## REG-2425-MOCK-SET 3-MATH-CP 2

**Answers:**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. A | 2. B | 3. A | 4. C | 5. B | 6. B | 7. A | 8. A | 9. D | 10. D |
| 11. B | 12. C | 13. C | 14. A | 15. B | 16. B | 17. A | 18. B | 19. C | 20. D |
| 21. D | 22. C | 23. C | 24. D | 25. D | 26. D | 27. C | 28. A | 29. D | 30. B |
| 31. B | 32. D | 33. A | 34. B | 35. D | 36. C | 37. D | 38. C | 39. A | 40. A |
| 41. A | 42. C | 43. C | 44. B | 45. D | | | | | |

**Suggested Solutions:**

1. A

$$\begin{aligned}(-2)^{2017}\left(\frac{1}{4}\right)^{2016} &= -\frac{2^{2017}}{2^{2\times 2016}} \\ &= -\frac{1}{2^{4032-2017}} \\ &= -\frac{1}{2^{2015}}\end{aligned}$$

2. B

Coefficient of $q^3 = (-1)(1) = -1$
Coefficient of $p^2q = (1)(1) + (-1)(1) = 0$
Only option B satisfies these.

3. A

$$\begin{aligned}\frac{c}{a-1} - \frac{ab}{1-a} &= 3 \\ c + ab &= 3a - 3 \\ a(b-3) &= -3 - c \\ a &= \frac{3+c}{3-b}\end{aligned}$$

4. C

Compare coefficients of $x$ and constant term,

$-a + 5a = 9 - 1$ and $5 - 5a^2 = -9 - b$

$4a = 8$ $\qquad$ $b = 6$

$a = 2$

5. B

$$-f(2)+f(-2) = -(2+4-4)+(2-4-4)$$
$$= -8$$

6. B

$$y=(qx-p)(x+1)+3 = q\left(x-\frac{p}{q}\right)(x+1)+3$$

I. ✓. Coefficient of $x^2 = q > 0$ since the graph open upwards.

II. ✓. $y$-intercept $= -p+3 < 0 \quad \Rightarrow \quad p > 3$

III. ✗. When $x=-1$, $y = 0+3 = 3 \neq 0$.

7. A

$$\frac{6x-1}{7} > 5 \quad \text{or} \quad 4-3(1-x) > 7$$
$$\frac{6x}{7} > \frac{36}{7} \qquad\qquad 3x > 6$$
$$x > 6 \qquad\qquad x > 2$$

Thus, $x > 2$.

8. A

$$p(-k) = -k^3+k^3-4k-16 = 0$$
$$k = -4$$

Remainder $= (-2)^3 - 4(-2)^2 + 4(-2) - 16$
$= -48$

9. D

$$\text{Amount} = 94\,000\left(1+\frac{4\%}{12}\right)^{3\times 12}$$
$$\approx \$105\,964$$

10. D

Let $\begin{cases} 3b-4c=1 \\ 4b-3c=2 \end{cases}$. Then $b=\frac{5}{7}$ and $c=\frac{2}{7}$.

We have $a = c\times\frac{2}{1} = \frac{4}{7}$.

$(a+b):(b+c) = \frac{9}{7}:\frac{7}{7} = 9:7$

11. B

Let $z = \dfrac{kx^3}{\sqrt{y}}$, where $k$ is a non-zero constant.

Then $k = \dfrac{\sqrt{y}z}{x^3}$.

So, $\dfrac{yz^2}{x^6} = k^2$ is a constant.

12. C

$n < \dfrac{2.5 \times 1000}{39.5}$

$n < 63.3$

The greatest possible value of $n$ is 63.

13. C

The numbers are formed by +6, +10, +14, . . .

The sequence of numbers of dots is 1, 7, 17, 31, 49, 71, 97, . . .

Required number is 97.

14. A

$34\pi = \pi(r)\left(\dfrac{17r}{8}\right)$

$r = 4$

$\text{Volume} = \dfrac{1}{3}\pi(4)^2\sqrt{8.5^2 - 4^2}$

$= 40\pi \text{ cm}^3$

15. B

$\sin \angle ODC = \dfrac{OC}{OD}$

$= \dfrac{1}{2}$

$\angle ODC = 30^\circ$

$\angle DFE = 75^\circ - 30^\circ = 45^\circ$ and $\angle CFB = \angle DFE = 45^\circ$

