

REG-AOT-2425-ASM-SET 1-MATH

Suggested solutions

Multiple Choice Questions

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|-------|-------|-------|-------|-------|
| 1. D | 2. C | 3. B | 4. C | 5. B |
| 6. B | 7. B | 8. B | 9. C | 10. B |
| 11. D | 12. B | 13. C | 14. D | 15. B |
| 16. C | 17. B | 18. C | 19. A | 20. A |
| 21. D | 22. C | 23. C | 24. B | |

1. D

$$BD = \sqrt{12^2 + 5^2} = 13 \text{ cm}$$

$$BH = \sqrt{5^2 + 8^2} = \sqrt{89} \text{ cm}$$

$$\tan \angle FBG = \frac{12}{8} \quad \text{and} \quad \tan \angle DEB = \frac{13}{8}$$

$$\angle FBG \approx 56.3^\circ \quad \angle DEB \approx 58.4^\circ$$

$$\tan \angle EBH = \frac{12}{\sqrt{89}} \quad \text{and} \quad \tan \angle ACB = \frac{12}{5}$$

$$\angle EBH \approx 51.8^\circ \quad \angle ACB \approx 67.4^\circ$$

2. C

$$BD = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{6} \text{ m}$$

$$DH = \sqrt{(\sqrt{3})^2 + 1^2} = 2 \text{ m}$$

$$BH = \sqrt{(\sqrt{3})^2 + 1^2} = 2 \text{ m}$$

$$BD^2 = DH^2 + BH^2 - 2(DH)(BH) \cos \angle BHD$$

$$\angle BHD \approx 76^\circ$$

3. B

Since BV is perpendicular to the plane VAC , $\angle BVA = \angle BVC = 90^\circ$.

$$AB = BC = \sqrt{6^2 + 8^2} = 10 \text{ cm and } BM = BN = 5 \text{ cm}$$

$\triangle BMN$ is equilateral. So, $MN = 5 \text{ cm}$.

$$\angle VBM = \tan^{-1} \frac{8}{6}$$

$$VM^2 = 6^2 + 5^2 - (2)(6)(5) \cos \angle VBM$$

$$VM = 5$$

So, $VM = VN = MN = 5 \text{ cm}$, and the required area is $\frac{1}{2}(5)^2 \sin 60^\circ = \frac{25\sqrt{3}}{4} \text{ cm}^2$.

4. C

Let the length of edges be 2. Consider the triangular pyramid at a corner.

Length of bold edges = $\sqrt{1^2 + 1^2} = \sqrt{2}$. Volume = $\frac{1}{3} \left(\frac{1}{2} (1)(1) \right) (1) = \frac{1}{6}$.

Let the height of pyramid with respect to bold edge base be h .

$$\frac{1}{6} = \frac{1}{3} \left[\frac{1}{2} (\sqrt{2})^2 \sin 60^\circ \right] h$$

$$h = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Distance between opposite corners of the cube = $\sqrt{2^2 + 2^2 + 2^2} = \sqrt{12} = 2\sqrt{3}$

Required distance = $2\sqrt{3} - 2h = \frac{4\sqrt{3}}{3}$

5. B

Let $VA = 2$ cm.

Note that $VA = VC$ and $\angle AVC = 60^\circ$. $\triangle VAC$ is an equilateral triangle.

$$AB = AC \cos 45^\circ$$

$$= \sqrt{2} \text{ cm}$$

$$AB^2 = VA^2 + VB^2 - 2(VA)(VB) \cos \angle AVB$$

$$\angle AVB \approx 41.4^\circ$$

6. B

Let the length of the cube be 2.

