Solution	Marks
REG-2425-MOCK-SET 3-MATH-CP 1	
Suggested solutions	
1. $2(3a-11)=3a-5b$	
$6a-22=3a-5b$	1M
$3a=22-5b$	1M
$a=\dfrac{22-5b}{3}$	1A
2. $\dfrac{m^6n^{-3}}{(m^5n^{-4})^2}=\dfrac{m^6n^{-3}}{m^{10}n^{-8}}$	1M
$=\dfrac{n^{-3+8}}{m^{10-6}}$	1M
$=\dfrac{n^5}{m^4}$	1A
3. $\dfrac{5}{3k+2}-\dfrac{4}{2k+7}=\dfrac{5(2k+7)-4(3k+2)}{(3k+2)(2k+7)}$	1M
$=\dfrac{10k+35-12k-8}{(3k+2)(2k+7)}$	1M
$=\dfrac{-2k+27}{(3k+2)(2k+7)}$	1A
4. (a) $25x^2-4=(5x+2)(5x-2)$	1A
(b) $5x^2y-17xy+6y=y(5x^2-17x+6)$	
$=y(x-3)(5x-2)$	1A
(c) $5x^2y-17xy+6y-25x^2+4=y(x-3)(5x-2)-(5x+2)(5x-2)$	1M
$=(5x-2)(xy-3y-5x-2)$	1A
5. (a) $-3(x-4)\geq\dfrac{5x+3}{6}$	
$-\dfrac{23x}{6}\geq-\dfrac{23}{2}$	
$x\leq 3$	1A
(b) $6x+24>0$	
$x>-4$	1A
So, $-4<x\leq 3$.	1M
There are 7 integers.	1A
6. (a) Cost $=\dfrac{7000}{1+40\%}$	1M
$=\$5000$	1A
(b) Percentage profit $=\dfrac{7000(1-12\%)-5000}{5000}\times 100\%$	1M
$=23.2\%$	1A

Solution	Marks
7. Let the present age of Irene be x. Then the present age of Peter is $\frac{4x}{3}$.	1A
$\frac{x-7}{\frac{4x}{3}-7}=\frac{2}{3}$	1M+1A
$3x-21=\frac{8x}{3}-14$	
$x=21$	1A
She is 21 years old now.	
8. (a) $57=\frac{41+47+\ldots+(70+a)}{12}$	1M
$a=5$	1A
(b) Range $=75-41=34$ kg	1A
Interquartile range $=66-49.5=16.5$ kg	1A
Standard deviation ≈ 10.7 kg	1A
9. (a) Let $\angle AOE=x$.	
Then $\angle AEO=x$ and $\angle DBE=\frac{x}{2}$.	1A
$x+\frac{x}{2}=48^\circ$	
$x=32^\circ$	1A
(b) $\angle OAB=32^\circ+32^\circ=64^\circ$ and $\angle AOB=180^\circ-2\times 64^\circ=52^\circ$.	1A
$\frac{\overset{\frown}{AB}}{AE}=\frac{2(AO)\pi\times\frac{52^\circ}{360^\circ}}{AO}$	1M
$\approx 0.908<1$	
So, $\overset{\frown}{AB}<AE$. The claim is agreed.	1A
10. (a) Let $f(x)=a+bx^2$, where a and b are non-zero constants.	1A
$\begin{cases}206=a+b\\254=a+9b\end{cases}$	1M
Solving, we have $a=200$ and $b=6$.	1A
Thus, $f(x)=200+6x^2$.	
(b) $200+6x^2=80x$	1M
$6x^2-80x+200=0$	
$x=\frac{10}{3}$ or 10	1A

