REG-2425-MOCK-SET 5-MATH-CP 2

Answers:

1. A	2. B	3. C	4. A	5. A	6. B	7. C	8. D	9. D	10. A
11. B	12. D	13. A	14. D	15. C	16. D	17. A	18. B	19. A	20. A
21. A	22. D	23. A	24. D	25. C	26. C	27. A	28. C	29. B	30. D
31. B	32. B	33. A	34. D	35. B	36. A	37. B	38. D	39. B	40. D
41. A	42. B	43. B	44. D	45. C					

Suggested Solutions:

1. A

$$\frac{9^{3x+1}}{27^{2x+1}} = \frac{3^{6x+2}}{3^{6x+3}}$$
$$= \frac{1}{3}$$

2. B

$$2a^2 - ab - b^2 - 4a - 2b$$
$$= (2a+b)(a-b) - 2(2a+b)$$
$$= (2a+b)(a-b-2)$$

3. C

Solve $\begin{cases} 2x + y = 5 \\ 3x - 2y + 1 = 5 \end{cases}$, we have $x = 2$ and $y = 1$.

4. A

Put $x = -1$.

$$0 - 3 = (-1+2)^2 + \beta$$
$$\beta = -4$$

5. A

$$x = 2 - \frac{y+1}{y}$$
$$xy = 2y - (y+1)$$
$$y(x-1) = -1$$
$$y = \frac{1}{1-x}$$

6. B

A. ✗. 0.001 is of 3 decimal places.

B. ✓.

C. ✗. $x = 0.001$ (correct to 3 decimal places)

D. ✗. 0.0012 has only 2 significant figures.

7. C

$$-3(4-x) \le 9 \quad \text{or} \quad \frac{7x+2}{5} < -8$$
$$3x \le 21 \qquad \frac{7x}{5} < -\frac{42}{5}$$
$$x \le 7 \qquad x < -6$$

Thus, $x \le 7$.

The greatest integer is 7.

8. D

$$f(\alpha) - f(\alpha-1) = 5[(\alpha)^2 - (\alpha-1)^2] - (1-1)$$
$$= 5(2\alpha - 1)$$
$$= 10\alpha - 5$$

9. D

$$g\left(\frac{1}{2}\right) = \frac{k}{8} - \frac{5}{4} - k + 3 = 0$$
$$k = 2$$
$$g(-2) = 2(-8) - 5(4) - 4(-2) + 3 = -25$$

10. A

I. ✗. When $x = 3$, $y = (-3+1)^2 + 2 = 6 \neq -2$.

II. ✓. Coefficient of $x^2 = (-1)^2 = 1 > 0$. The graph opens upwards.

III. ✗. y-intercept $= (0+1)^2 + 2 = 3 \neq 2$

11. B

Let the cost of the handbag be $$x$.

Percentage profit $= \dfrac{x(1+50\%)(1-20\%)-x}{x} \times 100\%$

$= 20\%$

12. D

Let $a = 6$, then $b = \dfrac{2a}{3} = 4$ and $c = \dfrac{2a}{4} = 3$.

Thus, $a : b : c = 6 : 4 : 3$.

13. A

Let $p = \dfrac{kr}{q^2}$, where k is a non-zero constant.

Percentage change

$$= \frac{\frac{k(0.9r)}{(1.2q)^2} - \frac{kr}{q^2}}{\frac{kr}{q^2}} \times 100\%$$

$= -37.5\%$

> Let $p = \dfrac{kr}{q^2}$, where k is a non-zero constant.
>
> $$\frac{p_2}{p_1} = \frac{1-10\%}{(1+20\%)^2}$$
>
> $= 0.625$
>
> p is decreased by 37.5%.

14. D

The numbers are formed by +2, +4, +6, . . .

The sequence is 4, 6, 10, 16, 24, 34, 46, 60.

Required number is 60.

15. C

$(2q)^2(3p) = 648$

$pq^2 = 54$

Required volume $= \dfrac{1}{3}(3q)^2(2p)$

$= 6pq^2$

$= 324\,\text{cm}^3$

16. D

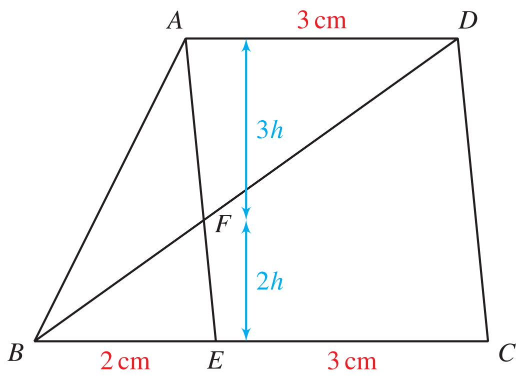
Let $BE = 2$ cm. Then $CE = AD = 3$ cm.

