M2REG-LIM-2425-ASM-SET 1-MATH
建議題解

1. $\lim_{x\to 4}\frac{x^2-3x-4}{x^2-6x+8}=\lim_{x\to 4}\frac{(x-4)(x+1)}{(x-4)(x-2)}$ 1M

$=\lim_{x\to 4}\frac{x+1}{x-2}$ 1M

$=\frac{4+1}{4-2}$

$=\frac{5}{2}$ 1A

2. $\lim_{x\to 3}\frac{x^3-27}{x^3-x^2-5x-3}=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{(x-3)(x^2+2x+1)}$ 1M

$=\lim_{x\to 3}\frac{x^2+3x+9}{x^2+2x+1}$ 1M

$=\frac{3^2+3(3)+9}{3^2+2(3)+1}$

$=\frac{27}{16}$ 1A

3. $\lim_{x\to 0}\frac{8^x-1}{2^x-1}=\lim_{x\to 0}\frac{(2^x-1)(2^{2x}+2^x+1)}{2^x-1}$ 1M

$=\lim_{x\to 0}(2^{2x}+2^x+1)$ 1M

$=2^0+2^0+1$

$=3$ 1A

4. $\lim_{h\to -2}\frac{(x-2h^2)^2-(x+4h)^2}{h+2}=\lim_{h\to -2}\frac{[(x-2h^2)+(x+4h)][(x-2h^2)-(x+4h)]}{h+2}$ 1M

$=\lim_{h\to -2}\frac{-2h(2x+4h-2h^2)(h+2)}{h+2}$

$=\lim_{h\to -2}-2h(2x+4h-2h^2)$ 1M

$=-2(-2)[2x+4(-2)-2(-2)^2]$

$=8x-64$ 1A

5. $\lim_{x\to h}\dfrac{(x+h)^3+(x-3h)^3}{x-h}=\lim_{x\to h}\dfrac{(2x-2h)[(x+h)^2-(x+h)(x-3h)+(x-3h)^2]}{x-h}$ 1M

$=2\lim_{x\to h}[(x+h)^2-(x+h)(x-3h)+(x-3h)^2]$ 1M

$=2(4h^2+4h^2+4h^2)$

$=24h^2$ 1A

$\lim_{x\to 2h}\dfrac{\frac{1}{x+h}-\frac{1}{4x-5h}}{x-2h}=\lim_{x\to 2h}\dfrac{(4x-5h)-(x+h)}{(x-2h)(x+h)(4x-5h)}$ 1M

$=\lim_{x\to 2h}\dfrac{2x-6h}{(x+h)(4x-5h)(x-2h)}$

$=\lim_{x\to 2h}\dfrac{3}{(x+h)(4x-5h)}$ 1M

$=\dfrac{3}{(2h+h)(8h-5h)}$

$=\dfrac{1}{3h^2}$ 1A

因此，$\left[\lim_{x\to h}\dfrac{(x+h)^3+(x-3h)^3}{x-h}\right]\cdot\left[\lim_{x\to 2h}\dfrac{\frac{1}{x+h}-\frac{1}{4x-5h}}{x-2h}\right]=24h^2\cdot\dfrac{1}{3h^2}=8$。 1A

6. $\lim_{x\to 3}\dfrac{\sqrt{3x+7}-4}{3-\sqrt{x+6}}=\lim_{x\to 3}\dfrac{(\sqrt{3x+7}-4)(\sqrt{3x+7}+4)(3+\sqrt{x+6})}{(3-\sqrt{x+6})(\sqrt{3x+7}+4)(3+\sqrt{x+6})}$ 1M

$=\lim_{x\to 3}\dfrac{(3x-9)(3+\sqrt{x+6})}{(3-x)(\sqrt{3x+7}+4)}$

$=\lim_{x\to 3}\dfrac{-3(3+\sqrt{x+6})}{\sqrt{3x+7}+4}$ 1M

$=\dfrac{-3(3+3)}{4+4}$

$=-\dfrac{9}{4}$ 1A

7. (a) $\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}=\lim_{x\to 4}\dfrac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)}$ 1M

