1. B

$8 + x > 3x - 7$ and $\frac{3-x}{2} \geq 4$

$-2x > -15$ $\qquad -\frac{x}{2} \geq \frac{5}{2}$

$x < \frac{15}{2}$ $\qquad x \leq -5$

Thus, $x < \frac{15}{2}$.

2. C

$2(1-x) > 6x$ and $x \leq \frac{4x+1}{-2}$

$-8x > -2$ $\qquad 3x \leq -\frac{1}{2}$

$x < \frac{1}{4}$ $\qquad x \leq -\frac{1}{6}$

Thus, $x \leq -\frac{1}{6}$.

The greatest value of x is -1.

3. D

$\frac{6-x}{2} \leq x - 3$ or $9 - 2x \geq 1$

$-\frac{3x}{2} \leq -6$ $\qquad -2x \geq -8$

$x \geq 4$ $\qquad x \leq 4$

Thus, x can be any real number.

4. A

$-3x - 2 > \frac{x+10}{2}$ or $-9 - 2x > -1$

$-\frac{7x}{2} > 7$ $\qquad -2x > 8$

$x < -2$ $\qquad x < -4$

Thus, $x < -2$.

5. B

$2x + 9 < 1$ or $1 - \frac{x}{2} \geq 0$

$2x < -8$ $\qquad -\frac{x}{2} \geq -1$

$x < -4$ $\qquad x \leq 2$

Thus, $x \leq 2$.

6. C

$2(x+1)+5 \le 3$ or $\frac{4x-1}{11} < 1$

$x \le -2$ $\quad$ $x < 3$

Thus, we have $x < 3$.

The greatest integer is 2.

7. B

$4-x < 2-3x$ or $x+3 > 2x-5$

$2x < -2$ $\quad$ $-x > -8$

$x < -1$ $\quad$ $x < 8$

Thus, $x < 8$.

8. A

$\frac{5y+3}{2} \le 3y+2$ and $3y+2 < 2y+5$

$-\frac{y}{2} \le \frac{1}{2}$ $\quad$ $y < 3$

$y \ge -1$

Thus, $-1 \le y < 3$.

9. B

$\frac{x+1}{2}+\frac{x}{3} \ge 8$ and $2x+3 < 4x-5$

$\frac{5x}{6} \ge \frac{15}{2}$ $\quad$ $-2x < -8$

$x \ge 9$ $\quad$ $x > 4$

Thus, $x \ge 9$.

10. C

$-3(4-x) \le 9$ or $\frac{7x+2}{5} < -8$

$3x \le 21$ $\quad$ $\frac{7x}{5} < -\frac{42}{5}$

$x \le 7$ $\quad$ $x < -6$

Thus, $x \le 7$.

The greatest integer is 7.

11. A

$-5x < \frac{2}{3}$ and $\frac{2}{3} < 4x$

$x > -\frac{2}{15}$ $\quad$ $x > \frac{1}{6}$

Thus, $x > \frac{1}{6}$.

12. B

$-\frac{4}{3}(x-5) \ge 8$ or $2x-1 \le -5$

$x \le -1$ $\quad$ $x \le -2$

Thus, we have $x \le -1$.

13. A

$-4x < 6 - x$ and $5(x+1) > 17 + x$

$-3x < 6$ $\quad$ $4x > 12$

$x > -2$ $\quad$ $x > 3$

Thus, $x > 3$.

14. D

$-5 - 3x > 1$ and $\frac{2x}{3} - 1 > 3$

$x < -2$ $\quad$ $x > 6$

No solution.

15. B

$2 - 2x \ge \frac{4-x}{3}$ or $\frac{x}{2} + \frac{1}{3} < \frac{1}{6}$

$x \le \frac{2}{5}$ $\quad$ $x < -\frac{1}{3}$

Thus, $x \le \frac{2}{5}$.

16. C

$5x - 11 < 9$ or $4 - 3x > 7$

$5x < 20$ $\quad$ $-3x > 3$

$x < 4$ $\quad$ $x < -1$

Thus, $x < 4$.