Solution	Marks
11. (a) $(c+1)+4+a=8+(b-a)+c$	1M
$2a-b=3$	
Since $a>5$ and $b<11$, $(a,\ b)=(6,\ 9)$.	1A+1A
(b) (i) 1	1A
(ii) 6	1A
(c) Required probability $=\dfrac{3+1}{2+4+6+8+3+1}$	1M
$=\dfrac{1}{6}$	1A
12. (a) $f(-1)=0=-4(-1)^3+(a+2)(-1)^2+2(-1)-3b$	1M
$a-3b=-4$	
$f(2)=9=-4(2)^3+(a+2)(2)^2+2(2)-3b$	1M
$4a-3b=29$	
Solving, we have $a=11$ and $b=5$.	1A
(b) $f(x)=-4x^3+13x^2+2x-15$	
$=(-x^2+2x+3)(4x-5)$	1M
So, $g(x)=4x-5$.	
$kx(4x-5)=(-x^2+2x+3)(4x-5)$	1M
$(4x-5)(x^2+(k-2)x-3)=0$	1M
$x=\dfrac{5}{4}$ or $x^2+(k-2)x-3=0$	
$\Delta=(k-2)^2-4(1)(-3)=(k-2)^2+12>0$ for all real values of k.	
$x^2+(k-2)x-3=0$ has two distinct real roots.	
The claim is agreed.	1A

Solution	Marks
13. (a) We have $\angle DAE = \angle BAE$ and $\angle CBE = \angle ABE$.	
$\angle DAB + \angle ABC = 180°$	
$2\angle BAE + 2\angle ABE = 180°$	
$\angle BAE + \angle ABE = 90°$	
$\angle BAE + \angle ABE + \angle AEB = 180°$	
$90° + \angle AEB = 180°$	1M
$\angle AEB = 90°$	
Thus, $\triangle ABE$ is a right-angled triangle.	1A
(b) (i) $\triangle BPE$	1A
$\angle BCE = 180° - \angle ADE$ *(int ∠s, AD//BC)*	
$\angle BPE = 180° - \angle APE$ *(adj. ∠s on st. line)*	
$= 180° - \angle ADE$ *(corr. ∠s, ≅ △s)*	
$= \angle BCE$	
$\angle CBE = \angle PBE$ *(given)*	
$BE = BE$ *(common side)*	
$\triangle BCE \cong \triangle BPE$ *(AAS)*	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) $AB = \sqrt{12^2 + 5^2}$	1M
$= 13\text{ cm}$	
We have $AD = AP$ and $BC = BP$.	
$AD + BC = AP + BP$	
$= AB$	
$= 13\text{ cm}$	1A

Solution	Marks
14. (a) $BG = \sqrt{13^2 - (29-24)^2}$	1M
$= 12\text{ cm}$	
Capacity $= \dfrac{(29+24)(12)}{2}(24)$	1M
$= 7632\text{ cm}^2$	1A
(b) (i) Base radius of the cone is 12 cm.	
Slant height $= \sqrt{12^2 + 16^2}$	1M
$= 20\text{ cm}$	
Curved surface area of the cone $= \pi(12)(20)$	1M
$= 240\pi\text{ cm}^2$	
Required area $= 240\pi\left[1 - \left(\dfrac{16-12}{16}\right)^2\right]$	1M
$= 225\pi\text{ cm}^2$	1A
(ii) Required volume $= 7632 - \left[\dfrac{1}{3}\pi(12)^2(16) - \dfrac{1}{3}\pi(3)^2(16-12)\right]$	1M
$\approx 5257\text{ cm}^3$	1A
15. (a) Required number $= C_5^8 C_2^5$	1M
$= 560$	1A
(b) Required number $= C_3^8 C_4^5 + C_2^8 C_5^5$	1M
$= 308$	1A
16. (a) $f(x) = \dfrac{x^2}{16} - \dfrac{x}{2} + 11$	
$= \dfrac{1}{16}(x^2 - 8x + 16) + 10$	1M
$= \dfrac{1}{16}(x-4)^2 + 10$	
The coordinates of vertex are $(4,\ 10)$.	1A
(b) $g(x) = -f(x) = -\dfrac{1}{16}(x-4)^2 - 10$	1M
$h(x) = -\dfrac{1}{16}(x-4)^2 + 6$	1M
y-intercept $= -\dfrac{1}{16}(0-4)^2 + 6 = 5$	1A

