

M2REG-LIM-2425-ASM-SET 3-MATH

建議題解

$$1. \lim_{x \rightarrow 0} \frac{\sin x + \sin 3x}{x} = \lim_{x \rightarrow 0} \frac{2 \sin 2x \cos x}{x} \quad 1M$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \cdot 4 \cos x \right) \quad 1M$$

$$= 1 \cdot 4$$

$$= 4$$

1A

$$2. \lim_{x \rightarrow 0} \frac{\sin 3x + \sin 2x}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{3x} \cdot 3 + \frac{\sin 2x}{2x} \cdot 2 \right) \quad 1M$$

$$= 1(3) + 1(2)$$

$$= 5$$

1A

$$3. \lim_{x \rightarrow 0} \frac{4x \cot 2x}{\cos x} = \lim_{x \rightarrow 0} \frac{4x \cos 2x}{\cos x \sin 2x} \quad 1M$$

$$= \lim_{x \rightarrow 0} \left(\frac{2x}{\sin 2x} \cdot \frac{2 \cos 2x}{\cos x} \right)$$

$$= 1 \cdot 2$$

$$= 2$$

1A

$$4. \lim_{x \rightarrow 0} \frac{7x^2}{1 - \cos 7x} = \lim_{x \rightarrow 0} \left(\frac{7x^2}{1 - \cos 7x} \cdot \frac{1 + \cos 7x}{1 + \cos 7x} \right) \quad 1M$$

$$= \lim_{x \rightarrow 0} \frac{7x^2(1 + \cos 7x)}{1 - \cos^2 7x}$$

$$= \lim_{x \rightarrow 0} \frac{7x^2(1 + \cos 7x)}{\sin^2 7x}$$

$$= \lim_{x \rightarrow 0} \left[\left(\frac{7x}{\sin 7x} \right)^2 \cdot \frac{1 + \cos 7x}{7} \right] \quad 1M$$

$$= 1^2 \cdot \frac{1 + 1}{7}$$

$$= \frac{2}{7}$$

1A

$$5. \lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{x^2} = \lim_{x \rightarrow 0} \frac{-2 \sin 4x \sin x}{x^2} \quad 1M$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \cdot \frac{\sin x}{x} \cdot (-2)(4) \right) \quad 1M$$

$$= (1)(1)(-8)$$

$$= -8$$

1A

$$6. \lim_{x \rightarrow 0} \frac{1 - \cos x}{1 - \cos 5x} = \lim_{x \rightarrow 0} \frac{1 - \left(1 - 2 \sin^2 \frac{x}{2}\right)}{1 - \left(1 - 2 \sin^2 \frac{5x}{2}\right)} \quad 1M$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{\sin^2 \frac{5x}{2}} \quad 1M$$

$$= \lim_{x \rightarrow 0} \left[\frac{\sin \frac{x}{2}}{\left(\frac{x}{2}\right)} \right]^2 \left[\frac{\left(\frac{5x}{2}\right)}{\sin \frac{5x}{2}} \right]^2 \cdot \frac{1}{25}$$

$$= 1^2 \cdot 1^2 \cdot \frac{1}{25} \quad 1A$$

$$= \frac{1}{25}$$

$$7. \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x \sin 3x} = \lim_{x \rightarrow 0} \frac{1 - \left(1 - 2 \sin^2 \frac{3x}{2}\right)}{x \sin 3x} \quad 1M$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{3x}{2}}{x \sin 3x}$$

$$= \lim_{x \rightarrow 0} \left\{ \left[\frac{\sin \frac{3x}{2}}{\left(\frac{3x}{2}\right)} \right]^2 \cdot \frac{3x}{\sin 3x} \cdot \frac{3}{2} \right\} \quad 1M$$

$$= 1^2 \cdot 1 \cdot \frac{3}{2} \quad 1A$$

$$= \frac{3}{2}$$

$$8. \quad (a) \lim_{u \rightarrow 0} \frac{u \sin u}{1 - \cos 3u} = \lim_{u \rightarrow 0} \frac{u \sin u}{1 - \left(1 - 2 \sin^2 \frac{3u}{2}\right)} \quad 1M$$

$$= \lim_{u \rightarrow 0} \frac{u \sin u}{2 \sin^2 \frac{3u}{2}}$$

$$= \lim_{u \rightarrow 0} \frac{\left(\frac{3u}{2}\right)^2}{\sin^2 \frac{3u}{2}} \cdot \frac{\sin u}{u} \cdot \frac{2}{9} \quad 1M$$

$$= \frac{2}{9} \quad 1A$$

(b) 設 $u = \frac{\pi}{2} - x$.