$$MN = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$MP = DE = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$NP = \sqrt{NH^2 + HP^2} = \sqrt{1^2 + (1^2 + 2^2)} = \sqrt{6}$$

$$MN^2 = MP^2 + NP^2 - 2(MP)(NP) \cos \angle MPN$$

$$\angle MPN = 30^\circ$$

7. B

$$AC = AD \cos \angle DAC$$

$$= x \cos \beta$$

$$BC = AC \sin \angle CAB$$

$$= x \sin \alpha \cos \beta$$

8. B

$$ME = MF = \frac{1}{2} \sqrt{4^2 + 3^2 + 12^2}$$

$$= 6.5 \text{ cm}$$

$$4^2 = ME^2 + MF^2 - 2(ME)(MF) \cos \theta$$

$$\theta \approx 36^\circ$$

9. C

Let $PQ = 1$. Then $PR = \frac{1}{\sin 48^\circ}$, $QR = \frac{1}{\tan 48^\circ}$, $PS = \frac{1}{\sin 60^\circ}$ and $QS = \frac{1}{\tan 60^\circ}$.

$$RS^2 = QR^2 + QS^2 - 2(QR)(QS) \cos 105^\circ = PR^2 + PS^2 - 2(PR)(PS) \cos \angle RPS$$

$$\angle RPS \approx 56^\circ$$

10. B

$$BC = AC \sin 60^\circ$$

$$= 4\sqrt{3} \text{ m}$$

$$\tan \theta = \frac{6}{4\sqrt{3}}$$

$$\theta \approx 41^\circ$$

11. D

Let the lengths of edges be 1. Let R be the mid-point of CD and Q be the mid-point of AC such that $PQ \perp QR$. Let S be the mid-point of HE such that $PS \perp HE$.

$$QR = 0.5 \text{ and } PR = \sqrt{1^2 - 0.5^2} = \sqrt{0.75}$$

$$\text{So, } PQ = \sqrt{(\sqrt{0.75})^2 - 0.5^2} = \sqrt{0.5}.$$

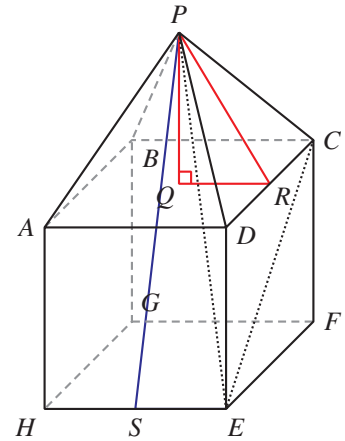
$$PS = \sqrt{(\sqrt{0.5} + 1)^2 + 0.5^2} \approx 1.78$$

$$PE = \sqrt{PS^2 + 0.5^2} \approx 1.85 \text{ } CE = \sqrt{1^2 + 1^2} = \sqrt{2}$$

In $\triangle PCE$,

$$1^2 = PE^2 + CE^2 - 2(PE)(CE) \cos \angle PEC$$

$$\angle PEC \approx 32.4^\circ$$



12. B

$$PE = \frac{1}{2} \sqrt{4^2 + 4^2}$$

$$= \sqrt{8} \text{ cm}$$

$$PD = \sqrt{PE^2 + 4^2}$$

$$= 2\sqrt{6} \text{ cm}$$

13. C

$$BM = MG = \sqrt{4^2 + 6^2 + 5^2} = \sqrt{77} \text{ cm}$$

$$10^2 = BM^2 + MG^2 - 2(BM)(MG) \cos \angle BMG$$

$$\cos \angle BMG = \frac{27}{77}$$

14. D

$$\angle DAC = \angle ACB = \alpha$$

$$AD = 2 \cos \alpha \text{ cm}$$

$$CD = 2 \sin \alpha \text{ cm}$$

$$CE = CD \cos \beta = 2 \sin \alpha \cos \beta \text{ cm}$$

$$\begin{aligned} \text{Required volume} &= \left(\frac{1}{2} (CD)(CE) \sin \beta \right) (AD) \\ &= 4 \sin^2 \alpha \cos \alpha \sin \beta \cos \beta \text{ cm} \end{aligned}$$

15. B

Let $BG = 1$.

