REG-2425-MOCK-SET 2-MATH-CP 2

Answers:

1. C	2. D	3. C	4. C	5. B	6. A	7. C	8. D	9. C	10. A
11. C	12. A	13. B	14. D	15. D	16. A	17. B	18. A	19. B	20. D
21. B	22. C	23. A	24. B	25. D	26. A	27. D	28. B	29. B	30. D
31. B	32. C	33. D	34. C	35. D	36. A	37. C	38. C	39. B	40. B
41. D	42. A	43. A	44. A	45. D					

Suggested Solutions:

1. $\boxed{\text{C}}$

$$\left(\frac{2^{888}}{4^{333}}\right)8^{111} = 2^{888-2\times333+3\times111}$$
$$= 2^{555}$$

2. $\boxed{\text{D}}$

$$\frac{x+y}{x} = \frac{1-y}{y}$$
$$xy + y^2 = x - xy$$
$$x(2y-1) = -y^2$$
$$x = \frac{y^2}{1-2y}$$

3. $\boxed{\text{C}}$

$$(3a+2b)^2 - (2a-3b)^2 = [(3a+2b)+(2a-3b)][(3a+2b)-(2a-3b)]$$
$$= (5a-b)(a+5b)$$

4. $\boxed{\text{C}}$

$$\left(\frac{\pi}{5}\right)^3 \approx 0.248\,050\,213$$
$$= 0.2481 \quad \text{(correct to 4 significant figures)}$$

5. $\boxed{\text{B}}$

$$2f(-1)+3 = 2\left[(-1)^{2012} + 2012(-1) + 2012\right] + 3$$
$$= 2(1-2012+2012)+3$$
$$= 5$$

6. A

$0 = k^3 - k(k^2) + 2k - 4$

$k = 2$

Remainder $= (-k)^3 - k(-k)^2 + 2(-k) - 4$

$= -2k^3 - 2k - 4$

$= -24$

7. C

$2(1 - x) > 6x$ and $x \leq \dfrac{4x + 1}{-2}$

$-8x > -2$ $3x \leq -\dfrac{1}{2}$

$x < \dfrac{1}{4}$ $x \leq -\dfrac{1}{6}$

Thus, $x \leq -\dfrac{1}{6}$.

The greatest value of x is -1.

8. D

$a^2 + a(a) - 8 = 0$

$a^2 = 4$

$a = \pm 2$

When $a = 2$, the roots of the equation $x^2 + 2x - 8 = 0$ are -4 and 2.

When $a = -2$, the roots of the equation $x^2 - 2x - 8 = 0$ are 4 and -2.

The possible value of the other root is -4 or 4.

9. C

The coordinates of vertex are $(-a,\ b)$.

I. ✓. From the vertex, we have $-a < -1$. Therefore, $a > 1$.

II. ✗. From the vertex, we have $b < 0$.

III. ✓. y-intercept $= (0 + a)^2 + b = a^2 + b < 1$

10. A

Cost $= \dfrac{6400 - 420}{1 + 15\%}$

$= \$5200$

11. C

Let A m^2 be the required area.

$$\frac{A \times 100^2}{25} = \left(\frac{4000}{1}\right)^2$$
$$A = 40\,000$$

12. A

Let $x = \frac{k\sqrt{z}}{y}$, where k is a non-zero constant.

Then $k = \frac{xy}{\sqrt{z}}$.

Thus, $\frac{x^2y^2}{z} = k^2$ is a constant.

13. B

The numbers of dots are formed by +4, +4, +4, . . .

The sequence of numbers of dots is 2, 6, 10, 14, 18, 22, 26, 30.

Required number is 30.

14. D

Since $\triangle ABC \cong \triangle DEC$, we have $CE = BC = 5$ cm and $AC = CD = 12$ cm.

Consider $\triangle CDE$.

$$DE^2 = CE^2 + CD^2$$
$$DE = \sqrt{5^2 + 12^2}$$
$$= 13\text{ cm}$$

Consider $\triangle ACD$.

$$AD^2 = AC^2 + CD^2$$
$$AD = \sqrt{12^2 + 12^2}$$
$$= 12\sqrt{2}\text{ cm}$$

Required perimeter $= (12 - 5) + 13 + 12\sqrt{2}$

≈ 37.0 cm

15. D

Since $AB = BE$, we have $\angle AEB = \angle BAE$.

$$\angle AEB + \angle BAE = \angle ABF$$
$$2\angle AEB = 132°$$
$$\angle AEB = 66°$$

Since $AD // FC$, we have $\angle EAD = \angle AEB = 66°$.
Since $AE = DE$, we have $\angle ADE = \angle EAD = 66°$.