17. C

The graph of $y = f(x)$ lies below the x-axis.

$$\Delta = (2k)^2 - 4(-1)(-4) < 0$$
$$4k^2 - 16 < 0$$
$$-2 < k < 2$$

The greatest integral value of k is 1.

18. A

The equation $2x^2 + 2kx + k + 12 = 0$ has at most one real root.

$$\begin{aligned} \Delta = (2k)^2 - 4(2)(k+12) &\le 0 \\ 4k^2 - 8k - 96 &\le 0 \\ -4 \le k &\le 6 \end{aligned}$$

19. A

$$\begin{aligned} \pi^{2x} - 9\pi^x + 20 &< 2 \\ (\pi^x)^2 - 9\pi^x + 18 &< 0 \\ 3 < \pi^x &< 6 \\ \log 3 < x \log \pi &< \log 6 \\ \frac{\log 3}{\log \pi} < x &< \frac{\log 6}{\log \pi} \\ \log_\pi 3 < x &< \log_\pi 6 \end{aligned}$$

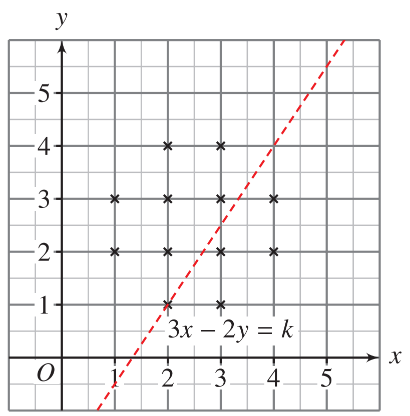
20. A

The graph $y = -x^2 - 2cx + c - 20$ lies on or below the x-axis.
The equation $-x^2 - 2cx + c - 20 = 0$ has repeated real roots or no real roots.

$$\begin{aligned} \Delta = (-2c)^2 - 4(-1)(c-20) &\le 0 \\ 4c^2 + 4c - 80 &\le 0 \\ -5 \le c &\le 4 \end{aligned}$$

21. B

Draw the line $3x - 2y = k$, where k is a constant.



$3x - 2y$ attains its minimum at the point $(1,\ 3)$.

Required value $= 3(1) - 2(3) = -3$

22. B

The value of $18 + x - 4y$ is smaller when x is smaller and y is larger.

The minimum value of $18 + x - 4y$ is attained at the top left corner, which is P or S.

The coordinates of P and S are $(3,\ 6)$ and $(1,\ 5)$ respectively.

	$P\,(3,\ 6)$	$S\,(1,\ 5)$
$18 + x - 4y$	-3	-1

The minimum value is -3.

23. D

Check if the point satisfies each of the inequalities.

Point	$(1,\ 1)$	$(4,\ 6)$	$(7,\ 0)$
$x \geq 0$	✓	✓	✓
$y \geq 0$	✓	✓	✓
$x - y \geq -2$	✓	✓	✓
$3x + 2y \leq 24$	✓	✓	✓

All points lie in D.

24. D

Maximum value of $3x - 2y + 15$ occurs at the bottom right corners, $B\,(3,\ 3)$ or $C\,(2,\ 0)$.

$(x,\ y)$	$B\,(3,\ 3)$	$C\,(2,\ 0)$
$3x - 2y + 15$	18	21

Maximum value = 21

25. D

Label the inequalities as follows:

① $y \leq 7$

② $7x + 16y - 70 \geq 0$

③ $7x + 9y - 70 \leq 0$

Lines	Coordinates	Check	$14x + ky$
① and ②	$(-6,\ 7)$	③ ✓	$-84 + 7k$
① and ③	$(1,\ 7)$	② ✓	$14 + 7k$
② and ③	$(10,\ 0)$	① ✓	140

Since the least value of $14x + ky$ is 140, we have $14 + 7k > -84 + 7k \geq 140$.

$$-84 + 7k \geq 140$$
$$k \geq 32$$

26. B

Compute the intercepts of all corresponding straight lines.

