

REG-AOT-2425-ASM-SET 6-MATH

Suggested solutions

Conventional Questions

1. (a) $PS^2 = 24^2 + 18^2 - 2(24)(18) \cos 50^\circ$ 1M
 $PS \approx 18.6 \text{ cm}$ 1A
- (b) Let M be the mid-point of QR . The angle required is $\angle SMP$. 1M
 $SM = \sqrt{18^2 - \left(\frac{20}{2}\right)^2} = 4\sqrt{14} \text{ cm}$ 1M
 $PM = \sqrt{24^2 - 10^2} = 2\sqrt{119} \text{ cm}$
 $PS^2 = SM^2 + PM^2 - 2(SM)(PM) \cos \angle SMP$ 1M
 $\angle SMP \approx 57.0^\circ$ 1A
- (c) $\angle PAS = \cos^{-1} \frac{AP^2 + AS^2 - SP^2}{2(AP)(AS)}$
 By symmetry of the figure, $\angle PAS$ is the largest when A is at the mid-point of QR such that AP and AS are perpendicular to QR and are shortest.
 When A moves from Q to mid-point M , $\angle PAS$ increases from 50° to 57.0° . 1A
 When A moves from mid-point M to R , $\angle PAS$ decreases from 57.0° to 50° . 1A
2. (a) $AM = 10 \sin 60^\circ \approx 8.66 \text{ cm}$ 1A
 $\angle MAP = \frac{60^\circ}{4} = 15^\circ$
 $PQ = 2PM$
 $= 2 \times AM \tan 15^\circ$ 1M
 $\approx 4.64 \text{ cm}$ 1A
- (b) As the planes APB and AQC are identical and vertical, vertical distance from M to the ground is equal to the vertical distance from P to the ground.
 With reference of the second figure, the vertical distance = PM . 1M
 Vertical distance = $10 \sin 60^\circ \tan 15^\circ$
 $\approx 2.32 \text{ cm}$ 1A
- (c) Let P' and Q' be the projections of P and Q on the horizontal ground respectively.
 $AP' = AQ' = AM \approx 8.66 \text{ cm}$ 1A
 $P'Q' = PQ \approx 4.64 \text{ cm}$
 In $\triangle AP'Q'$,
 $P'Q'^2 = AP'^2 + AQ'^2 - 2(AP')(AQ') \cos \angle P'AQ'$ 1M
 $\angle P'AQ' \approx 31.1^\circ$
 Thus, $\angle BAC \approx 31.1^\circ$ 1A
3. (a) $\frac{\sin \angle ABD}{19} = \frac{\sin 80^\circ}{30}$ 1M
 $\angle ABD \approx 38.6^\circ \text{ or } 141^\circ \text{ (rejected)}$ 1A

