M2REG-VEC-2425-ASM-SET 1-MATH
Suggested solutions

1. (a) $\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR}$

$= \frac{2}{3}\mathbf{p} + \frac{3}{4}\mathbf{q}$ 1A

(b) $\overrightarrow{QR} = \overrightarrow{QC} + \overrightarrow{CR}$

$= \frac{2}{3}\mathbf{p} - \frac{1}{4}\mathbf{q}$ 1A

(c) $\overrightarrow{PS} = \overrightarrow{PA} + \overrightarrow{AS}$

$= \frac{1}{3}\mathbf{p} - \frac{1}{2}\mathbf{q}$ 1A

(d) $\overrightarrow{QP} = \overrightarrow{QB} + \overrightarrow{BP}$

$= -\frac{1}{3}\mathbf{p} - \frac{1}{2}\mathbf{q}$ 1A

2. (a) $\overrightarrow{AF} = \frac{2}{3}(\overrightarrow{AB} + \overrightarrow{BC})$

$= \frac{2}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$ 1A

(b) $\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$

$= \frac{3}{4}\mathbf{p} + \frac{2}{5}\mathbf{q}$ 1A

(c) $\overrightarrow{DF} = \overrightarrow{AF} - \overrightarrow{AD}$

$= \overrightarrow{AF} - \frac{1}{4}\overrightarrow{AB}$

$= \left(\frac{2}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}\right) - \frac{1}{4}\mathbf{p}$

$= \frac{5}{12}\mathbf{p} + \frac{2}{3}\mathbf{q}$ 1A

(d) $\overrightarrow{FE} = \overrightarrow{DE} - \overrightarrow{DF}$

$= \frac{1}{3}\mathbf{p} - \frac{4}{15}\mathbf{q}$ 1A

3. (a) $\overrightarrow{BE} = \overrightarrow{BA} + \frac{1}{2}\overrightarrow{AC} = -\mathbf{p} + \frac{1}{2}\mathbf{q}$ 1A

$\overrightarrow{CD} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} = -\mathbf{q} + \frac{1}{2}\mathbf{p}$ 1A

(b) $2\overrightarrow{BE} + \overrightarrow{CD} = -2\mathbf{p} + \frac{1}{2}\mathbf{p}$ 1M

$\mathbf{p} = -\frac{4}{3}\overrightarrow{BE} - \frac{2}{3}\overrightarrow{CD}$ 1A

$\overrightarrow{BE} + 2\overrightarrow{CD} = \frac{1}{2}\mathbf{q} - 2\mathbf{q}$

$\mathbf{q} = -\frac{2}{3}\overrightarrow{BE} - \frac{4}{3}\overrightarrow{CD}$ 1A

4. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -5\mathbf{a} + 5\mathbf{b}$ 1A

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB} = 5\mathbf{a} + \frac{3}{4}(-5\mathbf{a} + 5\mathbf{b}) = \frac{5}{4}\mathbf{a} + \frac{15}{4}\mathbf{b}$ 1A

$\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AP}$ 1M

$= \frac{1}{2}\left(\frac{3}{4}\overrightarrow{AB}\right)$

$= -\frac{15}{8}\mathbf{a} + \frac{15}{8}\mathbf{b}$ 1A

(b) $\overrightarrow{MC} = -\frac{5}{2}\mathbf{a} + k\mathbf{b}$

$$\frac{\left(-\frac{15}{8}\right)}{\left(-\frac{5}{2}\right)} = \frac{\left(\frac{15}{8}\right)}{k}$$ 1M

$k = \frac{5}{2}$ 1A

5. (a) $\overrightarrow{EF} = \frac{1}{5}\overrightarrow{AC} - \frac{1}{4}\overrightarrow{AD} = \frac{1}{5}(\mathbf{a} + \mathbf{b}) - \frac{1}{4}\mathbf{b} = \frac{1}{5}\mathbf{a} - \frac{1}{20}\mathbf{b}$ 1A

$\overrightarrow{FB} = \overrightarrow{AB} - \overrightarrow{AF} = \mathbf{a} - \frac{1}{5}(\mathbf{ab}) = \frac{4}{5}\mathbf{a} - \frac{1}{5}\mathbf{b}$ 1A

(b) $\overrightarrow{FB} = \frac{4}{5}\mathbf{a} - \frac{1}{5}\mathbf{b} = 4\left(\frac{1}{5}\mathbf{a} - \frac{1}{20}\mathbf{b}\right) = 4\overrightarrow{EF}$ 1M

Therefore, $FB // FB$ and E, F and B are collinear, and $EF : FB = 1 : 4$. 1+1A

6. (a) $\overrightarrow{SP} = \overrightarrow{SA} + \overrightarrow{AP}$

$= \frac{1}{2}\overrightarrow{DA} + \frac{1}{2}\overrightarrow{AB}$ 1M

$= \frac{1}{2}(\overrightarrow{DA} + \overrightarrow{AB})$

$2\overrightarrow{SP} = \overrightarrow{DB}$ 1

(b) $\overrightarrow{RQ} = \overrightarrow{RC} + \overrightarrow{CQ}$

$= \frac{1}{2}\overrightarrow{DC} + \frac{1}{2}\overrightarrow{CB}$ 1M

$= \frac{1}{2}(\overrightarrow{DC} + \overrightarrow{CB})$

$\overrightarrow{RQ} = \frac{1}{2}\overrightarrow{DB}$

$= \overrightarrow{SP}$

We have $RQ // SP$ and $RQ = SP$.