Line	x-intercept	y-intercept
$x + y = 4$	4	4
$3x + 2y = 6$	2	3
$x = y$	0	0

$x + y \leq 4$: on the left of the straight line $x + y = 4$
$3x + 2y \geq 6$: on the right of the straight line $3x + 2y = 6$
$x \leq y$: on the left of the straight line $x = y$
$x \geq 0$: on the right of the y-axis
$y \geq 0$: above the x-axis
The answer is B.

27. B

Label the inequalities as follows:

① $x + y - 4 \leq 0$

② $x + 3y - 6 \geq 0$

③ $x \geq 0$

Lines	Coordinates	Check	$mx + y + 1$
① and ②	$(3,\ 1)$	③ ✓	$3m + 2$
① and ③	$(0,\ 4)$	② ✓	5
② and ③	$(0,\ 2)$	① ✓	3

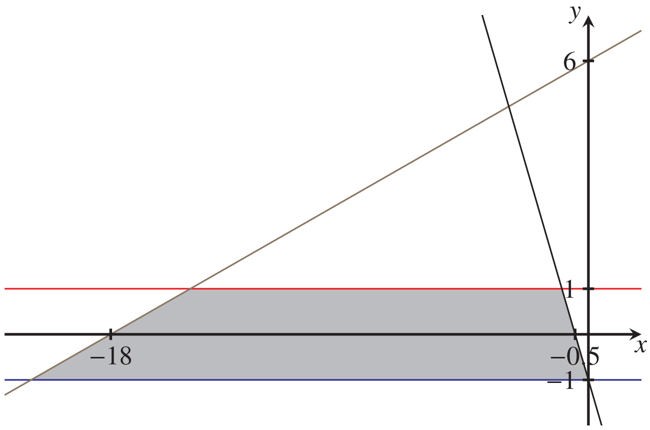
Since $mx + y + 1$ attains its maximum value at $(3,\ 1)$ only, we have $3 < 5 < 3m + 2$.

$$5 < 3m + 2$$
$$m > 1$$

28. A

Line	x-intercept	y-intercept
$x - 3y + 18 = 0$	-18	6
$2x + y + 1 = 0$	-0.5	-1
$y = -1$		-1
$y = 1$		1

Sketch the solution region using the intercepts.



The value of $5x - 2y + k$ is larger when x is larger and y is smaller.
$5x - 2y + k$ attains its maximum value at the bottom right corner, which is $(0,\ -1)$.

$$5(0) - 2(-1) + k = 12$$
$$k = 10$$

29. D

Label the inequalities as follows:

① $y \le 2$

② $2x + 5y - 10 \ge 0$

③ $x + y - 5 \le 0$

Lines	Coordinates	Check	$\alpha x + 4y$
① and ②	$(4,\ 2)$	③ ✓	$4\alpha + 8$
① and ③	$(3,\ 2)$	② ✓	$3\alpha + 8$
② and ③	$(5,\ 0)$	① ✓	5α

The least value of $\alpha x + 4y$ is 8.

$$4\alpha + 8 \geq 8 \quad \text{and} \quad 3\alpha + 8 \geq 8 \quad \text{and} \quad 5\alpha \geq 8$$
$$\alpha \geq 0 \qquad\qquad \alpha \geq 0 \qquad\qquad \alpha \geq \frac{8}{5}$$

Thus, $\alpha \geq \dfrac{8}{5}$.

Since one of the value is equal to 8, we have $\alpha = \dfrac{8}{5}$.