$\triangle ADF \sim \triangle EBF$ (ratio = 3 : 2)

Required ratio

$$= \frac{(3)(5h)}{2} : \left[(3)(5h) - \frac{(3)(3h)}{2}\right]$$

$$= 5 : 7$$



17. A

$\angle AOB = 2\angle ACB = 40°$

$\angle BOC = 140° - 40° = 100°$

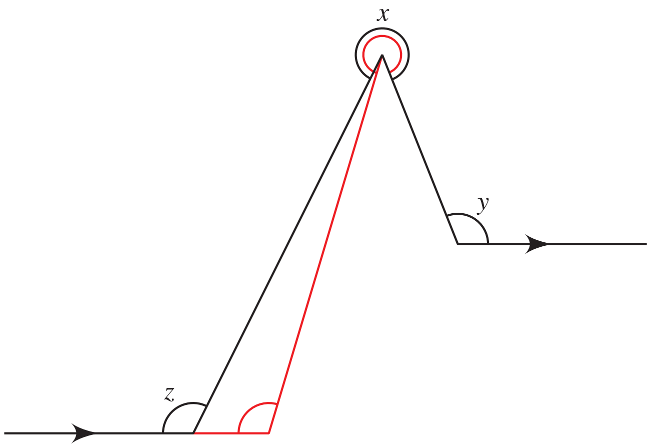
Required area

$$= \pi(6)^2 \times \frac{100°}{360°} - \frac{1}{2}(6)(6)\sin 100°$$

$$\approx 14\text{ cm}^2$$

18. B

Move the point correspond to angle z to the right.



Note that x becomes larger and z becomes smaller.

I. ✗.

II. ✗.

The result of $y + z - x$ becomes smaller.

The equation cannot hold in both cases.

III. ✓.

Break the angle x into three parts using a line parallel to the other two given parallel lines.

$$(180^\circ - z) + 180^\circ + (180^\circ - y) = x$$
$$x + y + z = 540^\circ$$

19. A

Note that $\triangle ADE$ is an equilateral triangle.

We have $\angle EAD + \angle ADE = \angle DEA = 60^\circ$.

Since $AB = AC$, we have $\angle CBA = \angle ACB$.

$$\angle CBA + \angle ACB + \angle BAC = 180^\circ$$
$$2\angle ACB + (60^\circ + 32^\circ) = 180^\circ$$
$$\angle ACB = 44^\circ$$

Consider $\triangle CDE$.

$$\angle DCE + \angle DEC = \angle ADE$$
$$44^\circ + \angle DEC = 60^\circ$$
$$\angle DEC = 16^\circ$$

20. A

Let $AB = 1$. Then $CD = AB = 1$.

$$\frac{BF}{CE} = \frac{1}{\sin\beta} \div \frac{1}{\sin\alpha}$$
$$= \frac{\sin\alpha}{\sin\beta}$$

21. A

Let $BD = x$ cm.

Note that $\triangle ABC \sim \triangle EFC$.

$$\frac{FC}{BC} = \frac{EF}{AB}$$
$$\frac{FC}{20} = \frac{x}{10}$$
$$FC = 2x \text{ cm}$$

Consider the area of $BDEF$.

$$x(20 - 2x) = 42$$
$$-2x^2 + 20x - 42 = 0$$
$$x = 3 \quad \text{or} \quad 7$$

The shortest possible length of BD is 3 cm.

22. D

I. ✓. Since $ABCD$ is a parallelogram, $\angle BAD = \angle BCD$.

$$360° - \angle BCD = 2\angle BAD$$
$$360° - \angle BAD = 2\angle BAD$$
$$\angle BAD = 120°$$

II. ✓. Since $ABCD$ is a parallelogram, $AB = CD$ and $AD = BC$.
Since $BC = CD$, the four sides of $ABCD$ are equal and it is a rhombus.

III. ✓. Since $AB = AD$, $\overset{\frown}{AB} : \overset{\frown}{AD} = 1 : 1$.

23. A

x-intercept $= \dfrac{5}{b}$ and y-intercept $= \dfrac{5}{a}$.
Note that a and b are positive.

$$\frac{5}{b} < 2 \quad \text{and} \quad \frac{1}{2} \times \frac{5}{b} \times \frac{5}{a} > 4$$
$$b > \frac{5}{2} \qquad\qquad ab < \frac{25}{8}$$

I. ✓.

II. ✓.

III. ✗. Take $a = 1$ and $b = 3$. These values of a and b satisfy all the conditions above but $2a < b$.

24. D

Two lines are parallel.

$$\frac{-2}{3} = \frac{-6}{k}$$
$$k = 9$$

25. C

$(-2,\ -5) \quad \longleftarrow \quad B\,(-2,\ 5) \quad \longleftarrow \quad A\,(-2,\ 2) \quad = \quad \left(2\sqrt{2},\ 135°\right)$

26. C

The locus of P is a pair of straight lines, $y = -1$ and $y = 11$.