Thus, $PQRS$ is a parallelogram. 1

7. (a) $\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$

$= 4\mathbf{a} - 4\mathbf{b}$ 1A

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

$= \overrightarrow{AB} - \frac{1}{2}\overrightarrow{CB}$

$= 4\mathbf{a} - \frac{1}{2}(4\mathbf{a} - 4\mathbf{b})$

$= 2\mathbf{a} + 2\mathbf{b}$ 1A

$\overrightarrow{FD} = \overrightarrow{AD} - \overrightarrow{AF}$

$= \frac{1}{4}\overrightarrow{AB} - \frac{1}{4}\overrightarrow{AC}$

$= \mathbf{a} - \mathbf{b}$ 1A

(b) $\overrightarrow{AG} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DF}$

$= \frac{1}{4}(4\mathbf{a}) - \frac{1}{2}(\mathbf{a} - \mathbf{b})$

$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ 1M

$\overrightarrow{AG} = k\overrightarrow{AE}$

$\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = 2k\mathbf{a} + 2k\mathbf{b}$ 1M

$k = \frac{1}{4}$ 1A

8. (a) $3\mathbf{x} = \mathbf{y}$

$6\mathbf{a} + 3n\mathbf{b} = 2k\mathbf{a} + 3\mathbf{b}$

Therefore, $\begin{cases} 6 = 2k \\ 3n = 3 \end{cases}$. 1M

Thus, $k = 3$ and $n = 1$. 1A

(b) $3\mathbf{x} = 5\mathbf{y}$

$3k\mathbf{a} + 3n\mathbf{b} = 15\mathbf{a} + \frac{30}{k}\mathbf{b}$

Therefore, $\begin{cases} 3k = 15 \\ 3n = \frac{30}{k} \end{cases}$. 1M

Thus, $k = 5$ and $n = 2$. 1A

9. $\mathbf{x} = 4\mathbf{y}$

$k^2\mathbf{a} + n^2\mathbf{b} = (4k - 4)\mathbf{a} - (8n + 16)\mathbf{b}$

$k^2 = 4k - 4$ and $n^2 = -(8n + 16)$ 1M

$k^2 - 4k + 4 = 0$ $\quad$ $n^2 + 8n + 16 = 0$

$k = 2$ $\quad$ $n = -4$ 1A+1A

10. (a) $\overrightarrow{AQ} = \frac{1}{5}\overrightarrow{AC} = \frac{1}{5}(\overrightarrow{AB} + \overrightarrow{AD}) = \frac{1}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$ 1A

$\overrightarrow{DQ} = \overrightarrow{AQ} - \overrightarrow{AD} = \frac{1}{5}\mathbf{a} - \frac{4}{5}\mathbf{b}$ 1A

$\overrightarrow{DP} = \overrightarrow{DA} + \frac{1}{4}\overrightarrow{AB} = \frac{1}{4}\mathbf{a} - \mathbf{b}$ 1A

(b) $\overrightarrow{DQ} = r\overrightarrow{DP}$

$\frac{1}{5}\mathbf{a} - \frac{4}{5}\mathbf{b} = \frac{r}{4}\mathbf{a} - r\mathbf{b}$

Therefore, $\begin{cases} \frac{1}{5} = \frac{r}{4} \\ -\frac{4}{5} = -r \end{cases}$. 1M

Thus, $r = \frac{4}{5}$. 1A

11. (a) $\overrightarrow{OP} = h\left(\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = h\mathbf{a} + \frac{h}{2}\mathbf{b}$ 1A

(b) (i) $\overrightarrow{MP} = k\overrightarrow{MA} = k\left(\frac{1}{2}\mathbf{a} - \mathbf{b}\right) = \frac{k}{2}\mathbf{a} - k\mathbf{b}$ 1A

(ii) $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \frac{1}{2}\mathbf{a} + \mathbf{b} + \frac{k}{2}\mathbf{a} - k\mathbf{b} = \frac{1+k}{2}\mathbf{a} + (1-k)\mathbf{b}$ 1A

(c) By (a) and (b), $\begin{cases} h = \frac{1+k}{2} \\ \frac{h}{2} = 1 - k \end{cases}$. 1M

Solving, we have $h = \frac{4}{5}$ and $k = \frac{3}{5}$.

Therefore, $h : k = 4 : 3$. 1A

 END OF PAPER