30. B

Label the inequalities as follows:

① $y - 8 \leq 0$

② $4x - y - 20 \leq 0$

③ $8x + 9y - 40 \geq 0$

Lines	Coordinates	Check	$-5x + \alpha y$
① and ②	$(7,\ 8)$	③ ✓	$-35 + 8\alpha$
① and ③	$(-4,\ 8)$	② ✓	$20 + 8\alpha$
② and ③	$(5,\ 0)$	① ✓	-25

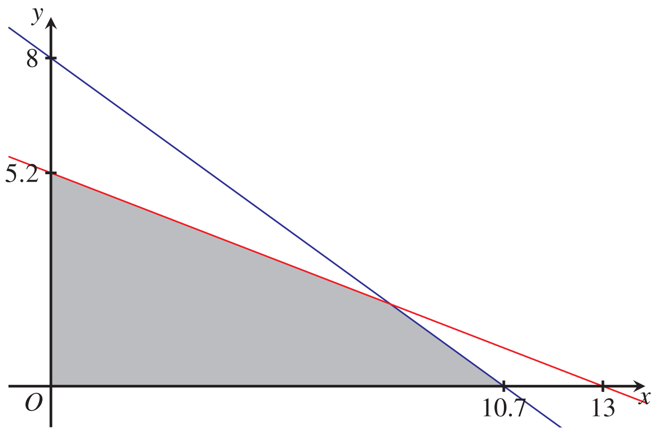
Since the least value of $-5x + \alpha y$ is -25, we have $20 + 8\alpha > -35 + 8\alpha \geq -25$.

$$-35 + 8\alpha \geq -25$$
$$\alpha \geq 1.25$$

31. A

Line	x-intercept	y-intercept
$3x + 4y - 32 = 0$	10.7	8
$2x + 5y - 26 = 0$	13	5.2
$x = 0$	0	
$y = 0$		0

Sketch the solution region using the intercepts.



The value of $7x + 12y + k$ is larger when x is larger and y is larger.

$7x + 12y + k$ attains its maximum value at the top right corner, which is $\left(0,\ \frac{26}{5}\right)$, $(8,\ 2)$ or $\left(\frac{32}{3},\ 0\right)$.

$(x,\ y)$	$\left(0,\ \frac{26}{5}\right)$	$(8,\ 2)$	$\left(\frac{32}{3},\ 0\right)$
$7x + 12y + k$	$\frac{312}{5} + k$	$80 + k$	$\frac{224}{3} + k$

The greatest value is $80 + k$.

$$80 + k = 55$$
$$k = -25$$

32. B

The corresponding system of inequalities is

$$\begin{cases} x \geq y + 1 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$x \geq y + 1$: on the right of the straight line $y = x - 1$

$x \geq 0$: on the right of the y-axis

$y \geq 0$: above the x-axis

The answer is B.

33. B

Label the inequalities as follows:

① $x \geq -4$

② $x - 2y + 10 \leq 0$

③ $y \leq 5 - x$

Lines	Coordinates	Check	$x + 2y$
① and ②	$(-4,\ 3)$	③ ✓	2
① and ③	$(-4,\ 9)$	② ✓	14
② and ③	$(0,\ 5)$	① ✓	10

The minimum value of $x + 2y$ is 2.
The maximum value of k is 2.

34. A

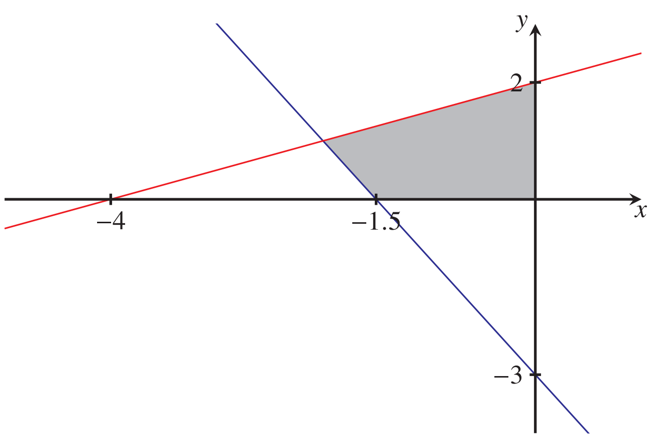
The equations of the three boundaries are $x = -1$, $y = 2$ and $x + y = 4$.
We have $x \geq -1$, $y \geq 2$ and $x + y \leq 4$.

