Suggested solutions

Conventional Questions

1. (a) Let $f(x) = a + bx^2$, where a and b are non-zero constants.   1A

$$\begin{cases} -20 = a + 4b \\ 108 = a + 36b \end{cases}$$ 1M

Solving, we have $a = -36$ and $b = 4$.   1A

$f(0) = -36 + 4(0)^2 = -36$ 1A

(b) 3 and -3 1A

(c) $f(x) = -8x - c$

$4x^2 + 8x + (c - 36) = 0$

The equation $4x^2 + 8x + (c - 36) = 0$ has no real roots.

$8^2 - 4(4)(c - 36) < 0$ 1M

$c > 40$ 1A

2. (a) Let $f(x) = a + b(x + 1)^2$, where a and b are non-zero constants.   1A

$20 = a + b(2 + 1)^2$ 1M

$12 = a + b(-2 + 1)^2$

Solving, we have $a = 11$ and $b = 1$.   1A

Thus, $f(x) = 11 + (x + 1)^2$.

(b) (Cancelled)

3. (a) Let $P = a + bV$, where a and b are non-zero constants.   1A

$90 = a + 30b$ 1M

$114 = a + 50b$

Solving, we have $a = 54$ and $b = \dfrac{6}{5}$.   1A

Required volume $= \dfrac{144 - 54}{\left(\frac{6}{5}\right)}$

$= 75 \, \text{m}^3$ 1A

(b) Ratio of the volume of X to the volume of Y

$= \left(\sqrt{\dfrac{1}{16}}\right)^3$ 1M

$= \dfrac{1}{64}$

Volume of $Y = 75(64)$

$$= 4800 \, \text{m}^3$$

Required cost $= 54 + \dfrac{6}{5}(4800)$

$$= \$5814 \qquad\qquad\qquad \text{1A}$$

4.   (a) Let $F = a + bv^2$, where a and b are non-zero constants.   1A

$$60 = a + b(20)^2 \qquad\qquad\qquad \text{1M}$$
$$85 = a + b(30)^2$$

Solving, we have $a = 40$ and $b = \dfrac{1}{20}$.

Required resistance $= 40 + \dfrac{1}{20}(50)^2$

$$= 165 \text{ units} \qquad\qquad\qquad \text{1A}$$

  (b) Let u km/h be the original speed of the car.

$$40 + \frac{1}{20}(2v)^2 = 2 \times \left[40 + \frac{1}{20}v^2 \right] \qquad\qquad \text{1M+1A}$$

$$\frac{v^2}{10} = 40$$

$$v = 20 \quad \text{or} \quad -20 \text{ (rejected)} \qquad\qquad \text{1A}$$

5.   (a) Let $f(x) = a + bx$, where a and b are non-zero constants.   1A

$$\begin{cases} 22 = a + 7b \\ 4 = a - 2b \end{cases} \qquad\qquad\qquad \text{1M}$$

Solving, we have $a = 8$ and $b = 2$.   1A

Thus, $f(x) = 8 + 2x$.

  (b) (i) The coordinates of B and C are $(-4, \ 0)$ and $(0, \ 8)$ respectively.   1M

$BD = \sqrt{(-4 - 0)^2 + (0 - 3)^2} = 5$

$CD = 8 - 3 = 5$

Thus, $BD = CD$.   1

    (ii) Slope of $L_1 \times$ slope of $L_2 = 2 \times \left(-\dfrac{1}{2} \right) = -1$

We have $L_1 \perp L_2$.   1M

Since $BD = CD$, we have $BM = CM$.

Since $CF : CM : BM = 2 : 1 : 1$, we have $FM : BM = 3 : 1$.   1M

Required ratio

$= FM : BM$

$= 3 : 1$ 1A

6. (a) Volume of milk $= \frac{1}{3}\pi(78)^2\sqrt{130^2 - 78^2}$ 　　　　　　1M

$$= 210\,912\pi \text{ cm}^3$$ 　　　　　　1A

(b) Let the height of the frustum inside the hemispherical vessel be h cm.

$$h^2 + \left(\frac{AC}{2}\right)^2 = 75^2$$ 　　　　　　1M

$$h^2 + 60^2 = 75^2$$

$$h = 45$$

Volume of the frustum

$$= \frac{1}{3}(96)(72)(150)\left[1 - \left(\frac{150-45}{150}\right)^3\right]$$ 　　　　　　1M

$$= 227\,059.2 \text{ cm}^3$$

Capacity of the vessel

$$= \frac{2}{3}\pi(75)^3 - 227\,059.2$$ 　　　　　　1M

$$\approx 656\,514 \text{ cm}^3$$ 　　　　　　1A

$$< 210\,912\pi \text{ cm}^3$$

The milk will overflow.