$$BC = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$$

$$AB = \frac{BC}{\tan 45^\circ} = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \tan \theta &= \frac{AB}{BG} \\ &= \frac{\left(\frac{1}{\sqrt{3}} \right)}{1} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

16. C

Let $PQ = 2$. Then $RS = 3$.

$$RQ = \frac{PQ}{\tan 45^\circ} = 2$$

$$QS = \frac{PQ}{\tan 30^\circ} = 2\sqrt{3}$$

Consider $\triangle QRS$.

$$RS^2 = RQ^2 + QS^2 - 2(RQ)(QS) \cos \angle RQS$$

$$\angle RQS \approx 60^\circ$$

17. B

Let the length of cube be 2.

$$BM = 1 \text{ and } MG = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$MD = \sqrt{1^2 + 2^2} = \sqrt{5} \text{ and } ME = \sqrt{MD^2 + 2^2} = 3$$

$$EG = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$MG^2 = ME^2 + EG^2 - 2(ME)(EG) \cos \theta$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

18. C

Let $BC = 1$.

$$AB = 1 \tan 45^\circ = 1 \text{ and } BG = 1 \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

$$\begin{aligned} \tan \theta &= \frac{AB}{AF} \\ &= \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} \\ &= \sqrt{3} \end{aligned}$$

19. A

Let $CD = 1$.

$$BD = \frac{1}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$$

$$DH = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

$$CH = \frac{1}{\tan 45^\circ} = 1$$

$$BC = CD \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$BH = \sqrt{CH^2 + BC^2} = \sqrt{\frac{4}{3}}$$

$$BH^2 = DH^2 + BD^2 - 2(DH)(BD) \cos \angle BDH$$

$$\angle BDH \approx 52^\circ$$

20. A

$$CM = 30 \times \frac{7}{3+7} = 21 \text{ cm and } MH = 30 - 21 = 9 \text{ cm}$$

$$MA = \sqrt{12^2 + 16^2 + 21^2} = 29 \text{ cm and } MG = \sqrt{12^2 + 9^2} = 15 \text{ cm}$$

$$AG = \sqrt{30^2 + 16^2} = 34 \text{ cm}$$

$$AG^2 = AM^2 + MG^2 - 2(AM)(MG) \cos \theta$$

$$\cos \theta = \frac{-3}{29}$$

21. D

Let $AD = 1$. Then $AM = \tan \theta$.

$$DE = AH = \tan \beta. \quad AB = EF = DE \tan \alpha = \tan \beta \tan \alpha.$$

Since M is the mid-point of AB , $2AM = AB$ and $2 \tan \theta = \tan \beta \tan \alpha$

22. C

$$AD = BD = \frac{3}{\cos 30^\circ} = 2\sqrt{3} \text{ m}$$

$$AB = \sqrt{AD^2 + BD^2} = 2\sqrt{6} \text{ m}$$

$$DC = 3 \tan 30^\circ = \sqrt{3} \text{ m}$$

$$AC = \sqrt{DC^2 + AD^2} = \sqrt{15} \text{ m}$$

$$3^2 = AC^2 + AB^2 - 2(AC)(AB) \cos \angle BAC$$

$$\angle BAC \approx 38^\circ$$

23. C

Let $OA = 1$ m.

Then $OB = \frac{1}{\tan 45^\circ} = 1$ m and $OC = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$ m.

$$\tan \angle OCB = \frac{OB}{OC}$$

$$\angle OCB = 60^\circ$$

24. B

Let $AB = 1$.

In $\triangle ABD$, $AD = \frac{1}{\sin 30^\circ} = 2$ and $BD = \frac{1}{\tan 30^\circ} = \sqrt{3}$.

