x+2)(4x^2 - 8x + 9)$ 1M+1A

(c) Remainder $= f(-1-1)$

$= f(-2)$

$= 4(-2)^3 - 7(-2) + 18$ 1M

$= 0$ 1A

12. (a) Let $p(x) = 2(x+3)^2(Ax + B) + 22$, where A and B are constants. 1A

$\begin{cases} 0 = 2(-4+3)^2(-4A + B) + 22 & \text{1M} \\ -5 = 2\left(-\frac{3}{2} + 3\right)^2\left(-\frac{3A}{2} + B\right) + 22 & \text{1M} \end{cases}$

Solving, we have $A = 2$ and $B = -3$. 1A

We have $p(x) = 2(x+3)^2(2x - 3) + 22$.

(b) $0 = p(x) + k(x^2 + 6x + 8)$

$= 4x^3 + 18x^2 - 32 + k(x^2 + 6x + 8)$

$= 2(x+4)(2x^2 + x - 4) + k(x+2)(x+4)$ 1M

$= (x+4)(4x^2 + (k+2)x + (2k-8))$

$x = -4$ or $4x^2 + (k+2)x + (2k-8) = 0$

The equation $4x^2 + (k+2)x + (2k-8) = 0$ has two distinct real roots.

$\Delta = (k+2)^2 - 4(4)(2k-8) > 0$ 1M

$k^2 - 28k + 132 > 0$

$k < 6$ or $k > 22$

When $x = -4$ is a root of $4x^2 + (k+2)x + (2k-8) = 0$.

$4(-4)^2 + (k+2)(-4) + (2k-8) = 0$ 1M

$k = 24$

We have $k \neq 24$ for three distinct real roots.

Thus, we have $k < 6$ or $22 < k < 24$ or $k > 24$. 1A

13. (a) Let $g(x) = (2x^2 - 9x + 14)(Ax + B) + 3x - 1$, where A and B are constants. 1M

$$\begin{cases} 33 = [2(2)^2 - 9(2) + 14](2A + B) + 3(2) - 1 \\ 21 = [2(-1)^2 - 9(-1) + 14](-A + B) + 3(-1) - 1 \end{cases}$$ 1M

Solving, we have $A = 2$ and $B = 3$.

Required quotient is $2x + 3$. 1A

(b)
$$(2x^2 - 9x + 14)(2x + 3) + 3x - 1 = 2(x + 2)(3x - 1)$$
$$(2x^2 - 9x + 14)(2x + 3) + (3x - 1)[1 - 2(x + 2)] = 0$$
$$(2x + 3)[(2x^2 - 9x + 14) - (3x - 1)] = 0$$ 1M
$$(2x + 3)(2x^2 - 12x + 15) = 0$$

$2x + 3 = 0$ or $2x^2 - 12x + 15 = 0$

$x = -\dfrac{3}{2}$ $\qquad$ $x = \dfrac{12 \pm \sqrt{12^2 - 4(2)(15)}}{2(2)}$ 1M

$= \dfrac{6 \pm \sqrt{6}}{2}$

$\dfrac{6 \pm \sqrt{6}}{2}$ are not rational numbers.

The claim is disagreed. 1A

14. (a) $f(3) = 3^3 - 8(3) - 3 = 0$

Thus, $x - 3$ is a factor of $f(x)$. 1

(b) (i) $g(2) = 0 + 2a + b = 0$ 1M

$$2a + b = 0$$

$g(1) = 2(1 - 8)(1 - 2) + a + b = 20$ 1M

$$a + b = 6$$

Solving, we have $a = -6$ and $b = 12$. 1A

(ii) $g(x) = 2x(x^2 - 8)(x - 2) - 6x + 12 = 0$

$$(x - 2)[2x(x^2 - 8) - 6] = 0$$ 1M
$$(x - 2)(2x^3 - 16x - 6) = 0$$
$$2(x - 2)(x - 3)(x^2 + 3x + 1) = 0$$ 1M

$x = 2$ or $x = 3$ or $x^2 + 3x + 1 = 0$

$x = 2$ or 3 or $\dfrac{-3 \pm \sqrt{3^2 - 4}}{2}$ 1M

$x = 2$ or 3 or $\dfrac{-3 \pm \sqrt{5}}{2}$

Since $\dfrac{-3 \pm \sqrt{5}}{2}$ is not rational, the claim is disagreed. 1A

END OF PAPER