The claim is agreed. 　　　　　　1A

7. (a) $\cos\dfrac{\angle AOB}{2} = \dfrac{10-2}{10}$ 　　　　　　1M

$$\angle AOB \approx 73.7°$$

Required area

$$= \pi(10)^2 \times \frac{\angle AOB}{360°}$$ 　　　　　　1M

$$\approx 64.4 \text{ cm}^2$$ 　　　　　　1A

(b) $AB = 2\sqrt{10^2 - 8^2} = 12$ cm

Volume of water

$$= \left(\pi(10)^2 \times \frac{\angle AOB}{360°} - \frac{(12)(8)}{2}\right)(30)$$ 　　　　　　1M

$$\approx 491 \text{ cm}^3$$

$$CD = 12 \times \frac{5}{4} = 15 \text{ cm}$$ 　　　　　　1M

$$\sin\frac{\angle COD}{2} = \frac{\left(\frac{15}{2}\right)}{10}$$

$$\angle COD \approx 97.2°$$

Volume of the oil

$$= \left[\pi(10)^2 \times \frac{\angle COD}{360°} - \frac{(15)(\sqrt{10^2 - 7.5^2})}{2}\right](30) - (\text{volume of water})$$ 1M

$\approx 565\,\text{cm}^3$

$> 491\,\text{cm}^3$

The volume of the oil is greater than the volume of the water. 1A

8. (a) Let the radius of the circular cone be r cm.

$$2\pi r = 2\pi(60) \times \frac{216°}{360°}$$ 1M

$$r = 36$$ 1A

Required radius is 36 cm.

(b) Height of circular cone $= \sqrt{60^2 - 36^2} = 48$ cm. 1A

Let the radius of the hemisphere be R cm.

$$\frac{48 - (R + 6)}{48} = \frac{R}{36}$$ 1M

$$6(42 - R) = 8R$$

$$R = 18$$

Volume of the decoration

$$= \frac{\pi}{3}(36)^2(48)\left[1 - \left(\frac{48 - 24}{48}\right)^3\right] - \frac{2\pi}{3}(18)^3$$ 1M+1A

$$= 14256\pi\,\text{cm}^3$$

$\approx 0.0448\,\text{m}^3 > 0.04\,\text{m}^3$

The claim is agreed. 1A

9. (a) Let the radius of the sphere be r cm.

$$4\pi r^2 = 144\pi$$

$$r = 6 \quad \text{or} \quad -6 \text{ (rejected)}$$

Required volume $= \dfrac{4\pi}{3}(6)^3$ 1M

$$= 288\pi\,\text{cm}^3$$ 1A

(b) Volume of water $= \pi(16)^2(14) - 288\pi$ 1M

$$= 3296\pi\,\text{cm}^3$$

Required depth $= \dfrac{3296\pi}{\pi(16)^2}$ 1M

$$= \frac{103}{8}\,\text{cm}$$ 1A

(c) Base radius of cone $= \dfrac{48\pi}{2\pi} = 24\,\text{cm}$

Slant height of cone = $\dfrac{720\pi}{\pi(24)} = 30\,\text{cm}$

Volume of cone = $\dfrac{\pi}{3}(24)^2\sqrt{30^2 - 24^2}$ 1M+1M

$= 3456\pi\,\text{cm}^3$

$> 3296\pi\,\text{cm}^3$

The water will not overflow.   1A

10. (a) Mean = $\dfrac{140 + 141 + \ldots + 188}{22}$

$= 161\,\text{cm}$ 1A

Inter-quartile range = $173 - 150$

$= 23\,\text{cm}$ 1A

Standard deviation $\approx 14.8\,\text{cm}$ 1A

(b) One of the left student has a height of $154\,\text{cm}$.

Let x cm be the height of the other student.

$161 \times 22 - 154 - x = (161 + 1.2)(20)$ 1M

$x = 144$

New inter-quartile range = $\dfrac{173 + 178}{2} - \dfrac{150 + 153}{2} = 24\,\text{cm}$ 1M

$24 - 23 = 1$

The inter-quartile range is increased by $1\,\text{cm}$.   1A

11. (a) $(50 + b) - (20 + a) = 36$ 1M

$b - a = 6$

Mean = $\dfrac{(20 + a) + 23 + \ldots + (50 + b)}{18} > 38$ 1M

$a + b > 8$

We have $(a,\ b) = (3,\ 9)$ or $(2,\ 8)$.   1A+1A

(b) Suppose $a = 3$ and $b = 9$.

Standard deviation is 10.30.   1M

Suppose $a = 2$ and $b = 8$.

Standard deviation is 10.27.

The greatest possible standard deviation is 10.30.   1A

12. (a) $2a - (a - 32) = 4(118 - a)$ 1M

$a = 88$

Lower quartile is $88.   1A

Range = $2a - (a - 32)$

$= \$120$ 1A

(b) $\dfrac{219\,468 + 102 \times 92 + 54h + 54k}{2017 + 210} \geq 108$ 1M

$$k \geq 216 - h$$

Note that $h \leq 110$.

$$k \geq 216 - 110$$
$$k \geq 106$$

The values of h and k are greater than 105.   1M

There are 108 (54 + 54) new books with selling prices greater than $105.

There area 102 new books with selling prices less than $105.

The new median of the selling books is not less than the median before adding 210 new books.   1M

The new median is not less than $105.

The claim is disagreed.   1A