REG-AOT-2425-ASM-SET 3-MATH
Suggested solutions

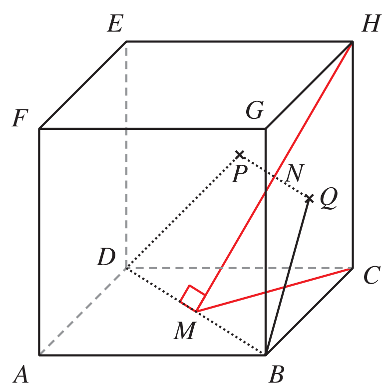
Multiple Choice Questions

1. C	2. D	3. B	4. C	5. D
6. D	7. A	8. B	9. B	10. D
11. A	12. B	13. C	14. B	15. C
16. B	17. D	18. C	19. B	20. D
21. C	22. B	23. D	24. C	

1. C

Let the length of cube be 2. M and N be mid-points of BD and PQ respectively.

Note that since D, P, H are collinear and B, Q, H are collinear, we have M, N and H are collinear.
Note that $MH \perp BD$ and $CM \perp BD$.
The angle required is $\angle CMH$.
$CM = \frac{1}{2}\sqrt{2^2 + 2^2} = \sqrt{2}$.
$\angle CMH = \tan^{-1} \frac{2}{\sqrt{2}} \approx 55°$.



2. D

Let E be a point on AC such that $VE \perp AC$.
Required angle is $\angle VED$.
Since the pyramid is symmetric about the plane VAC,
we have $\angle VEB = \angle VED = \frac{180°}{2} = 90°$.

3. B

Let M be a point on BC such that $AM \perp BC$.

Note that AMD is a plane of reflection of the tetrahedron and it is perpendicular to the plane BCD.

We have $\angle AMD = 80^\circ$ and the required angle is $\angle ADM$.

Consider $\triangle ABC$.

$AM = 56 \sin 60^\circ = 28\sqrt{3}$ cm

Consider $\triangle BCD$.

$DM = \sqrt{60^2 - 28^2} = \sqrt{2816}$ cm

Consider $\triangle AMD$.

$AD^2 = AM^2 + DM^2 - 2(AM)(DM)\cos 80^\circ$

$AD \approx 65.4$ cm

$AM^2 = AD^2 + DM^2 - 2(AD)(DM)\cos\angle ADM$

$\angle ADM \approx 47^\circ$

4. C

Let M and N be mid-point of AB and CD respectively.

Let X be the mid-point of MN such that VX is perpendicular to the plane $ABCD$.

We have $\theta = \angle MVN$ and $\dfrac{\theta}{2} = \angle VNX$.

$DX = \dfrac{1}{2}\sqrt{4^2 + 2^2} = \sqrt{5}$ cm

$VX = \sqrt{7^2 - DX^2} = 2\sqrt{11}$ cm

$$\tan\frac{\theta}{2} = \frac{MX}{VX} = \frac{2}{2\sqrt{11}} = \frac{\sqrt{11}}{11}$$

5. D

Let E be a point on CD such that $BE \perp CD$. Then $\theta = \angle AEB$.

$CD = \sqrt{24^2 + 7^2} = 25$ cm. By considering the area of $\triangle BCD$,

$$\frac{(25)(BE)}{2} = \frac{(24)(7)}{2}$$

$$BE = \frac{168}{25}$$

$\tan\theta = \dfrac{7}{BE} = \dfrac{25}{24}$

6. D

Let D be a point on BC such that $PD \perp BC$.

Required angle is $\angle ADP$.

$BC = \sqrt{4^2 + 3^2} = 5\text{ m}$

$$\frac{(PD)(BC)}{2} = \frac{(PB)(PC)}{2}$$

$$PD = 2.4\text{ m}$$

$$\tan\angle ADP = \frac{5}{2.4} = \frac{25}{12}$$

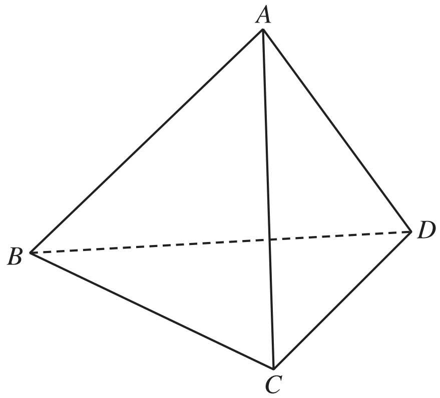
7. A

Note that $\alpha = \angle GFH = 45^\circ$, $\beta = 90^\circ$.

Thus, $\alpha < 60^\circ < \beta$.

8. B

Refer to the figure. Let E be the mid-point of BC and length of each side be x cm.



