REG-AOT-2425-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. C	3. C	4. B	5. C
6. B	7. B	8. A	9. B	10. B
11. D	12. D	13. D	14. A	15. C
16. C	17. A	18. D	19. B	20. C
21. A	22. B	23. B	24. B	25. B
26. C	27. B	28. A	29. B	30. D
31. A	32. C	33. A	34. D	35. D
36. A				

1. A

Required angle is $\angle BDF$.
$BD = \sqrt{4^2 + 8^2} = \sqrt{80}\,\text{cm}$
$BF = \sqrt{4^2 - 3^2} = \sqrt{7}\,\text{cm}$
$\sin\angle BDF = \dfrac{\sqrt{7}}{\sqrt{80}}$
$\angle BDF \approx 17^\circ$

2. C

We have $\theta = \angle BEG$.
$EG = \sqrt{8^2 + 6^2} = 10\,\text{cm}$
$BE = \sqrt{10^2 + 4^2} = \sqrt{116}\,\text{cm}$
$\cos\theta = \dfrac{10}{\sqrt{116}}$
$= \dfrac{5}{\sqrt{29}}$

3. C

A. $\tan\angle ACE = \dfrac{AE}{CE}$
B. $\tan\angle AQE = \dfrac{AE}{EQ}$
C. $\tan\angle ADE = \dfrac{AE}{DE}$
D. $\tan\angle PCR = \dfrac{PR}{CR} = \dfrac{AE}{CR}$

Since $DE < EQ = RC < CE$, we have $\tan\angle ADE$ being the greatest among all.
$\angle ADE$ is therefore the greatest angle among all.
The answer is C.

4. B

A. Angle between DH and $EFGH$ is $\angle DHE$.

B. Angle between CF and $EFGH$ is $\angle CFH$.

C. Note that the projection of M on $EFGH$ is the mid-point of HE.
Angle between MH and $EFGH$ is $\angle MHE$.

D. Note that the projection of K on $EFGH$ is the mid-point of EG.
Angle between KG and $EFGH$ is $\angle KGE$.

Comparing, we have $\angle CFH$ is the smallest.
The answer is B.

5. C

Note that $\theta = \angle BEG$.

$$EG = \sqrt{6^2 + 8^2}$$
$$= 10\,\text{cm}$$
$$\tan\theta = \frac{BG}{EG}$$
$$= \frac{2}{5}$$

6. B

A. Required angle is $\angle EBF$.

B. Required angle is $\angle ENF$.

C. Let Q be a point on $ABGF$ such that PQ is perpendicular to $ABGF$.
Required angle is $\angle PFQ$, which is equal to $\angle FPE$.

D. Let K be the mid-point of BG.
Required angle is $\angle MNK$, which is equal to $\angle CAB$.

By simple observation, we have $\angle ENF$ being the greatest angle among all.
The answer is B.

7. B

I. ✓. Note that $\triangle AHF \cong \triangle DGE$, we have $\angle AHF = \angle DGE$.

II. ✗. $\angle AGH = 90^\circ$ while $\angle DGE < 90^\circ$.

III. ✓. Note that $\triangle BEG \cong \triangle DGE$, we have $\angle BEG = \angle DGE$.

8. A

A. Angle between AC and $BCHG$ is $\angle ACB$.

B. Angle between DH and $BCHG$ is $\angle DHC$.

C. Angle between DG and $BCHG$ is $\angle DGC$.

D. Let Y be the mid-point of GH.
Angle between XB and $BCHG$ is $\angle XBY$.

By simple observation, we have $\angle ACB$ being the greatest angle among all.
The answer is A.

9. B

We have $\angle VAC = 60°$.
Since $VA = VC$, we have $\angle VCA = \angle VAC = 60°$ and $\triangle VAC$ is equilateral.
$AC = \sqrt{1^2 + 1^2} = \sqrt{2}$ m
$VB = VA = VC = AC = \sqrt{2}$ m
In $\triangle VAB$,

$$1^2 = VA^2 + VB^2 - 2(VA)(VB)\cos\angle AVB$$
$$\angle AVB \approx 41°$$

10. B

Let Q be a point on BG such that $PQ \perp BG$.
Then $\theta = \angle PCQ$.
$CQ = \sqrt{2^2 + 5^2} = \sqrt{29}$ cm

$$\cos\theta = \frac{CQ}{PC}$$
$$= \frac{\sqrt{29}}{\sqrt{14^2 + (\sqrt{29})^2}}$$
$$= \frac{\sqrt{29}}{15}$$

11. D

Let Q be the mid-point of AD.
Then $\theta = \angle PEQ$.