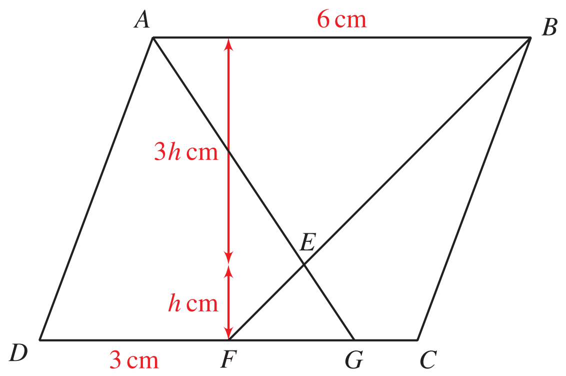
$\angle CBF = 180^\circ - 90^\circ - 45^\circ = 45^\circ$ and $\angle OAB = \angle OBA = 45^\circ$

$\angle DOA = 75^\circ - 45^\circ = 30^\circ$

Area of sector $= \pi(12)^2 \times \dfrac{30^\circ}{360^\circ} = 12\pi \text{ cm}^2$

16. [B]

Let $CG = 1$ cm. Then we have the lengths as shown in the figure.




[Point $E$] Note that $\triangle EFG \sim \triangle EBA$ (ratio 1 : 3).

Consider the area of $BCGE$.

$$1265 = \frac{(3)(4h)}{2} - \frac{(2)(h)}{2}$$
$$h = 253$$

$$\text{Required area} = \frac{(6)(3h)}{2} = 2277\ \text{cm}^2$$

17. [A]

Let $\angle EAD = x$.

In $\triangle ABD$, we have $\angle ABD = \angle ADB$.

$$\angle ABD + \angle ADB = \angle EAD$$
$$\angle ABD = \frac{x}{2}$$

Since $DC // EB$, we have $\angle BDC = \angle ABD = \frac{x}{2}$.

In $\triangle BCD$, we have $\angle DBC = \angle DCB = 4\angle BDC = 4\left(\frac{x}{2}\right) = 2x$.

$$2x + 2x + \frac{x}{2} = 180^\circ$$
$$x = 40^\circ$$

18. B

Note that $\triangle ADE \sim \triangle ACF$.

$$\frac{AD}{AC} = \frac{AE}{AF}$$
$$\frac{AD}{AD+10} = \frac{6}{6+6}$$
$$AD = 10\text{ cm}$$

In $\triangle ADE$, we have $DE = \sqrt{10^2 - 6^2} = 8\text{ cm}$.

$$\frac{CF}{DE} = \frac{AF}{AE}$$
$$\frac{CF}{8} = \frac{6+6}{6}$$
$$CF = 16\text{ cm}$$

In $\triangle BCF$, we have $BC = \sqrt{16^2 + 12^2} = 20\text{ cm}$.

19. C

$BE = \sqrt{20^2 - 16^2} = 12\text{ cm}$ and $DE = \sqrt{13^2 - 12^2} = 5\text{ cm}$

Perimeter = 5 + 13 + (5 + 16) + 20 = 58 cm

20. D

$DE = \frac{1}{2}AD = 1\text{ cm}$

$DG = \frac{1}{2}CD = 1\text{ cm}$

Note that $\triangle CDE \cong \triangle ADG$.

We have $\angle DAG = \angle DCE$.

Consider $\triangle ADG$.

$$AG^2 = AD^2 + DG^2$$
$$AG = \sqrt{2^2 + 1^2}$$
$$= \sqrt{5}\text{ cm}$$

Note that $\triangle CFG \sim \triangle ADG$.

$$\frac{\text{area of } \triangle CFG}{\text{area of } \triangle ADG} = \left(\frac{CG}{AG}\right)^2$$
$$\frac{\text{area of } \triangle CFG}{\frac{1}{2}(2)(1)} = \left(\frac{1}{\sqrt{5}}\right)^2$$
$$\text{area of } \triangle CFG = \frac{1}{5}\text{ cm}^2$$

21. D

Let $AD = 1$.

$\angle AFD = \alpha$, $AF = \dfrac{1}{\sin\alpha}$ and $DF = \dfrac{1}{\tan\alpha}$

$CF = 1 - \dfrac{1}{\tan\alpha}$ and $EF = \dfrac{CF}{\sin(180° - \beta)} = \dfrac{1 - \frac{1}{\tan\alpha}}{\sin\beta}$

$$\begin{aligned}\frac{AF}{EF} &= \frac{1}{\sin\alpha} \div \frac{1 - \frac{1}{\tan\alpha}}{\sin\beta} \\ &= \frac{\sin\beta}{\sin\alpha - \cos\alpha}\end{aligned}$$

22. C

$\angle CAD = \angle BAC \times \dfrac{16}{24} = 42°$

$\angle ACB = 180° - 90° - 63° = 27°$ and $\angle ADE = \angle ACB = 27°$

$\angle AED = 180° - 27° - 42° = 111°$

23. C

There are 6 axes of reflectional symmetry.