In $\triangle AED$, $AE = DE = x\sin 60^\circ = \frac{\sqrt{3}x}{2}$ cm.

$$x^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$

$$\angle AED = \cos^{-1}\frac{1}{3}$$

Consider the volume of the tetrahedron.

$$\frac{1}{3}\left(\frac{x^2}{2}\sin 60^\circ\right)(AE\sin\angle AED) = 576$$

$$x \approx 17.0$$

Let X be the projection of A on the plane BCD.

Note that D, X, E are collinear.

Consider $\triangle AXE$.

Required height $= AE\sin\angle AED$

$\approx 13.9\text{ cm}$

9. B

Let M and N be mid-points of BC and AD respectively.

Required angle is $\angle MVN$.

$MV = NV = \sqrt{8^2 - \left(\frac{4}{2}\right)^2} = \sqrt{60}\,\text{cm}$

$6^2 = MV^2 + NV^2 - 2(MV)(NV)\cos\angle MVN$

$\angle MVN \approx 46°$

10. D

Let K be a point on EG such that $AK \perp EG$.

Required angle is $\angle AKF$.

Consider the area of $\triangle EFG$.

$$\frac{1}{2}(8)(6) = \frac{1}{2}(EG)(FK)$$

$$24 = \frac{1}{2}\sqrt{8^2 + 6^2}(EK)$$

$$EK = 4.8\,\text{cm}$$

$\tan\theta = \frac{12}{4.8}$

$\theta \approx 68°$

11. A

Let M and N be mid-points of BD and AD respectively.

Required angle is $\angle MVN$.

$MV = NV = \sqrt{15^2 - 4^2} = \sqrt{209}\,\text{cm}$

$MN = 6\,\text{cm}$

$MN^2 = MV^2 + NV^2 - 2(MV)(NV)\cos\angle MVN$

$\angle MVN \approx 24°$

12. B

Required angle is $\angle CBH$ (or $\angle DAE$).

$\angle CBH = 45°$

13. C

$15^2 = 9^2 + 12^2 \quad \Rightarrow \quad \angle BDC = 90°$.

Since $\triangle ABD$ is vertical and CD is horizontal, $AD \perp CD$.

Therefore, $\theta = \angle ADB = \tan^{-1}\frac{12}{9}$ and so $\sin\theta = \frac{4}{5}$.

14. B

Let $BC = 1$.

$AE = BF = 1\sin 60^\circ = \dfrac{\sqrt{3}}{2}$

$AC = \dfrac{AE}{\sin 30^\circ} = \sqrt{3}$

$\cos\angle ACB = \dfrac{BC}{AC} = \dfrac{1}{\sqrt{3}}$

15. C

Let K be a point on AC such that $DK \perp AC$.

We have $x = \angle DKE$.

Consider the area of $\triangle ACD$.

$$\frac{(AD)(CD)}{2} = \frac{(AC)(DK)}{2}$$
$$\frac{(3)(4)}{2} = \frac{\sqrt{3^2+4^2}(DK)}{2}$$
$$DK = 2.4\,\text{cm}$$

Consider $\triangle DKE$.

$$\begin{aligned}\tan x &= \frac{DE}{DK}\\ &= \frac{2}{2.4}\\ &= \frac{5}{6}\end{aligned}$$

16. B

Note that $\angle BCT = \angle BCA = 90^\circ$.

We have BC is perpendicular to the plane ACT.

Let D be a point on AT such that $CD \perp AT$.

Then $\theta = \angle BDC$.

Consider the area of $\triangle ACT$.

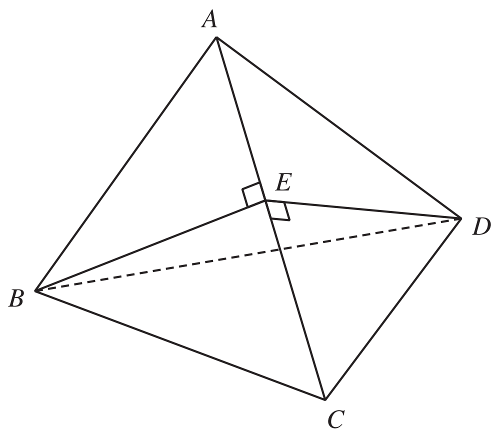
$$\frac{(40)(30)}{2} = \frac{\sqrt{40^2+30^2}(CD)}{2}$$
$$CD = 24\,\text{cm}$$

Consider $\triangle BCD$.

$$\begin{aligned}\tan\theta &= \frac{20}{24}\\ &= \frac{5}{6}\end{aligned}$$

17. D

Since $BE \perp AC$ and $DE \perp AC$, required angle is $\angle BED$.