Thus, we have $\begin{cases} x + 1 \geq 0 \\ y - 2 \geq 0 \\ x + y - 4 \leq 0 \end{cases}$.

35. B

Line	x-intercept	y-intercept
$x = 0$	0	
$y = 0$		0
$2x + y + 3 = 0$	-1.5	-3
$x - 2y + 4 = 0$	-4	2

Sketch the solution region using the intercepts.



The value of $3x - 2y$ is smaller when x is smaller and y is larger.

$3x - 2y$ attains its least value at the top left corner, which is $(-2,\ 1)$ or $(0,\ 2)$.

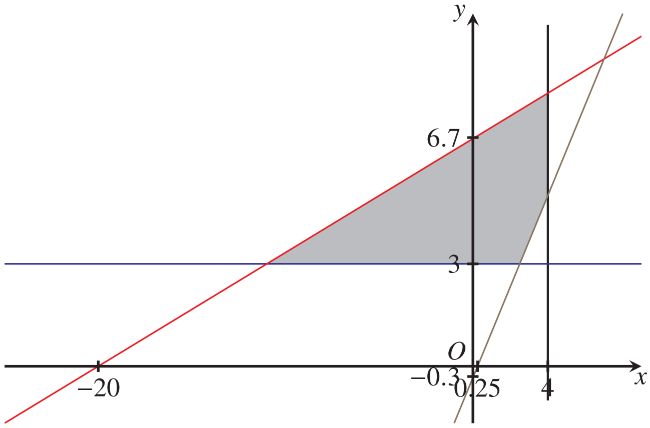
$(x,\ y)$	$(-2,\ 1)$	$(0,\ 2)$
$3x - 2y$	-8	-4

The least value is -8.

36. C

Line	x-intercept	y-intercept
$x = 4$	4	
$y = 3$		3
$x - 3y + 20 = 0$	-20	6.7
$4x - 3y - 1 = 0$	0.25	-0.3

Sketch the solution region using the intercepts.



The value of $4y - 6x + 12$ is smaller when x is larger and y is smaller.

$4y - 6x + 12$ attains its least value at the bottom right corner, which is $(2.5,\ 3)$ or $(4,\ 5)$.

$(x,\ y)$	$(2.5,\ 3)$	$(4,\ 5)$
$4y - 6x + 12$	9	8

The least value is 8.

37. C

The value of $4x + 3y$ is larger when x and y are larger.

The maximum value is attained at the top right corner, which is $(2,\ 2)$ or $(0,\ 4)$.

	(2, 2)	(0, 4)
$4x + 3y$	14	12

The greatest value is 14.

38. C

Label the inequalities as follows:

① $2x - y \le 0$

② $4x - y \ge 0$

③ $4x + y \le 24$

Lines	Coordinates	Check	$y - 3x + 10$
① and ②	(0, 0)	③ ✓	10
① and ③	(4, 8)	② ✓	6
② and ③	(3, 12)	① ✓	13

The greatest value is 13.

39. C

Label the inequalities as follows:

① $x + 2y \le 22$

② $4x - 3y \le 22$

③ $7x + 3y \ge 22$

Lines	Coordinates	Check	$4x + 3y - k$
① and ②	(10, 6)	③ ✓	$58 - k$
① and ③	(−2, 12)	② ✓	$28 - k$
② and ③	(4, −2)	① ✓	$10 - k$

The greatest value is $58 - k$.

$$58 - k = 5$$
$$k = 53$$

40. D

Label the inequalities as follows:

① $3x + 11 \geq 4y$

② $5x + 3y - 30 \leq 0$

③ $2x + 7y - 12 \geq 0$

Lines	Coordinates	Check	$8x + 9y$
① and ②	$(3,\ 5)$	③ ✓	69
① and ③	$(-1,\ 2)$	② ✓	10
② and ③	$(6,\ 0)$	① ✓	48

The greatest value is 69.

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