$$\tan\theta = \frac{PQ}{EQ}$$
$$= \frac{y}{\sqrt{x^2 + (2z)^2}}$$
$$= \frac{y}{\sqrt{x^2 + 4z^2}}$$

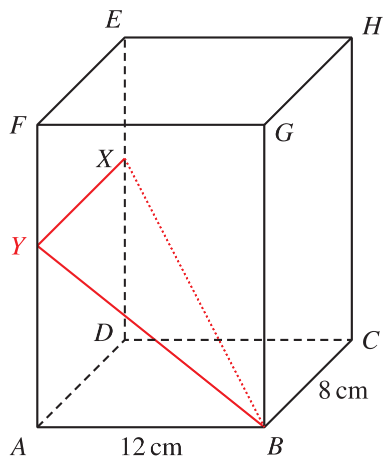
12. D

Let Y be a point on AF such that $XY \perp AF$.

Then $\theta = \angle XBY$.

$BY = \sqrt{9^2 + 12^2} = 15$ cm

$\theta = \tan^{-1} \frac{8}{15}$ and $\cos\theta = \frac{15}{17}$



13. D

Let G be the mid-point of BC.

Required angle is $\angle ADG$.

$AG = 6\sin 60^\circ = 3\sqrt{3}$ cm

$DG = \sqrt{3^2 + 10^2} = \sqrt{109}$ cm

$\tan\angle ADG = \frac{3\sqrt{3}}{\sqrt{109}}$

$\angle ADG \approx 26.5^\circ$

14. A

Note that $\theta = \angle BKA$.

$AK = \sqrt{12^2 + 9^2} = 15$ cm

$BK = \sqrt{15^2 + 8^2} = 17$ cm

$\cos\theta = \frac{AK}{BK}$

$= \frac{15}{17}$

15. C

Let K be the mid-point of EF. The required angle is $\angle MNK$.

In $\triangle DEF$,

$$\frac{7}{\sin 50^\circ} = \frac{6}{\sin \angle DFE}$$

$$\angle DFE \approx 41.0^\circ \quad \text{or} \quad 139^\circ \text{ (rejected)}$$

$$\angle DEF = 180^\circ - \angle DFE - 50^\circ \approx 89.0^\circ$$

$$NK^2 = \left(\frac{6}{2}\right)^2 + \left(\frac{7}{2}\right)^2 - 2\left(\frac{6}{2}\right)\left(\frac{7}{2}\right)\cos \angle DEF$$

$$NK \approx 4.57\text{ cm}$$

In $\triangle MNK$,

$$\tan \angle MNK = \frac{5}{NK}$$

$$\angle MNK \approx 48^\circ$$

16. C

A. $\angle HDE = 45^\circ$

B. Note that $\triangle BHD$ is an equilateral triangle.
$\angle BHD = 60^\circ$

C. Let the length of cube be 1.

$$\tan \angle AHB = \frac{AB}{BH} = \frac{1}{\sqrt{1^2 + 1^2}}$$

$$\angle AHB \approx 35.3^\circ$$

D. Note that HG is perpendicular to the plane $CDEH$.
$\angle DHG = 90^\circ$

The answer is C.

17. A

$BD = \sqrt{25^2 - 15^2} = 20\text{ m}$

$$29^2 = 21^2 + 20^2 - 2(21)(20)\cos \angle BDC$$

$\angle BDC = 90^\circ$

Since $BD \perp AD$ and $BD \perp CD$, $BD \perp \triangle ACD$.

Thus, the projection of B on the plane ACD is D, and the required angle is $\angle BAD$.

$\angle BAD = \tan^{-1} \dfrac{20}{15} \approx 53^\circ$

18. D

$PQ = CE = 40\sin 10°$

$DP = \frac{90}{1+2} = 30\,\text{m}$ and $AP = \sqrt{30^2 + 40^2} = 50\,\text{m}.$

$\sin\theta = \frac{40\sin 10°}{50}$

$= \frac{4\sin 10°}{5}$

19. B

Required angle is $\angle ACF$.

$AC = \frac{AD}{\sin 60°} = \frac{10}{\sqrt{3}}\,\text{cm}$

$CD = \frac{AD}{\tan 60°} = \frac{5}{\sqrt{3}}\,\text{cm}$

$AF = DE = CD\sin 30° = \frac{5}{2\sqrt{3}}\,\text{cm}$

$\sin\angle ACF = \frac{AF}{AC}$

$\angle ACF \approx 14°$

20. C

The projection of A on $EFGH$ is F.

Required angle is $\angle AEF$.

21. A

Required angle is $\angle AHD$.