$=\lim_{x\to 4}\dfrac{(x-4)(\sqrt{x}+2)}{x-4}$

$=\lim_{x\to 4}(\sqrt{x}+2)$ 1M

$=\sqrt{4}+2$

$=4$ 1A

(b) $\lim_{x\to 0}\dfrac{x}{\sqrt{x+9}-3}=\lim_{x\to 0}\dfrac{x(\sqrt{x+9}+3)}{(\sqrt{x+9}-3)(\sqrt{x+9}+3)}$ 1M

$=\lim_{x\to 0}\dfrac{x(\sqrt{x+9}+3)}{(x+9)-9}$

$=\lim_{x\to 0}(\sqrt{x+9}+3)$ 1M

$=\sqrt{9}+3$

$=6$ 1A

8. $\lim\limits_{x\to 0}\dfrac{\sqrt{9+x}-3}{\sqrt{1+x}-1}=\lim\limits_{x\to 0}\dfrac{(\sqrt{9+x}-3)(\sqrt{9+x}+3)(\sqrt{1+x}+1)}{(\sqrt{1+x}-1)(\sqrt{9+x}+3)(\sqrt{1+x}+1)}$ 1M

$=\lim\limits_{x\to 0}\dfrac{x(\sqrt{1+x}+1)}{x(\sqrt{9+x}+3)}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}+1}{\sqrt{9+x}+3}$ 1M

$=\dfrac{1+1}{3+3}$

$=\dfrac{1}{3}$ 1A

9. $\lim\limits_{h\to 0}\dfrac{(x-h)\sqrt{x+h}-x\sqrt{x}}{\sqrt{x-h}-\sqrt{x}}=\lim\limits_{h\to 0}\dfrac{[(x-h)\sqrt{x+h}-x\sqrt{x}][(x-h)\sqrt{x+h}+x\sqrt{x}](\sqrt{x-h}+\sqrt{x})}{(\sqrt{x-h}-\sqrt{x})[(x-h)\sqrt{x+h}+x\sqrt{x}](\sqrt{x-h}+\sqrt{x})}$ 1M

$=\lim\limits_{h\to 0}\dfrac{[(x-h)^2(x+h)-x^3](\sqrt{x-h}+\sqrt{x})}{-h[(x-h)\sqrt{x+h}+x\sqrt{x}]}$

$=\lim\limits_{h\to 0}\dfrac{(-hx^2-h^2x+h^3)(\sqrt{x-h}+\sqrt{x})}{-h[(x-h)\sqrt{x+h}+x\sqrt{x}]}$ 1A

$=\lim\limits_{h\to 0}\dfrac{(x^2+hx-h^2)(\sqrt{x-h}+\sqrt{x})}{(x-h)\sqrt{x+h}+x\sqrt{x}}$ 1M

$=\dfrac{x^2(2\sqrt{x})}{2x\sqrt{x}}$

$=x$ 1A

10. $\lim\limits_{x\to a}\dfrac{x^2-25}{\sqrt{x^2-9}-4}=\lim\limits_{x\to a}\dfrac{(x^2-25)(\sqrt{x^2-9}+4)}{(\sqrt{x^2-9}-4)(\sqrt{x^2-9}+4)}$ 1M

$8=\lim\limits_{x\to a}(\sqrt{x^2-9}+4)$ 1M

$=\sqrt{a^2-9}+4$ 1A

$a^2-9=16$

$a=\pm 5$ 1A

11. $\lim\limits_{x\to -2}\dfrac{x^2+6x+8}{2-\sqrt{x^3+12}}=\lim\limits_{x\to -2}\dfrac{(x+2)(x+4)(2+\sqrt{x^3+12})}{(2-\sqrt{x^3+12})(2+\sqrt{x^3+12})}$ 1M

$=\lim\limits_{x\to -2}\dfrac{(x+2)(x+4)(2+\sqrt{x^3+12})}{-x^3-8}$

$=\lim\limits_{x\to -2}\dfrac{(x+2)(x+4)(2+\sqrt{x^3+12})}{-(x+2)(x^2-2x+4)}$ 1M

$=\lim\limits_{x\to -2}\dfrac{(x+4)(2+\sqrt{x^3+12})}{-(x^2-2x+4)}$ 1M

$=\dfrac{(-2+4)(2+\sqrt{-8+12})}{-(4+4+4)}$

$=-\dfrac{2}{3}$ 1A

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