- (b) (i) $25^2 = 19^2 + 30^2 - 2(19)(30) \cos \angle CDA$ 1M
 $\angle CDA \approx 56.1^\circ$ 1A
- (ii) Let E be a point on BD such that $AE \perp BD$.
Let F be a point on CD such that $EF \perp BD$.
Required angle is $\angle AEF$. 1A
 $AE = 19 \sin 80^\circ \approx 18.7 \text{ cm}$ 1M
 $DE = 19 \cos 80^\circ \approx 3.30 \text{ cm}$
 $\angle EDF = \angle ABD \approx 38.6^\circ$
 $EF = DE \tan EDF \approx 2.63 \text{ cm}$
 $DF = \frac{DE}{\cos \angle EDF} \approx 4.22 \text{ cm}$
 $AF^2 = DF^2 + 19^2 - 2(DF)(19) \cos \angle CDA$ 1M
 $AF \approx 17.0 \text{ cm}$
In $\triangle AEF$,
 $AF^2 = AE^2 + EF^2 - 2(AE)(EF) \cos \angle AEF$
 $\angle AEF \approx 46.6^\circ$ 1A
Required angle is 46.6° .
4. (a) $AD = 20 \cos 50^\circ \approx 12.9 \text{ cm}$ 1A
 $DB = \sqrt{30^2 - (20 \sin 50^\circ)^2} \approx 25.8 \text{ cm}$ 1A
- (b) (i) $AB^2 = 20^2 + 30^2 - 2(20)(30) \cos 45^\circ$ 1M
 $AB \approx 21.2 \text{ cm}$ 1A
- (ii) Required angle is $\angle ADB$.
 $AB^2 = AD^2 + DB^2 - 2(AD)(DB) \cos \angle ADB$ 1M
 $\angle ADB \approx 55.1^\circ$ 1A
- (iii) $CD = 20 \sin 50^\circ \approx 15.3 \text{ cm}$
Volume of tetrahedron $= \frac{1}{3} \times \frac{1}{2} (AD)(DB) \sin \angle ADB (CD)$ 1M
 $\approx 695 \text{ cm}^3 > 650 \text{ cm}^3$
The claim is disagreed. 1A
5. (a) $\frac{12\sqrt{3}}{\sin 120^\circ} = \frac{12}{\sin \angle ABC}$ 1M
 $\angle ABC = 30^\circ$ or 150° (rejected) 1A
 $\angle ACB = 180^\circ - 30^\circ - 120^\circ = 30^\circ$.
 $AB^2 = 12^2 + (12\sqrt{3})^2 - 2(12)(12\sqrt{3}) \cos 30^\circ$
 $AB = 12 \text{ cm}$ 1A
- (b) Let M be the mid-point of BC . The angle required is $\angle VMA$. 1A
 $VM = \sqrt{15^2 - (6\sqrt{3})^2} = 3\sqrt{13} \text{ cm}$
 $AM = \sqrt{12^2 - (6\sqrt{3})^2} = 6 \text{ cm}$

- $$15^2 = 6^2 + (3\sqrt{13})^2 - 2(6)(3\sqrt{13}) \cos \angle VMA \quad 1M$$
- $$\angle VMA \approx 124^\circ \quad 1A$$
- (c) (i) $QR = \sqrt{12^2 + 9^2} = 15 \text{ cm} \quad 1A$
 Consider the circumcircle PQR . Since $\angle QPR = 90^\circ$, QR is a diameter and M is the centre. 1M
 Thus, $PM = \frac{QR}{2} = 7.5 \text{ cm} \quad 1A$
- (ii) $\triangle QRV$ is equilateral. $VM = VR \sin 60^\circ = \frac{15\sqrt{3}}{2} \text{ cm}. \quad 1A$
 $15^2 = VM^2 + (7.5)^2 - 2(VM)(7.5) \cos \angle PMV \quad 1M$
 $\angle PMV = 90^\circ$
 Since $VM \perp QR$ and $VM \perp PM$, VM is perpendicular to the plane PQR .
 The claim is agreed. 1A
6. (a) $\angle BDC = \frac{360^\circ - 150^\circ}{2} = 105^\circ \quad 1A$
 $50^2 = 32^2 + BD^2 - 2(32)(BD) \cos 105^\circ \quad 1M$
 $0 = BD^2 - 64(BD) \cos 105^\circ - 1476$
 $BD \approx 31.0 \text{ cm} \quad \text{or} \quad -47.6 \text{ cm (rejected)} \quad 1A$
- (b) (i) $AC^2 = 32^2 + 32^2 - 2(32)^2 \cos 40^\circ \quad 1M$
 $AC \approx 21.9 \text{ cm}$
 Required distance is 21.9 cm. 1A
- (ii) Let E be the mid-point of AC .
 Required angle is $\angle BED$. 1A
 $DE = 32 \cos \frac{40^\circ}{2} \approx 30.1 \text{ cm} \quad 1M$
 $BE = \sqrt{50^2 - \left(\frac{AC}{2}\right)^2} \approx 48.8 \text{ cm}$
 $BD^2 = DE^2 + BE^2 - 2(DE)(BE) \cos \angle BED$
 $\angle BED \approx 37.7^\circ \quad 1A$
- (iii) Area of $\triangle ACD = \frac{1}{2}(32)(32) \sin 40^\circ \quad 1M$
 $= 512 \sin 40^\circ \text{ cm}^2$
 Volume $= \frac{1}{3}(512 \sin 40^\circ)(BE \sin \angle BED) \quad 1M$
 $\approx 3270 \text{ cm}^3 \quad 1A$
Alternative method:
 Volume $= \frac{1}{3} \left(\frac{1}{2}(AC)(BE) \right) (DE \sin \angle BED) \quad 1M+1M$
 $\approx 3270 \text{ cm}^3 \quad 1A$
- (c) Let D' and P' be the projections of D and P on plane ABC respectively.
 Since $\triangle BDD' \sim \triangle BPP'$, we have $\frac{PP'}{DD'} = \frac{BP}{BD} = \frac{1}{2}$. 1M