$DH = \sqrt{6^2 + 8^2} = 10\,\text{cm}$

$\tan\angle AHD = \frac{10}{10}$

$\angle AHD = 45°$

22. B

Let E be a point on $ABCD$ such that VE is perpendicular to the plane $ABCD$.

Required angle is $\angle VCE$.

$CE = \frac{1}{2}\sqrt{5^2 + 6^2} = \frac{\sqrt{61}}{2}\,\text{cm}$

$\cos\angle VCE = \frac{CE}{8}$

$\angle VCE \approx 61°$

23. B

Required angle is $\angle DBE$.

Let $AD = CE = DE = 1\,\text{cm}$.

$BE = \sqrt{1^2 + 1^2} = \sqrt{2}\,\text{cm}$

$\tan\angle DBE = \frac{1}{\sqrt{2}}$

$\angle DBE \approx 35°$

24. $\boxed{B}$

Required angle is $\angle YXH$.
Let $CH = 2\,\text{cm}$.
We have $YH = \dfrac{2}{2} = 1\,\text{cm}$ and $XH = \sqrt{1^2 + 2^2} = \sqrt{5}\,\text{cm}$.

$$\tan \angle YXH = \frac{1}{\sqrt{5}}$$
$$\angle YXH \approx 24^\circ$$

25. $\boxed{B}$

Let H be a point on CF such that $GH \perp CF$. The angle required is $\angle GEH$.

$GH = BC \times \dfrac{3}{2+3} = 28.8\,\text{cm}$

$FH = FC \times \dfrac{3}{5} = 8.4\,\text{cm}$ and $EH = \sqrt{40^2 + 8.4^2} = \sqrt{1670.56}\,\text{cm}$

Required angle $= \tan^{-1} \dfrac{28.8}{EH} \approx 35^\circ$

26. $\boxed{C}$

Let N be the mid-point of GH.
Required angle is MFN.

$$MN = 24\,\text{cm}$$
$$FN = \sqrt{5^2 + 8^2} = \sqrt{89}\,\text{cm}$$
$$\tan \angle MFN = \frac{24}{\sqrt{89}}$$
$$\angle MFN \approx 69^\circ$$

27. $\boxed{B}$

Since $VA = VB$, we have $\angle VAB = \dfrac{180^\circ - 60^\circ}{2} = 60^\circ$ and so all lateral faces are equilateral triangles.
Required angle is $\angle VAM$, where M is the projection of V on $ABCD$.
Let $AB = 2$. Then $VA = 2$ and $AM = \dfrac{1}{2}\sqrt{2^2 + 2^2} = \sqrt{2}$

Required angle $= \angle VAM = \cos^{-1} \dfrac{\sqrt{2}}{2} = 45^\circ$

28. $\boxed{A}$

Let the length of side of the cube be 2.

I. ✓. EF is perpendicular to $ABFG$. So, $\angle BFE = 90^\circ$.

II. ✓. AB is perpendicular to $ADEF$. So, $\angle BAE = 90^\circ$.

III. ✗. $BE = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12}$. $EM = BM\sqrt{2^2 + 1^2} = \sqrt{5}$
$EM^2 + BM^2 = 5 + 5 = 10 \neq BE^2$. So, $\angle BME \neq 90^\circ$.

29. B

Note that $GH = CF$ and $BF < AH < BH$.
We have $\frac{CF}{BF} > \frac{GH}{AH} > \frac{GH}{BH}$.
We have $\tan a > \tan c > \tan b$.

Thus, $a > c > b$.

30. D

Note that $DE < EG < FH$.
Since $\tan\alpha = \frac{AE}{EG}$, $\tan\beta = \frac{AE}{DE}$ and $\tan\gamma = \frac{BF}{FH} = \frac{AE}{FH}$,
we have $\tan\beta > \tan\alpha > \tan\gamma$.

Therefore, we have $\beta > \alpha > \gamma$.

31. A

I. ✓. VA is vertical and AB is horizontal.

II. ✗. Note that $\angle VAM = 90°$ and so $\angle VMA = 180° - 90° - \angle AVM < 90°$.

III. ✗.

32. C

$BD = \sqrt{25^2 - 15^2} = 20\text{ m}$

$$32^2 = 22^2 + 20^2 - 2(22)(20)\cos\angle BDC$$

$\angle BDC \approx 99.2°$

Let E be a point on CD produced such that $BE \perp CD$.

Required angle is $\angle BAE$.

