REG-AOT-2425-ASM-SET 4-MATH
Suggested solutions

Conventional Questions

1. (a) In $\triangle VBC$,

$$VD = 1.2\sin 60^\circ = \frac{3\sqrt{3}}{5}\text{ m}$$

In $\triangle ABC$,

$$AD = 1.2\sin 60^\circ = \frac{3\sqrt{3}}{5}\text{ m}$$

In $\triangle VAD$,

$$1.2^2 = \left(\frac{3\sqrt{3}}{5}\right)^2 + \left(\frac{3\sqrt{3}}{5}\right)^2 - 2\left(\frac{3\sqrt{3}}{5}\right)^2\cos\angle VDA \quad \text{1M}$$

$$\angle VDA \approx 70.528\,779\,37^\circ$$

In $\triangle VGD$,

$$VG = \frac{3\sqrt{3}}{5}\sin\angle VDA \quad \text{1M}$$

$$\approx 0.980\text{ m} \quad \text{1A}$$

(b) Required angle is also equal to the angle between VBC and ABC, i.e., $\angle VDA$. 1M+1A

Required angle is 70.5°. 1A

2. (a) $\dfrac{\sin\angle AVB}{22} = \dfrac{\sin 115^\circ}{36}$ 1M

$\angle AVB \approx 33.6^\circ$

$\angle VBA = 180^\circ - 115^\circ - \angle AVB \approx 31.4^\circ$ 1A

(b) In $\triangle VAB$,

$$VA^2 = 22^2 + 36^2 - 2(22)(36)\cos\angle VBA \quad \text{1M}$$

$$VA \approx 20.7\text{ cm}$$

$MP = NQ = \frac{1}{2}VA \approx 10.3\text{ cm}$ 1M

$PQ = MN = \frac{BC}{2} = 7\text{ cm}$

Since the solid is symmetric, $PQNM$ is a rectangle.

Required area $= (PQ)(MP)$ 1M

$\approx 72.4\text{ cm}^2$ 1A

3. (a) $\dfrac{AB}{\sin 65^\circ} = \dfrac{15}{\sin 58^\circ}$ 1M

$AB \approx 16.0\text{ cm}$ 1A

$AC^2 = AB^2 + 17^2 - 2(AB)(17)\cos 116°$ 1M

$AC \approx 28.0\,\text{cm}$ 1A

(b) $\angle BAD = 180° - 58° - 65° = 57°$

$AK = AB\cos 57° \approx 8.73\,\text{cm}$ 1M

$27^2 = AC^2 + 15^2 - 2(AC)(15)\cos\angle CAD$

$\angle CAD \approx 70.5°$

$AC\cos\angle CAD \approx 9.36\,\text{cm} \neq AK$

So, CK is not perpendicular to AD. 1M

The claim is disagreed. 1A

4. (a) The paper card is symmetric about AC.

So, $\angle BAC = 60°$ and $\angle ACB = 180° - 60° - 70° = 50°$. 1A

$$\frac{AC}{\sin 70°} = \frac{10}{\sin 50°}$$ 1M

$AC \approx 12.3\,\text{cm}$ 1A

$BC = \dfrac{10\sin 60°}{\sin 50°} \approx 11.3\,\text{cm}$ 1A

(b) (i) $AE = \sqrt{AB^2 - BE^2}$ and $CE^2 = \sqrt{BC^2 - BE^2}$

$AC^2 = AE^2 + CE^2$

$= BC^2 + AB^2 - 2BE^2$ 1M

$BE \approx 6.22\,\text{cm}$ 1A

(ii) Let h cm be the required distance.

$AE = \sqrt{10^2 - BE^2} \approx 7.83\,\text{cm}$ and $CE = \sqrt{BC^2 - BE^2} \approx 9.44\,\text{cm}$

$$\frac{1}{3} \times \frac{(AE)(DB)}{2} \times CE = \frac{1}{3} \times \frac{(AD)(AC)\sin 60°}{2} \times h$$ 1M+1M

$h \approx 8.66$ 1A

Required distance is 8.66 cm.

5. (a) (i) $QR^2 = 25^2 + 30^2 - 2(25)(30)\cos 95°$ 1M

$QR \approx 40.7\,\text{cm}$ 1A

(ii) $25^2 = 30^2 + QR^2 - 2(30)(QR)\cos\angle PQR$ 1M

$\angle PQR \approx 37.7°$ 1A

(b) Let R' and M' be the feet of perpendicular of R and M on the horizontal ground respectively.

$RR' = RP\sin 70° \approx 23.5\,\text{cm}$

Since $\triangle QMM' \sim \triangle QRR'$, $MM' = \dfrac{RR'}{2} \approx 11.7\,\text{cm}$ 1M

$$PM^2 = 30^2 + \left(\frac{QR}{2}\right)^2 - 2(30)\left(\frac{QR}{2}\right)\cos\angle PQR$$

$PM \approx 18.7\,\text{cm}$

Angle between PM and the horizontal ground is $\angle MPM'$.

$$\sin \angle MPM' = \frac{MM'}{PM} \quad \text{1M}$$

$$\angle MPM' \approx 39.0^\circ < 40^\circ$$

The claim is not correct. 1A

6. (a) Let M be the mid-point of AC.

$BM = 20 \sin 60^\circ = 10\sqrt{3}$ cm

$BE = BM \sin 60^\circ = 15$ cm 1A

$\angle BEC = 90^\circ$. So, BC is a diameter of the circumcircle of $\triangle BCE$. 1M

Thus, $DE = DB = DC = \dfrac{20}{2} = 10$ cm 1A

(b) $AE = \sqrt{AB^2 - BE^2}$ 1M

$= \sqrt{175}$ cm

$AD = BM = 10\sqrt{3}$ cm

$AE^2 = AD^2 + DE^2 - 2(AD)(DE) \cos \angle ADE$ 1M

$\angle ADE \approx 49.5^\circ \neq 90^\circ$

The claim is disagreed. 1A

7. (a) (i) $17^2 = 23^2 + 10^2 - 2(23)(10) \cos \angle CAB$ 1M

$\angle CAB \approx 42.3^\circ$ 1A

(ii) $\dfrac{AF}{\sin(180^\circ - 58^\circ - \angle CAB)} = \dfrac{10}{\sin 58^\circ}$ 1M

$AF \approx 11.6$ cm 1A

(b) (i) $BF = 23 - AF \approx 11.4$ cm

$AB^2 = AF^2 + BF^2 - 2(AF)(BF) \cos 75^\circ$ 1M

$AB \approx 14.0$ cm 1A

(ii) $AP = AF \cos \angle FAC \approx 8.57$ cm 1M

$17^2 = 10^2 + (AB)^2 - 2(10)(AB) \cos \angle BAC$

$\angle BAC \approx 88.6^\circ$

$BP^2 = AP^2 + AB^2 - 2(AP)(AB) \cos \angle BAC$

$BP \approx 16.2$ cm 1M

$AB^2 = AP^2 + BP^2 - 2(AP)(BP) \cos \angle APB$

$\angle APB \approx 59.6^\circ \neq 90^\circ$

The claim is disagreed. 1A

END OF PAPER