$$\begin{aligned}\frac{\text{volume of } ABCP}{\text{volume of } ABCD} &= \frac{\frac{1}{3}(\text{area of } \triangle ABC)(PP')}{\frac{1}{3}(\text{area of } \triangle ABC)(DD')} \\ &= \frac{PP'}{DD'} = \frac{1}{2}\end{aligned}$$

The claim is agreed.

1A

7. (a) $BD^2 = 20^2 + 25^2 - 2(20)(25) \cos 120^\circ$ 1M

$$BD = 5\sqrt{61} \text{ cm}$$

$$20^2 = 25^2 + BD^2 - 2(25)(BD) \cos \angle BDC$$

$$\angle BDC \approx 26.3^\circ$$

1A

$$\frac{AD}{BD} = \frac{BD}{25}$$

1M

$$AD = 61 \text{ cm}$$

1A

(b) (i) Let E be a point on DC produced such that $BE \perp CD$.

Let F be a point on AD such that $EF \perp CD$. Required angle is $\angle BEF$.

1A

$$BE = 20 \sin(180^\circ - 120^\circ) = 10\sqrt{3} \text{ cm}$$

$$\frac{25}{\sin \angle DAC} = \frac{61}{\sin 140^\circ}$$

1M

$$\angle DAC \approx 15.3^\circ$$

$$\angle ADC = 180^\circ - 140^\circ - \angle DAC \approx 24.7^\circ$$

$$DE = CD + 20 \cos(180^\circ - 120^\circ) = 35 \text{ cm}$$

$$EF = DE \tan \angle ADC \approx 16.1 \text{ cm}$$

1A

$$DF = \frac{DE}{\cos \angle ADC} \approx 38.5 \text{ cm}$$

$$\begin{aligned}BF^2 &= BD^2 + DF^2 - 2(BD)(DF) \cos \angle BDA \\ &= BD^2 + DF^2 - 2(BD)(DF) \cos \angle BDC\end{aligned}$$

$$BF \approx 17.7 \text{ cm}$$

$$BF^2 = BE^2 + EF^2 - 2(BE)(EF) \cos \angle BEF$$

$$\angle BEF \approx 63.7^\circ$$

1A

Required angle is 63.7° .

(ii) Let G be a point on the ground such that BG is perpendicular to the plane ACD .

Required angle is $\angle BDG$.

1A

$$BG = BE \sin \angle BEF \approx 15.5 \text{ cm}$$

1A

$$\sin \angle BDG = \frac{BG}{BD}$$

$$\angle BDG \approx 23.4^\circ$$

1A

Required angle is 23.4° .

(iii) Let the required distance be h cm.

$$\text{Area of } \triangle BCD = \frac{1}{2}(20)(25)(\sin 120^\circ) = 125\sqrt{3} \text{ cm}^2.$$

$$\text{Area of } \triangle ACD = \frac{1}{2}(AD)(25)(\sin \angle ADC) \approx 319 \text{ cm}^2$$

By considering the volume of the tetrahedron $ABCD$,

$$\frac{1}{3}(125\sqrt{3})(h) = \frac{1}{3}(\text{area of } \triangle ACD)(BG) \quad 1M$$

$$h \approx 22.9 \text{ cm} < 23 \text{ cm}$$

The claim is disagreed.

1A