27. A

$$x^2 + y^2 + 2x + 4y + \frac{4}{3} = 0$$

I. ✗. x-coordinate of centre $= -1$

II. ✓. $0^2 + 0^2 + 0 + 0 + \dfrac{4}{3} > 0$. The origin lies outside C.

III. ✗. Radius $= \sqrt{1^2 + 2^2 - \dfrac{4}{3}} \neq 1$

28. C

Required probability

$= 1 - \left(\frac{4}{7}\right)^2$

$= \frac{33}{49}$

29. B

Average of 60 and 70 is 65.

If there are more boys than girls, the mean score will be closer to the mean of boys.

The mean score will lie between 60 and 65.

Mathematical explanation

Let the ratio of number of boys to girls be $1 : \beta$, where $0 < \beta < 1$.

Mean $= \frac{60(1) + 70(\beta)}{1+\beta} = 60 + \frac{10\beta}{1+\beta} = 65 + \frac{5\beta - 5}{1+\beta}$

Since $\frac{10}{1+\beta} > 0 > \frac{5\beta-5}{1+\beta}$, the mean of the test marks lies between 60 and 65.

Only option B satisfies this.

30. D

A. ✗. Mode = 30

B. ✗. Median = 30

C. ✗. Lower quartile = 25

D. ✓.

31. B

Slope $= \frac{12-0}{0-3} = -4$

The equation of the relationship is

$$\frac{y^3 - 12}{\log_5 x - 0} = -4$$

$$y^3 - 12 = -4\log_5 x$$

Put $y = 2$.

$$2^3 - 12 = -4\log_5 x$$

$$\log_5 x = 1$$

$$x = 5$$

32. B

$y = f(x) \quad \longrightarrow \quad y = f(-x) \quad \longrightarrow \quad y = f(-2x)$

Reflect about y-axis.

Reduce along x-axis to $\frac{1}{2}$ times the original.

The answer is B.

33. A

$$\begin{aligned}\pi^{2x} - 9\pi^x + 20 &< 2 \\ (\pi^x)^2 - 9\pi^x + 18 &< 0 \\ 3 < \pi^x &< 6 \\ \log 3 < x\log\pi &< \log 6 \\ \frac{\log 3}{\log\pi} < x &< \frac{\log 6}{\log\pi} \\ \log_\pi 3 < x &< \log_\pi 6\end{aligned}$$

34. D

$512 = 200_{16}$

$11 \times 8^{16} = 11 \times 2^{48} = 11 \times 16^{12} = \text{B000000000000}_{16}$

The answer is D.

35. B

Put $k = 2$, we have $\frac{i^{2020}}{k + i^{2019}} = \frac{1}{2 - i} = \frac{2}{5} + \frac{1}{5}i$.

The imaginary part is $\frac{1}{5}$.

Check the value of each option when $k = 1$.

A. $\frac{1}{3}$

B. $\frac{1}{5}$

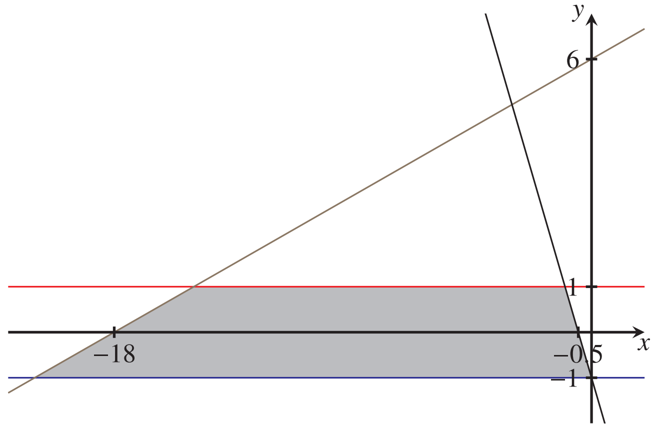
C. $\frac{2}{3}$

D. $\frac{2}{5}$

36. A

Line	x-intercept	y-intercept
$x - 3y + 18 = 0$	-18	6
$2x + y + 1 = 0$	-0.5	-1
$y = -1$		-1
$y = 1$		1

Sketch the solution region using the intercepts.