18. C

Let length of each edge be 2. Let K be a point on BV such that $AK \perp BV$ and $CK \perp BV$.
$AK = CK = 2\sin 60^\circ = \sqrt{3}$ and $AC = \sqrt{2^2 + 2^2} = 2\sqrt{2}$
In $\triangle AKC$, $\angle AKC$ is the required angle.

$$(2\sqrt{2})^2 = (\sqrt{3})^2 + (\sqrt{3})^2 - 2(\sqrt{3})^2 \cos\angle AKC$$
$$\angle AKC \approx 109^\circ$$

19. B

Let F be a point on AC such that $BF \perp AC$.
Required angle is $\angle BFD$.
Let $BC = 1$ cm.
Note that $\triangle ABC$ and $\triangle ACD$ are equilateral.
$DF = BF = 1\sin 60^\circ = \dfrac{\sqrt{3}}{2}$ cm
$BD = \sqrt{1^2 + 1^2} = \sqrt{2}$ cm
$$BD^2 = DF^2 + BF^2 - 2(DF)(BF)\cos\angle BFD$$
$\angle BFD \approx 109^\circ$

20. D

Let E be a point on CD such that $BE \perp CD$. Then $AE \perp CD$ and $\theta = \angle AEB$.
$\angle BCD = \tan^{-1}\dfrac{15}{8}$, and $BE = BC\sin\angle BCD = \dfrac{120}{17}$ m
$\tan\theta = \dfrac{8}{BE} = \dfrac{17}{15}$

21. C

Let K be a point on AM such that $DK \perp AM$.
Then $\alpha = \angle EMD$ and $\beta = \angle EKD$.

I. ✓. Since $\angle DKM = 90^\circ$, we have $DK < DM$ and $\alpha < \beta$.

II. ✗. Since $\alpha < \beta$, we have $\cos\alpha > \cos\beta$.

III. ✓. $\angle DAM = \angle BMA = \tan^{-1}\dfrac{5}{2.5} \approx 63.4^\circ$
$DK = AD\sin\angle DAM \approx 4.47\text{ cm}$
$\tan\beta = \dfrac{DE}{DK} = \dfrac{4}{\sqrt{5}}$

22. B

Refer to the figure. Let E be the mid-point of BC and length of each side be 12 cm.

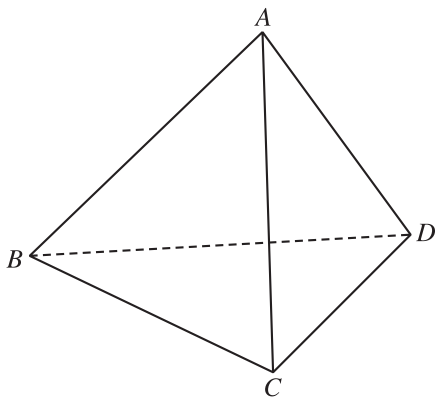
In $\triangle AED$, $AE = DE = 12\sin 60^\circ = 6\sqrt{3}\text{ cm}$

$$12^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$

$\angle AED = \cos^{-1}\dfrac{1}{3}$
Let X be the projection of A on the plane BCD. Then X is the centroid of $\triangle BCD$ and it lies on DE.
In $\triangle AEX$, $\angle AXE = 90^\circ$ and

height $= AE\sin\angle AED$
$= \sqrt{96}\text{ cm}$

Volume $= \dfrac{1}{2}(12)^2\sin 60^\circ(\sqrt{96}) \times \dfrac{1}{3}$
$= 144\sqrt{2}\text{ cm}^3$



23. D

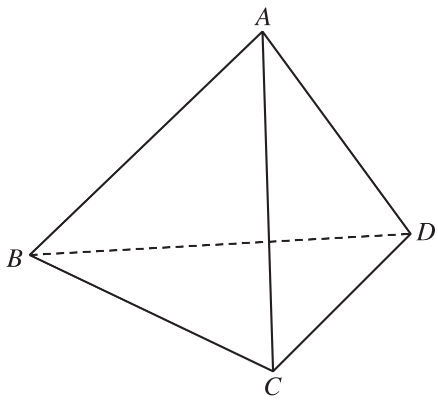
Refer to the figure. Let E be the mid-point of BC and length of each side be x cm.

In $\triangle AED$, $AE = DE = x\sin 60^\circ = \dfrac{\sqrt{3}x}{2}\text{ cm}$

$$x^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$

$\angle AED = \cos^{-1}\dfrac{1}{3}$
Let X be the projection of A on the plane BCD. Then X lies on DE and $AX \perp BD$.
In $\triangle AEX$,

$$AE\sin\angle AED = 4$$
$$x = 2\sqrt{6}$$



24. $\boxed{\text{C}}$

Let K be a point on VB such that $AK \perp VB$. We have $CK \perp VB$ also.
Required angle is $\angle AKC$.
$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2}\,\text{cm}$
$VA = VB = VC = \sqrt{8^2 + \left(\frac{12\sqrt{2}}{2}\right)^2} = \sqrt{136}\,\text{cm}$
In $\triangle VAB$,

$$\cos \angle ABV = \frac{\left(\frac{12}{2}\right)}{VB}$$
$$\angle ABV \approx 59.0^\circ$$

