REG-QE-2425-ASM-SET 3-MATH
Suggested solutions

Basic Questions

1. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. $x = 4$ or -6

3. $\Delta = 1^2 - 4(1)(k) > 0$
$$k < \frac{1}{4}$$

4. Consider the equation $2x^2 - 6x + 3k$.
$$\Delta = (-6)^2 - 4(2)(3k) = 0$$
$$k = \frac{3}{2}$$

5. Sum of roots $= -\dfrac{5}{-1} = 5$

6. $\alpha\beta = \dfrac{c}{a}$

7. $\alpha + \beta = -\dfrac{b}{a}$

8. $\dfrac{k}{4} = -\dfrac{3}{2}$
$k = -6$

9. $-\dfrac{k}{3} = 3$
$k = -9$

10. $\alpha + \beta = -\dfrac{-9}{3}$
$= 3$

Conventional Questions

11. (a) We have $\alpha + \beta = -\dfrac{2}{1} = -2$ and $\alpha\beta = \dfrac{-5}{1} = -5$. 1A

$$\begin{aligned} \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta && \text{1M} \\ &= (-2)^2 - 2(-5) \\ &= 14 && \text{1A} \end{aligned}$$

(b)
$$\begin{aligned} (2\alpha - \beta)(2\beta - \alpha) &= 4\alpha\beta - 2\alpha^2 - 2\beta^2 + \alpha\beta \\ &= 5\alpha\beta - 2(\alpha^2 + \beta^2) && \text{1M} \\ &= 5(-5) - 2(14) \\ &= -53 && \text{1A} \end{aligned}$$

12. (a)
$$\begin{aligned} -\frac{k+1}{3} &= -5 && \text{1M} \\ k &= 14 && \text{1A} \end{aligned}$$

(b)
$$\begin{aligned} \text{Product of roots} &= \frac{-k}{3} && \text{1M} \\ &= -\frac{14}{3} && \text{1A} \end{aligned}$$

13. (a) We have $\alpha + \beta = -\dfrac{-5}{1} = 5$ and $\alpha\beta = \dfrac{3}{1} = 3$. 1A

$$\begin{aligned} \alpha^2\beta + \alpha\beta^2 &= \alpha\beta(\alpha + \beta) \\ &= 5(3) \\ &= 15 && \text{1A} \end{aligned}$$

(b)
$$\begin{aligned} (\alpha + 3)(\beta + 3) &= \alpha\beta + 3\alpha + 3\beta + 9 \\ &= \alpha\beta + 3(\alpha + \beta) + 9 && \text{1M} \\ &= 3 + 3(5) + 9 \\ &= 27 && \text{1A} \end{aligned}$$

14. (a) We have $\alpha + \beta = -5$ and $\alpha\beta = 2$. 1A

$$\begin{aligned} \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\beta + \alpha}{\alpha\beta} \\ &= \frac{-5}{2} && \text{1A} \end{aligned}$$

(b)
$$\begin{aligned} \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= (-5)^2 - 2(2) \\ &= 21 && \text{1A} \end{aligned}$$

(c)
$$\begin{aligned} \frac{\alpha}{\beta} + \frac{\beta}{\alpha} &= \frac{\alpha^2 + \beta^2}{\alpha\beta} \\ &= \frac{21}{2} && \text{1A} \end{aligned}$$

15. (a) $\alpha(\alpha+3)=\dfrac{54}{1}$ 1M

$\alpha^2+3\alpha-54=0$

$\alpha=6 \quad \text{or} \quad -9$ 1A

(b) Suppose $\alpha=6$.

$6+(6+3)=-\dfrac{k}{1}$

$k=-15$ 1A

Suppose $\alpha=-9$.

$-9+(-9+3)=-\dfrac{k}{1}$

$k=15$ 1A

16. $\alpha(-2\alpha)=\dfrac{-36}{2}$ 1M

$\alpha^2=9$

$\alpha=\pm 3$ 1A

Suppose $\alpha=3$.

$3+(-2(3))=-\dfrac{-3k}{2}$

$k=-2$ 1A

Suppose $\alpha=-3$.

$-3+(-2(-3))=-\dfrac{-3k}{2}$

$k=2$ 1A

17. α and β are the roots of the equation $x^2+6x-3=0$.

We have $\alpha+\beta=-\dfrac{6}{1}=-6$ and $\alpha\beta=\dfrac{-3}{1}=-3$. 1A+1A

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ 1M

$=(-6)^2-2(-3)$

$=42$ 1A

18. α is a root of the equation.

$\alpha^2+7\alpha-13=0$ 1M

$\alpha^2=-7\alpha+13$

We have $\alpha+\beta=-\frac{7}{1}=-7$. $\alpha^2-7\beta=(-7\alpha+13)-7\beta$ 1A

$$=-7(\alpha+\beta)+13$$ 1M

$$=-7(-7)+13$$

$$=62$$ 1A

19. (a) α and β are roots of the equation $2x^2+8x-5=0$.

We have $\alpha+\beta=-\frac{8}{2}=-4$ and $\alpha\beta=\frac{-5}{2}$. 1A

$$\alpha-\beta=-\sqrt{(\alpha-\beta)^2}$$

$$=-\sqrt{(\alpha+\beta)^2-4\alpha\beta}$$ 1M

$$=-\sqrt{(-4)^2-4\left(-\frac{5}{2}\right)}$$

$$=-\sqrt{26}$$ 1A

(b) $\alpha^2+4\beta=\left(\frac{-8\alpha+5}{2}\right)+4\beta$ 1M

$$=-4(\alpha-\beta)+\frac{5}{2}$$

$$=-4(-\sqrt{26})+\frac{5}{2}$$

$$=4\sqrt{26}+\frac{5}{2}$$ 1A

20. (a) a and b are roots of the equation $x^2-12x+c=0$.

We have $a+b=-\frac{-12}{1}=12$ and $ab=\frac{c}{1}=c$. 1A

$$b-a=20$$

$$(b-a)^2=20^2$$

$$(a+b)^2-4\alpha\beta=400$$ 1M

$$12^2-4c=400$$

$$c=-64$$ 1A

(b) $x^2-12x-64=0$ 1M

$$x=16 \quad \text{or} \quad -4$$

Since $a<b$, we have $a=-4$ and $b=16$. 1A+1A

21. (a) Let the two roots be α and $4\alpha+2$.

$$\alpha+(4\alpha+2)=-\frac{-(5p+19)}{2}$$ 1M

$$\alpha=\frac{p+3}{2}$$

Consider the product of roots.

$$\alpha(4\alpha+2)=\frac{10p^2+4p+12}{2}$$

$$\left(\frac{p+3}{2}\right)\left[4\left(\frac{p+3}{2}\right)+2\right]=5p^2+2p+6 \qquad \text{1M}$$

$$(p+3)^2+(p+3)=5p^2+2p+6$$

$$-4p^2+5p+6=0$$

$$p=2 \quad \text{or} \quad -\frac{3}{4} \qquad \text{1A}$$

(b) When $p=2$,

$$2x^2-29x+60=0$$

$$x=\frac{5}{2} \quad \text{or} \quad 12 \qquad \text{1A}$$

When $p=-\frac{3}{4}$,

$$2x^2-\frac{61x}{4}+\frac{117}{8}=0$$

$$x=\frac{13}{2} \quad \text{or} \quad \frac{9}{8} \qquad \text{1A}$$

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