Solution	Marks
17. (a) $(x+2)(x-2)=8(x-1)$	
$x^2-8x+4=0$	
Thus, $p=8$ and $q=4$.	1A+1A
(b) Common ratio $=\dfrac{\log 8}{\log 4}=\dfrac{3\log 2}{2\log 2}=\dfrac{3}{2}$.	1M
$(\log 4)\left(\dfrac{3}{2}\right)^{\alpha}+(\log 4)\left(\dfrac{3}{2}\right)^{2\alpha}<\log 2^{2020}$	1M
$2(1.5)^{\alpha}+2(1.5)^{2\alpha}<2020$	
$(1.5)^{2\alpha}+1.5^{\alpha}-1010<0$	
$\dfrac{-1-\sqrt{4041}}{2}<1.5^{\alpha}<\dfrac{-1+\sqrt{4041}}{2}$	
Since $1.5^{\alpha}>0$,	
$0<1.5^{\alpha}<\dfrac{-1+\sqrt{4041}}{2}$	
$\alpha\log 1.5<\log\dfrac{-1+\sqrt{4041}}{2}$	1M
$\alpha<8.49$	
The greatest value of α is 8.	1A

Solution	Marks
18. (a) $(\sqrt{48})^2 = BC^2 + BC^2 - 2(BC)(BC)\cos 120°$	1M
$BC^2 = 16$	
$BC = 4$ cm or -4 cm (rejected)	1A
(b) (i) $\angle CBE = 60°$	
$\angle AED = 60°$	
$BE = CD = 4$ cm	
$AE = 12 - 4 = 8$ cm	
$AD^2 = 4^2 + 8^2 - 2(8)(4)\cos 60°$	1M
$AD = \sqrt{48}$ cm	
$AB = \sqrt{(\sqrt{48})^2 + (\sqrt{48})^2}$	
$= \sqrt{96}$ cm	1A
(ii) Let X be a point on AB such that $CX \perp AB$.	
$AC = \sqrt{4^2 + 48} = 8$ cm	
$AE = AC = 8$ cm	
$8^2 = 6^2 + AB^2 - 2(6)(AB)\cos\angle ABC$	
$\angle ABC \approx 52.5°$	
$EX = CE = BC\sin\angle ABC \approx 3.16$ cm	1M
Since $\angle CBE = 60°$ and $BC = DE$, we have $CE = 4$ cm.	
$4^2 = EX^2 + CX^2 - 2(EX)(CX)\cos\angle CXE$	1M
$\angle CXE \approx 78.5°$	1A
(iii) Let Y be a point on BC produced such that $DY \perp BC$.	
Required angle is $\angle AYD$.	
$DY = 4\sin 60° = 2\sqrt{3}$ cm	
$\tan\angle AYD = \dfrac{\sqrt{48}}{2\sqrt{3}}$	1M
$\angle AYD \approx 63.4°$	
$> 60°$	
The claims is agreed.	1A

Solution	Marks

19. (a) (i)

$CE = EB$ *(given)*

$OA = OB$ *(radii)*

$OE // AC$ *(mid-pt. theorem)*

$\angle BOE = \angle OAD$ *(corr. ∠s, AC//OE)*

$\angle DOE + \angle BOE = 2\angle OAD$ *(∠ at centre twice ∠ at ⊙ce)*

$\angle DOE + \angle BOE = 2\angle BOE$

$\angle DOE = \angle BOE$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(ii) $\angle DOE = \angle BOE$ *(proved)*

$OE = OE$ *(common side)*

$OD = OB$ *(radii)*

$\triangle DOE \cong \triangle BOE$ *(SAS)*

$\angle OBE = 90°$ *(tangent ⊥ radius)*

$\angle ODE = \angle OBE$ *(corr. ∠s, ≅ △s)*

$= 90°$

Thus, DE is the tangent to the circle at D. *(converse of tangent ⊥ radius)*.

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) $\angle ODE + \angle OBE = 90° + 90° = 180°$ 1M

Thus, O, B, E, D are concyclic.

Since $\angle OBE = 90°$, OE is a diameter of the circle $OBED$.

The coordinates of O are $(0,\ 8)$. 1M

Slope of $OE = \frac{8-0}{0+6} = \frac{4}{3}$

Slope of required tangent $= -\frac{3}{4}$ 1M

Required equation is

$$y - 0 = -\frac{3}{4}(x+6)$$ 1M

$3x + 4y + 18 = 0$ 1A