In $\triangle ABC$, $AC = \frac{1}{\sin 45^\circ} = \sqrt{2}$ and $BC = \frac{1}{\tan 45^\circ} = 1$.

$$CD^2 = 1^2 + (\sqrt{3})^2 - 2(1)(\sqrt{3}) \cos 150^\circ$$

$$CD = \sqrt{7}$$

$$CD^2 = AD^2 + AC^2 - 2(AD)(AC) \cos \angle CAD$$

$$\angle CAD \approx 100^\circ$$

Conventional Questions

25. (a) $BG = \sqrt{4^2 + 2^2}$ 1M
 $= 2\sqrt{5} \text{ cm}$ 1A
 $GD = BG = 2\sqrt{5} \text{ cm}$ 1A
 $DF = \sqrt{4^2 + 4^2}$
 $= \sqrt{32} \text{ cm}$
 $BD = \sqrt{4^2 + DF^2}$
 $= 4\sqrt{3} \text{ cm}$ 1A
- (b) $BD^2 = BG^2 + DG^2 - 2(BG)(DG) \cos \angle BGD$ 1M
 $\angle BGD \approx 102^\circ$ 1A
26. (a) $EI = \sqrt{90^2 + 60^2}$ 1M
 $= \sqrt{11\,700}$
 $\approx 108 \text{ cm}$ 1A
 $BE = \sqrt{90^2 + 60^2 + 60^2}$ 1M
 $= \sqrt{15\,300}$
 $\approx 124 \text{ cm}$ 1A
- (b) (i) $\angle BDC = 45^\circ$ 1A
 $\angle BDI = 45^\circ + 50^\circ = 95^\circ$ 1A
- (ii) $BD = \sqrt{60^2 + 60^2}$ 1M
 $= \sqrt{7200} \text{ cm}$
 $BI^2 = BD^2 + 60^2 - 2(BD)(60) \cos \angle BDI$ 1M
 $BI \approx 108 \text{ cm}$ 1A
- (c) Required angle is $\angle BEI$. 1A
 $BI^2 = BE^2 + EI^2 - 2(BE)(EI) \cos \angle BEI$ 1M
 $\angle BEI \approx 55.1^\circ$ 1A
27. (a) In $\triangle PSR$, $\angle PRQ = 45^\circ$,
 $PS = 10 \sin 45^\circ$
 $= 5\sqrt{2} \text{ cm}$ 1A
- In $\triangle PQR$,
 $QR = \sqrt{10^2 + 10^2}$
 $= 10\sqrt{2} \text{ cm}$ 1A

(b) (i) Let $s = \frac{5\sqrt{2} + 5\sqrt{2} + 8}{2} = 5\sqrt{2} + 4$.

Area of $\triangle PQR$

$$= \sqrt{(5\sqrt{2} + 4)(5\sqrt{2} + 4 - 5\sqrt{2})(5\sqrt{2} + 4 - 5\sqrt{2})(5\sqrt{2} + 4 - 8)}$$

1M+1A

$$= \sqrt{(5\sqrt{2} + 4)(4)(4)(5\sqrt{2} - 4)}$$

$$= \sqrt{16(50 - 16)}$$

$$= 4\sqrt{34} \text{ cm}^2$$

1A

(ii) Volume of $PQRS = \frac{1}{3} \times PS \times \text{Area of } \triangle QSR$

$$= \frac{1}{3} \times 5\sqrt{2} \times 4\sqrt{34}$$

1M

$$= \frac{40\sqrt{17}}{3} \text{ cm}^3$$

1A

(c) (i) Volume of $PQRS = \frac{1}{3} \times PS \times \text{Area of } \triangle QSR$

$$= \frac{1}{3} \times 5\sqrt{2} \times \frac{1}{2} \times (5\sqrt{2})^2 \sin \theta$$

1M

$$= \frac{125\sqrt{2} \sin \theta}{3} \text{ cm}^3$$

1A

(ii) Volume of $PQRS \propto \sin \theta$ 1M

Thus, the volume of $PQRS$ increases when θ increases from 45° to 90° ; 1A

decreases when θ increases from 90° to 135° . 1A