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\left(x - \frac{\pi}{2}\right) \cos x}{1 + \sin 3x} = \lim_{u \rightarrow 0} \frac{-u \cos \left(\frac{\pi}{2} - u\right)}{1 + \sin \left(\frac{3\pi}{2} - 3u\right)} \quad 1M$$

$$= \lim_{u \rightarrow 0} \frac{-u \sin u}{1 - \cos 3u}$$

$$= -\frac{2}{9} \quad 1A$$

9. Let $y = \frac{1}{x}$, then $x = \frac{1}{y}$.

$$\lim_{x \rightarrow \infty} (2x^2 + x - 5) \sin^2 \frac{3}{x} = \lim_{y \rightarrow 0} \left(\frac{2}{y^2} + \frac{1}{y} - 5 \right) \sin^2 3y \quad 1M$$

$$= \lim_{y \rightarrow 0} \left(\frac{2 \sin^2 3y}{y^2} + \frac{\sin^2 3y}{y} - 5 \sin^2 3y \right) \quad 1M$$

$$= \lim_{y \rightarrow 0} \left[\left(\frac{\sin 3y}{3y} \right)^2 \cdot 18 + \frac{\sin 3y}{3y} \cdot 3 \sin 3y - 5 \sin^2 3y \right] \quad 1A$$

$$= 1(18) + 1(0) - 5(0)$$

$$= 18$$

$$10. \lim_{x \rightarrow 0} \frac{x \left[1 + \tan \left(\frac{\pi}{3} + x \right) \right]}{\sqrt{3} - \tan \left(\frac{\pi}{3} + x \right)} \quad 1M$$

$$= \lim_{x \rightarrow 0} \frac{x \left(1 + \frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \tan x} \right)}{\sqrt{3} - \frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \tan x}} \quad 1M$$

$$= \lim_{x \rightarrow 0} \left[x \left(\frac{(1 - \sqrt{3}) \tan x + (1 + \sqrt{3})}{1 - \sqrt{3} \tan x} \right) \right] \div \left(\frac{(-3 - 1) \tan x + (\sqrt{3} - \sqrt{3})}{1 - \sqrt{3} \tan x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{x[(1 - \sqrt{3}) \tan x + (1 + \sqrt{3})]}{-4 \tan x} \quad 1M$$

$$= \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \cdot \frac{\cos x[(1 - \sqrt{3}) \tan x + (1 + \sqrt{3})]}{-4} \right)$$

$$= 1 \cdot \frac{1[0 + (1 + \sqrt{3})]}{-4} \quad 1A$$

$$= -\frac{1 + \sqrt{3}}{4}$$

$$11. (a) \lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{x^2} = \lim_{x \rightarrow \infty} \frac{3 + \frac{2}{x}}{1} \quad 1M$$

$$= 3 \quad 1A$$

$$(b) \lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x + 2}{x^2} \right)^{2x} = \lim_{x \rightarrow \infty} \left(1 + \frac{3x + 2}{x^2} \right)^{2x} \quad 1M$$

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{3x + 2}{x^2} \right)^{\frac{x^2}{3x+2} \cdot \frac{2(3x^2+2x)}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{3x + 2}{x^2} \right)^{\frac{x^2}{3x+2} \cdot \left(6 + \frac{4}{x} \right)} \quad 1M$$

$$= e^6 \quad 1A$$

$$\begin{aligned}
 12. \quad \lim_{x \rightarrow 0} (1 + 5x + 6x^2)^{\frac{1}{x}} &= \lim_{x \rightarrow 0} \left[(1 + 2x)^{\frac{1}{x}} (1 + 3x)^{\frac{1}{x}} \right] \\
 &= \lim_{x \rightarrow 0} \left\{ \left[(1 + 2x)^{\frac{1}{2x}} \right]^2 \left[(1 + 3x)^{\frac{1}{3x}} \right]^3 \right\} & 1M \\
 &= e^2 \cdot e^3 \\
 &= e^5 & 1A
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{xe^{2x}} &= \lim_{x \rightarrow 0} \left(\frac{e^{2x} - 1}{2x} \cdot \frac{2}{e^{2x}} \right) & 1M \\
 &= 1 \cdot \frac{2}{1} \\
 &= 2 & 1A
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x(e^x + 1)} &= \lim_{x \rightarrow 0} \left(\frac{e^{2x} - 1}{2x} \cdot \frac{2}{e^x + 1} \right) & 1M \\
 &= 1 \cdot \frac{2}{e^0 + 1} \\
 &= 1 & 1A
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \lim_{x \rightarrow 0} \frac{e^{2x} + e^{-2x} - 2}{x \sin 4x} &= \lim_{x \rightarrow 0} \frac{e^{4x} + 1 - 2e^{2x}}{xe^{2x} \sin 4x} \\
 &= \lim_{x \rightarrow 0} \frac{(e^{2x} - 1)^2}{xe^{2x} \sin 4x} & 1M \\
 &= \lim_{x \rightarrow 0} \left[\left(\frac{e^{2x} - 1}{2x} \right)^2 \cdot \frac{4x}{\sin 4x} \cdot \frac{1}{e^{2x}} \right] & 1M \\
 &= 1^2 \cdot 1 \cdot \frac{1}{e^0} \\
 &= 1 & 1A
 \end{aligned}$$