The value of $5x - 2y + k$ is larger when x is larger and y is smaller.
$5x - 2y + k$ attains its maximum value at the bottom right corner, which is $(0,\ -1)$.

$$5(0) - 2(-1) + k = 12$$
$$k = 10$$

37. B

I. ✓. General term $= (1 - 2^{-n}) - (1 - 2^{-(n-1)})$
$= 2^{-n}(-1 + 2)$
$= 2^{-n}$
We have $2^{-n} < 1$ for all positive integers n.

II. ✗. The nth term $= \dfrac{1}{2^n}$ is a rational number for all positive integers n.

III. ✓. $\log T_{n+1} - \log T_n = \log 2^{-n-1} - \log 2^{-n}$
$= (-n - 1)\log 2 + n \log 2$
$= -\log 2 = \text{constant}$
Thus, it is an arithmetic sequence.

38. D

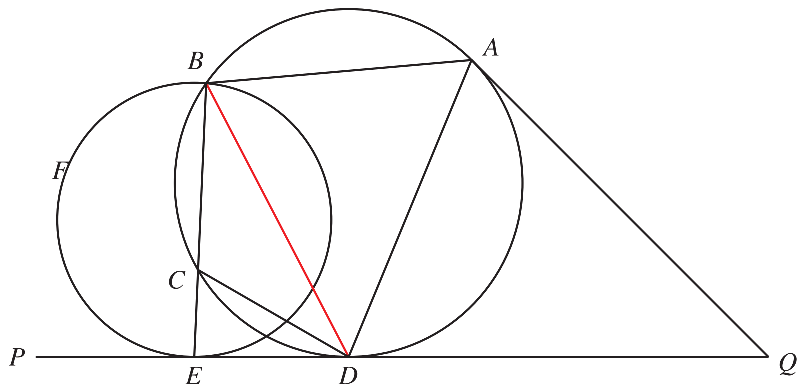
Solve the system $\begin{cases} x - y + m = 0 \\ x^2 + y^2 + 2x - 4y - 13 = 0 \end{cases}$ using the calculator program.

Value of m	Number of intersections	Sign of Δ
-9	0	$-$

Required range does not contain $m = -9$ and -9 is not a boundary value of the required range.
The answer is D.

39. B

Since $AQ = DQ$, we have $\angle ADQ = \dfrac{180° - 50°}{2} = 65°$.



$\angle ABD = \angle ADQ = 65°$
$\angle CBD = 100° - 65° = 35°$
$\angle CDE = \angle CBD = 35°$
$\angle CDB = \angle CBD = 35°$
$\angle PEB = \angle EBD + \angle BDE = 35° + (35° + 35°) = 105°$

40. D

$$2\cos^2\theta = 2 - \sin\theta$$
$$2(1 - \sin^2\theta) = 2 - \sin\theta$$
$$-2\sin^2\theta + \sin\theta = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \frac{1}{2}$$

When $\sin\theta = 0$, $\theta = 0°$ or $180°$ or $360°$.
When $\sin\theta = \frac{1}{2}$, $\theta = 30°$ or $150°$.
There are 5 roots.

41. A

The straight line $x - 2y + 10 = 0$ is perpendicular to the straight line $2x + y + a = 0$.
The triangle formed is a right-angled triangle. The orthocentre lies at the right-angled vertex.
When $x = -6$,

$$\begin{aligned}(-6) - 2y + 10 &= 0 \\ y &= 2\end{aligned}$$

Substitute $(-6,\ 2)$ into $2x + y + a = 0$,

$$\begin{aligned}2(-6) + (2) + a &= 0 \\ a &= 10\end{aligned}$$

42. B

$$\begin{aligned}\text{Required probability} &= \frac{1}{8} + \frac{1}{8} - \frac{6!}{8!} \\ &= \frac{13}{56}\end{aligned}$$

43. B

$$\begin{aligned}\text{Required number} &= C_2^3 C_1^5 C_1^2 + C_1^3 C_2^5 C_1^2 + C_1^3 C_1^5 C_2^2 \\ &= 105\end{aligned}$$

44. D

Let the mean and standard deviation be $\overline{x}$ marks and σ marks respectively.

$$\begin{cases}\dfrac{26 - \overline{x}}{\sigma} = -1 \\ \dfrac{92 - \overline{x}}{\sigma} = 0.5\end{cases}$$

Solving, we have $\overline{x} = 70$ and $\sigma = 44$.

45. C

Median $= 15 \times 2 + 3 = 33$

Interquartile range $= 10 \times 2 = 20$

Variance $= 40 \times 2^2 = 160$

END OF PAPER