M2REG-SPVP-2425-ASM-SET 1-MATH
Suggested solutions

1. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ -2 & -1 & -3 \end{vmatrix}$

$= 5\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}$ 1A

2. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & -3 \\ 4 & 3 & 1 \end{vmatrix}$

$= 11\mathbf{i} - 11\mathbf{j} - 11\mathbf{k}$ 1A

3. $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{a}+\mathbf{b}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} + (\mathbf{a}\times\mathbf{b})\cdot\mathbf{b}$ 1M

$= 0 + 0$

$= 0$ 1

4. $(\mathbf{a}+\mathbf{c})\times\mathbf{b} = (\mathbf{a}+\mathbf{c})\times(4\mathbf{a}-5\mathbf{c})$

$= -5\mathbf{a}\times\mathbf{c} + 4\mathbf{c}\times\mathbf{a}$

$= 5(\mathbf{c}\times\mathbf{a}) + 4(\mathbf{c}\times\mathbf{a})$ 1M

$= 9(\mathbf{c}\times\mathbf{a})$ 1

5. (a) $\mathbf{a}$ is parallel to $\mathbf{p}\times\mathbf{q}$.

We have $\dfrac{\lambda}{19} = \dfrac{-48}{12} = \dfrac{\mu}{2}$. 1M

Thus, $\lambda = -76$ and $\mu = -8$. 1A+1A

(b) $|\mathbf{p}\times\mathbf{q}| = \sqrt{76^2 + 48^2 + 8^2}$

$= \sqrt{8144}$ 1A

Let the required angle be θ.

$$\frac{|\mathbf{p}\times\mathbf{q}|}{\mathbf{p}\cdot\mathbf{q}} = \frac{\sqrt{8144}}{1}$$

$$\frac{|\mathbf{p}||\mathbf{q}|\sin\theta}{|\mathbf{p}||\mathbf{q}|\cos\theta} = \sqrt{8144}$$ 1M

$$\tan\theta = \sqrt{8144}$$

$$\theta \approx 89.4^\circ$$

Required angle is 89.4°. 1A

6. (a) $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 2 \\ 2 & -1 & n \end{vmatrix}$ 1M

$= (3n+2)\mathbf{i} + (-n+4)\mathbf{j} - 7\mathbf{k}$

Area of $\triangle ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{1}{2}\sqrt{(3n+2)^2 + (-n+4)^2 + 7^2}$ 1M

$= \frac{1}{2}\sqrt{10n^2 + 4n + 69}$ 1A

(b) Area of $\triangle ABC = \frac{1}{2}\sqrt{10n^2 + 4n + 69}$

$= \frac{1}{2}\sqrt{10\left[n^2 + 2(n)\left(\frac{1}{5}\right) + \left(\frac{1}{5}\right)^2\right] + \frac{343}{5}}$ 1M

$= \frac{1}{2}\sqrt{10\left(n + \frac{1}{5}\right)^2 + \frac{343}{5}}$ 1A

$\geq \frac{1}{2}\sqrt{\frac{343}{5}}$

> 0

A, B and C are not collinear for all real values of n. 1

7. (a) $\overrightarrow{AB} = -4\mathbf{i} + \mathbf{j} - \mathbf{k}$

$\overrightarrow{AC} = \mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & -1 \\ 1 & -4 & -2 \end{vmatrix}$

$= -6\mathbf{i} - 9\mathbf{j} + 15\mathbf{k}$ 1A

Required area $= \frac{1}{2}\sqrt{6^2 + 9^2 + 15^2}$

$= \frac{3\sqrt{38}}{2}$ 1A

(b) $AC = \sqrt{1^2 + 4^2 + 3^2}$

$= \sqrt{21}$

Let h be the required altitude.

$\frac{3\sqrt{38}}{2} = \frac{1}{2}(h)(AC)$ 1M

$h = \frac{\sqrt{798}}{7}$ 1A

Required altitude is $\frac{\sqrt{798}}{7}$.

8. (a) $\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + (t-2)\mathbf{k}$

$\overrightarrow{AD} = \mathbf{i} - \mathbf{j} - \mathbf{k}$

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & t-2 \\ 1 & -1 & -1 \end{vmatrix}$$

$= (t+1)\mathbf{i} + t\mathbf{j} + \mathbf{k}$ 1A

$|\overrightarrow{AB} \times \overrightarrow{AD}| = \sqrt{26}$

$\sqrt{(t+1)^2 + t^2 + 1^2} = \sqrt{26}$ 1A

$2t^2 + 2t + 2 = 26$

$t^2 + t - 12 = 0$

$t = 3$ or -4 (rejected) 1A

(b) Let (x, y, z) be the coordinates of C.

$\overrightarrow{AB} = \overrightarrow{DC}$

$2\mathbf{i} - 3\mathbf{j} + \mathbf{k} = (x+2)\mathbf{i} + (y-4)\mathbf{j} + (z-1)\mathbf{k}$ 1M

Solving, we have $x = 0$, $y = 1$ and $z = 2$.

The coordinates of C are $(0, 1, 2)$. 1A

9. (a) $104 = |\mathbf{m} - 2\mathbf{n}|^2$

$= (\mathbf{m} - 2\mathbf{n}) \cdot (\mathbf{m} - 2\mathbf{n})$ 1M

$= |\mathbf{m}|^2 - 4\mathbf{m} \cdot \mathbf{n} + 4|\mathbf{n}|^2$

$= |\mathbf{m}|^2 + 4|\mathbf{n}|^2$ 1M

$41 = (2\mathbf{m} + \mathbf{n}) \cdot (2\mathbf{m} + \mathbf{n})$

$= 4|\mathbf{m}|^2 + 4\mathbf{m} \cdot \mathbf{n} + |\mathbf{n}|^2$

$= 4|\mathbf{m}|^2 + |\mathbf{n}|^2$ 1A

Solving, we have $|\mathbf{m}| = 2$ and $|\mathbf{n}| = 5$. 1A+1A

(b) $\mathbf{p} \times \mathbf{q} = (4\mathbf{m} + \mathbf{n}) \times (\mathbf{m} - 2\mathbf{n})$

$= -8\mathbf{m} \times \mathbf{n} + \mathbf{n} \times \mathbf{m}$

$= 9\mathbf{n} \times \mathbf{m}$ 1M

Required area $= |\mathbf{p} \times \mathbf{q}|$

$= 9|\mathbf{n} \times \mathbf{m}|$

$= 9(5)(2)$ 1M

$= 90$ 1A

 END OF PAPER