24. D

I. ✓. Slope $= -\dfrac{a}{5} < 0 \quad \Rightarrow \quad a > 0$

II. ✓. $y$-intercept $= \dfrac{b}{5} < -1 \quad \Rightarrow \quad b < -5$

III. ✓. $x$-intercept $= \dfrac{b}{a} > -2$ and $a > 0 \quad \Rightarrow \quad b > -2a$

25. D

$L_1$ and $L_2$ are two perpendicular lines.

The locus of $P$ are the angle bisectors of the angles formed by $L_1$ and $L_2$.

The locus of $P$ is a pair of perpendicular lines.

26. D

The equation is in the form $2x - 3y + C = 0$, where $C$ is a constant.

Put $(5,\ 1)$ into the equation, we have $C = -7$.

27. C

$C_1$: $x^2 + y^2 - 8x - 6y + 20 = 0$
$C_2$: $x^2 + y^2 + 6x - 8y + \frac{33}{2} = 0$
The coordinates of $G_1$ and $G_2$ are $(4,\ 3)$ and $(-3,\ 4)$ respectively.

I. ✓.
Slope of $G_1O = \frac{3-0}{4-0} = \frac{3}{4}$
Slope of $G_2O = \frac{4-0}{-3-0} = -\frac{4}{3} = -1 \div \frac{3}{4}$
Thus, $G_1O$ is perpendicular to $G_2O$.

II. ✗.
Area of $C_1 = \pi\left(\sqrt{4^3 + 3^2 - 20}\right)^2 = 5\pi$
Area of $C_2 = \pi\left(\sqrt{3^2 + 4^2 - \frac{33}{2}}\right)^2 = \frac{17\pi}{2} > 5\pi$

III. ✓.
$OG_1 = \sqrt{4^2 + 3^2} = 5$
$OG_2 = \sqrt{3^2 + 4^2} = 5$

28. A

$$\text{Required probability} = \frac{3+2+2+1}{8 \times 7} = \frac{1}{7}$$

29. D

In the cumulative frequency curve, steeper $\Rightarrow$ more data in the corresponding class.
So, the data is more concentrated in the lower part.
Minimum, lower quartile, median and upper quartile will be closed to each other.

30. B

Mode = 60 $\Rightarrow$ at least three of $a$, $b$, $c$, $d$ are 60
Let $a = b = c = 60$.
$$70 = \frac{60 + 60 + 60 + d + \ldots + 81}{12}$$
$d = 76$
Median = 76

31. B

From $y = f(x)$,
reflect about $x$-axis $\longrightarrow$ $y = -f(x)$
translate leftwards by 2 units $\longrightarrow$ $y = -f(x+2)$

32. D

$1100001101011011_2$
$= 2^{15} + 2^{14} + 2^9 + 2^8 + 2^6 + 2^4 + 2^3 + 2 + 1$
$= (2+1) \times 2^{14} + (2^3 + 2^2 + 1) \times 2^6 + 2^4 + (2^2 + 1) \times 2 + 1$
$= 3 \times 2^{14} + 13 \times 2^6 + 2^4 + 5 \times 2 + 1$

33. A

For the point $\left(\frac{1}{3}, 0\right)$,

$$\log_8 x = \frac{1}{3} \qquad \text{and} \log_4 y = 0$$
$$x = 8^{\frac{1}{3}} = 2 \qquad y = 1$$

Only option A satisfies this.

34. B

$$\log_4 a = \frac{1}{c} \qquad \text{and} \qquad \log_{25} b = \frac{1}{c}$$
$$\frac{\log a}{2\log 2} = \frac{1}{c} \qquad \frac{\log b}{2\log 5} = \frac{1}{c}$$
$$\log a = \frac{2\log 2}{c} \qquad \log b = \frac{2\log 5}{c}$$

$$\begin{aligned} \log ab &= \log a + \log b \\ &= \frac{2\log 2}{c} + \frac{2\log 5}{c} \\ &= \frac{2\log 10}{c} \\ &= \frac{2}{c} \end{aligned}$$

35. D

Take $m = 1$, $i^7 + \dfrac{i^5 - 4}{m - i} = -\dfrac{5}{2} - \dfrac{5}{2}i$.
Only option D gives $-\dfrac{5}{2}$ when $m = 1$.