$BE = BD\sin(180° - \angle BDC) \approx 19.7\text{ m}$

$$\sin\angle BAE = \frac{BE}{25}$$

$\angle BAE \approx 52°$

33. A

Required angle is $\angle PHF$.
$PF = 12 \times \frac{2}{1+2} = 8\text{ cm}$ and $FH = \sqrt{8^2 + 6^2} = 10\text{ cm}$
$\tan\angle PHF = \frac{8}{10}$

$\angle PHF \approx 39°$

34. D

Let Q be a point on DE such that $NQ \perp DE$.

Then $\theta = \angle NPQ$.

$PQ = \dfrac{3}{2} = 1.5\,\text{cm}$

$$\begin{aligned}\tan\theta &= \frac{3}{1.5}\\ &= 2\end{aligned}$$

35. D

$$\begin{aligned}\text{Volume of tetrahedron } ABCD &= \frac{1}{3}\left[\frac{(3)(2)}{2}\right](4)\\ &= 4\,\text{cm}^3\end{aligned}$$

$BC = \sqrt{3^2 + 2^2} = \sqrt{13}\,\text{cm}$

$AC = \sqrt{2^2 + 4^2} = \sqrt{20}\,\text{cm}$

$AB = \sqrt{3^2 + 4^2} = 5\,\text{cm}$

Let $s = \dfrac{AB + AC + BC}{2} \approx 6.54\,\text{cm}$.

$$\begin{aligned}\text{Area of } \triangle ABC &= \sqrt{s(s-AB)(s-AC)(s-BC)}\\ &\approx 7.81\,\text{cm}^2\end{aligned}$$

Let the required angle be θ. Consider the volume of the tetrahedron.

$$4 = \frac{1}{3}(\text{area of } \triangle ABC)(CD\sin\theta)$$

$$\theta \approx 50^\circ$$

36. A

Let N be a point on EF such that $MN \perp EF$.

Required slope is $\tan\angle MDN$.

$DN = \sqrt{15^2 + 8^2} = 17\,\text{m}$

$$\begin{aligned}\text{Required slope} &= \tan\angle MDN\\ &= \frac{12}{17}\\ &\approx 0.7\end{aligned}$$

Conventional Questions

37. (a) $BC^2 = 30^2 + 30^2 - 2(30)(30)\cos 40°$ 1M

$BC \approx 20.5\text{ cm}$ 1A

(b) Since $\triangle ABC$ is equilateral, the circumcentre of $\triangle ABC$ coincides with centroid of $\triangle ABC$.

$r = \frac{2}{3} \times BC\sin 60°$ 1M

$\approx 11.8\text{ cm}$ 1A

(c) Required angle $= \cos^{-1}\frac{r}{30}$ 1M

$\approx 66.7°$ 1A

38. (a) $\angle BAC = 180° - 104° - 18° = 58°$

$\frac{AB}{\sin 18°} = \frac{56}{\sin 58°}$ 1M

$AB \approx 20.4\text{ cm}$ 1A

$\frac{AC}{\sin 104°} = \frac{56}{\sin 58°}$

$AC \approx 64.1\text{ cm}$ 1A

(b) $AP = \frac{AC}{4} \approx 16.0\text{ cm}$

$BP^2 = AP^2 + AB^2 - 2(AP)(AB)\cos 58°$ 1M

$BP \approx 18.1\text{ cm}$

Let Q and R be the projections of P and C on the horizontal ground respectively.

Required angle is $\angle PBQ$. 1M

$CR = 56\sin 37° \approx 33.7\text{ cm}$

$PQ = \frac{CR}{4} \approx 8.43\text{ cm}$ 1M

$\sin\angle PBQ = \frac{PQ}{BP}$

$\angle PBQ \approx 27.8° < 28°$

The claim is correct. 1A

39. (a) $\angle CTA = 180° - 42° - 30° = 108°$

$\frac{CA}{\sin 108°} = \frac{145}{\sin 42°}$ 1M

$CA \approx 206\text{ m}$ 1A

$AB^2 = 240^2 + AC^2 - 2(240)(AC)\cos 25°$ 1M

$AB \approx 102\text{ m}$ 1A

(b) Let T' be a point on AC such that $TT' \perp AC$.

The angle of elevation of T from P is $\angle TPT'$.

$\tan\angle TPT' = \frac{TT'}{T'P}$

The angle of elevation is greater when PT' is shorter. 1M

$$240^2 = AC^2 + AB^2 - 2(AC)(AB)\cos\angle CAB$$ 1M

$$\angle CAB \approx 96.4^\circ > 90^\circ$$

The length of $T'P$ is the shortest when P is at A.
The angle of elevation of T from P is the greatest when P is at A.
The claim is agreed. 1A

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