$AK = CK = 12 \sin \angle ABV \approx 10.3\,\text{cm}$
In $\triangle AKC$,

$$AC^2 = AK^2 + CK^2 - 2(AK)(CK)\cos \angle AKC$$
$$\angle AKC \approx 111^\circ$$

Conventional Questions

25. (a) In $\triangle ABC$,

$$AC^2 = 9^2 + 15^2 - 2(9)(15)\cos 120^\circ \qquad \text{1M}$$

$$AC = 21 \text{ cm} \qquad \text{1A}$$

In $\triangle ADC$, $\angle ADC = 180^\circ - 120^\circ = 60^\circ$

Since $AD = CD$, $\angle DAC = \angle ACD = \dfrac{180^\circ - 60^\circ}{2} = 60^\circ$ and hence $\triangle ACD$ is equilateral. 1M

So, $AD = AC = 21$ cm. 1A

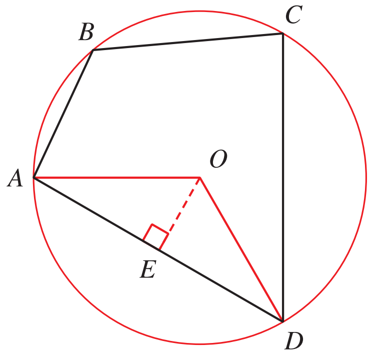
(b) Centre of the required circle is at the circumcentre O of $\triangle ACD$. Let E be the mid-point of AD.

$\angle OAE = \dfrac{60^\circ}{2} = 30^\circ$.

Radius $= OA = \dfrac{AE}{\cos 30^\circ}$ 1M

$= \dfrac{21}{2} \div \cos 30^\circ$

$= 7\sqrt{3}$ cm 1A



(c) Required angle is $\angle OEV$. 1A

$OE = OA \sin 30^\circ = \dfrac{7\sqrt{3}}{2}$ cm 1M

$\tan \angle OEV = \dfrac{15}{OA}$ 1M

$\angle OEV \approx 68.0^\circ$ 1A

Required angle is 68.0°.

26. (a) $AC^2 = 8^2 + 12^2 - 2(8)(12)\cos 100^\circ$ 1M

$AC \approx 15.5$ cm 1A

(b) (i) Let M be the mid-point of BD.

$DM = 3$ cm

In $\triangle AMD$,

$AM = \sqrt{8^2 - 3^2}$

$= \sqrt{55}$ cm 1M

In $\triangle CMD$,

$CM = \sqrt{12^2 - 3^2}$

$= \sqrt{135}$ cm

In $\triangle ACM$,

$$(\sqrt{55})^2 = AC^2 + (\sqrt{135})^2 - 2(AC)(\sqrt{135})\cos\angle ACM \quad \text{1M}$$

$$\angle ACM \approx 27.1^\circ$$

Required angle is 27.1°. 1A

(ii) Let Q be the foot of perpendicular from D to AC.

Since $\sin\dfrac{\angle DPB}{2} = \dfrac{DM}{PD}$,

$\angle DPB$ attains maximum when PD is minimum, i.e., $P = Q$. 1A

When P moves from A to Q, $\angle DPB$ increases.

When P moves from Q to C, $\angle DPB$ decreases. 1A

27. (a) (i) $CM = \sqrt{10^2 + 10^2}$

$= 10\sqrt{2}$ cm 1A

(ii) $\tan\angle ACB = \dfrac{20}{10}$

$\angle ACB \approx 63.4^\circ$ 1A

(iii) Area of $\triangle ACM = \dfrac{1}{2}(20 - 10)(10)$

$= 50\text{ cm}^2$ 1A

(b) (i) In $\triangle CDM$, $\angle DCM = \angle ACB - 45^\circ \approx 18.4^\circ$ 1A

$$DM^2 = CD^2 + (10\sqrt{2})^2 - 2(CD)(10\sqrt{2})\cos\angle DCM \quad \text{1M}$$

$CD \approx 7.45$ cm 1A

In $\triangle BCD$, $BD = \sqrt{10^2 - CD^2} \approx 6.67$ cm 1A

(ii) Let N be the foot of perpendicular from B to CM.

Required angle is $\angle BND$.

In $\triangle BNC$, $BN = 10\sin 45^\circ = 5\sqrt{2}$ cm

In $\triangle BND$,

$$\sin\angle BND = \frac{BD}{5\sqrt{2}} \quad \text{1M}$$

$\angle BND \approx 70.5^\circ$ 1A

Required angle is 70.5°.

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