36. C

Label the inequalities as follows:

① $7x + y \le 20$

② $2x + 3y \ge 3$

③ $5y - 3x \le 24$

| Lines | Coordinates | Check | $4y - 3x - 5$ |
|---|---|---|---|
| ① and ② | $(3, -1)$ | ③ ✓ | $-18$ |
| ① and ③ | $(2, 6)$ | ② ✓ | 13 |
| ② and ③ | $(-3, 3)$ | ① ✓ | 16 |

The greatest value is 16.

37. D

Note that $\log 9 - \log 3 = \log 27 - \log 9 = \log 81 - \log 27 = \log 3$.
$\log 3, \log 9, \log 27, \log 81$ is an arithmetic sequence.

38. C

$$4\cos^2\theta - 7\sin\theta - 7 = 0$$
$$4(1 - \sin^2\theta) - 7\sin\theta - 7 = 0$$
$$-4\sin^2\theta - 7\sin\theta - 3 = 0$$
$$\sin\theta = -1 \quad \text{or} \quad -\frac{3}{4}$$

$\sin\theta = -1$ has one root and $\sin\theta = -\frac{3}{4}$ has two roots.

39. A

$CM = 30 \times \frac{7}{3+7} = 21\text{ cm}$ and $MH = 30 - 21 = 9\text{ cm}$

$MA = \sqrt{12^2 + 16^2 + 21^2} = 29\text{ cm}$ and $MG = \sqrt{12^2 + 9^2} = 15\text{ cm}$

$AG = \sqrt{30^2 + 16^2} = 34\text{ cm}$

$AG^2 = AM^2 + MG^2 - 2(AM)(MG)\cos\theta$

$\cos\theta = \frac{-3}{29}$

40. A

$\angle AGE = 2 \times 66^\circ = 132^\circ$
$\angle CEA = 132^\circ \times \dfrac{6}{5+6} = 72^\circ$
$\angle CAT = \angle CEA = 72^\circ$ and $\angle ATC = 180^\circ - 2 \times 72^\circ = 36^\circ$

41. A

I. ✓. $G$ lies inside $\triangle OAB$, which is in the second quadrant. The $x$- and $y$-coordinates are not equal (one positive and one negative).

II. ✓. Let the radius of inscribed circle be $r$. Then the coordinates of $G$ are $(-r,\ r)$.

$$4r + (-r) = 3kb$$
$$r = kb$$

Using tangent properties, $OB$ is divided into two segments with lengths $b - r$ and $r$.
$OA$ is divided into two segments with lengths $10 - r$ and $r$.

$$(10 - r) + (b - r) = \sqrt{10^2 + b^2}$$
$$[10 + b(1 - 2k)]^2 = b^2 + 100$$
$$100 + 20b(1 - 2k) + b^2(1 - 2k)^2 = b^2 + 100$$
$$b^2(4k^2 - 4k) + 20b(1 - 2k) = 0$$
$$b = -\frac{20(1 - 2k)}{4k^2 - 4k} = \frac{5(1 - 2k)}{k(1 - k)}$$

Required distance $= r = kb = \dfrac{5(1 - 2k)}{1 - k}$

III. ✗. When $k = \dfrac{1}{6}$, $r = \dfrac{5(1 - 2k)}{1 - k} = 4$.
Equation of inscribed circle is $(x + 4)^2 + (y - 4)^2 = 4^2$.

$$(x + 4)^2 + (5 - 3x - 4)^2 = 16$$
$$10x^2 + 2x + 1 = 0$$

$\Delta = 2^2 - 4(10)(1) = -36 < 0$.
The straight line $3x + y = 5$ does not cut the inscribed circle of $\triangle OAB$ and hence is not a tangent.

42. C

Required number $= C_3^{10} 3!9! = 261\,273\,600$

43. C

Required probability $= 1 - \left(1 - \frac{2}{8}\right)\left(1 - \frac{3}{8}\right)\left(1 - \frac{4}{8}\right)$

$= \frac{49}{64}$

44. B

Let the standard deviation of the test scores be $\sigma$.

$$\frac{76 - 64}{\sigma} = 1.5$$

$$\sigma = 8$$

Standard score of Anson $= \frac{54 - 64}{8} = -1.25$

45. D

From new to old,

multiply by 4 and then subtract 3 from the resulting number.

Thus, mean of original set $= 4m - 3$ and variance $= 16v$.