$$\angle EAD + \angle ADE + \angle DEA = 180°$$
$$66° + 66° + \angle DEA = 180°$$
$$\angle DEA = 48°$$

Note that BEC is a straight line.

$$\angle AEB + \angle DEA + \angle DEC = 180°$$
$$66° + 48° + \angle DEC = 180°$$
$$\angle DEC = 66°$$

16. A

Let h cm be the base radius of the right circular cone.

$$\frac{4}{3}\pi(3)^3 = \frac{1}{3}\pi r^2(6)$$
$$r = \sqrt{18}$$

Percentage increase in total surface area

$$= \frac{[\pi r^2 + \pi r(\sqrt{r^2 + 6^2})] - 4\pi(3)^2}{4\pi(3)^2} \times 100\%$$

$\approx 36.6\%$

17. B

$$\text{Ratio of base radius} = \sqrt{\frac{4}{9}}$$
$$= \frac{2}{3}$$

$$\text{Required ratio} = \frac{2^3}{3^3 - 2^3}$$
$$= \frac{8}{19}$$

18. A

Consider $\triangle ABD$ and $\triangle ADE$.

$$\frac{\text{area of } \triangle ABD}{\text{area of } \triangle ADE} = \frac{BD}{DE}$$
$$\frac{\text{area of } \triangle ABD}{18} = \frac{1}{3}$$
$$\text{area of } \triangle ABD = 6\,\text{cm}^2$$

Note that $\triangle ABC \sim \triangle BEC$.

Let $x\,\text{cm}^2$ be the area of $\triangle BEC$.

$$\frac{\text{area of } \triangle BEC}{\text{area of } \triangle ABC} = \left(\frac{BE}{AB}\right)^2$$
$$\frac{x}{x + 18 + 6} = \left(\frac{1+3}{8}\right)^2$$
$$x = 8$$

Required area $= 18 + 6 + 8$

$= 32\,\text{cm}^2$

19. B

$AC = \sqrt{1^2 + 1^2} = \sqrt{2}\,\text{cm}$

$AD = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2\,\text{cm}$

Required area $= \dfrac{1^2}{2} + \dfrac{(\sqrt{2})^2}{2} + \dfrac{2^2}{2}$

$= \dfrac{7}{2}\,\text{cm}^2$

20. D

Note that $AB = AD = AE$.

A is the centre of the circumcircle of BDE.

Since BE is a diameter of circle BDE, we have $\angle BDE = 90°$.

$\angle ADE = \angle DEA = 35°$

$\angle ADB = 90° - 35° = 55°$

$\angle CBD = \angle ADB = 55°$

21. B

$\angle CBE = 90° - \alpha$ and $\angle BCE = 180° - 90° - (90° - \alpha) = \alpha$

$$CE = \frac{BE}{\tan\alpha} = \frac{\left(\frac{AB}{\cos\alpha}\right)}{\tan\alpha} = \frac{AB}{\cos\alpha \times \frac{\sin\alpha}{\cos\alpha}} = \frac{AB}{\sin\alpha}$$

22. C

$\angle ABC = 180° - 68° = 112°$

$\angle ABD = 90°$

$\angle CBD = 112° - 90° = 22°$

$\angle BDC = \angle CBD = 22°$

$\angle ADB = 68° - 22° = 46°$

23. A

Note that $\angle ACB = 90°$ and AB is a diameter of the circle.

$$\text{Required area} = \frac{1}{2} \times \left(\frac{25}{2}\right)^2 \pi - \frac{(7)(24)}{2}$$
$$\approx 161 \text{ cm}^2$$

24. B

$$(2,\ 120°) \quad = \quad \left(-1,\ \sqrt{3}\right) \quad \longrightarrow \quad \left(-1,\ -\sqrt{3}\right)$$

25. D

I. ✓. Slope of $L_1 = -\frac{1}{A} < 0$. So, $A > 0$.

II. ✓. Slope of $L_2 = -\frac{1}{C}$.

$$\left(\frac{-1}{A}\right)\left(\frac{-1}{C}\right) = -1$$
$$AC = -1$$

III. ✓. x-intercept of $L_1 = B$ and x-intercept of $L_2 = D$.

Thus, $B > D$.

26. A

$$\frac{a}{c} = \frac{b}{5} = \frac{2}{-1}$$

Thus, $a = -2c$ and $b = -10$.

$2a + 3b + 4c = 2(a + 2c) + 3b = -30$

27. D

$C: x^2 + y^2 - 3x + y - \dfrac{13}{2} = 0$

I. ✗.

The coordinates of the centre are $\left(\dfrac{3}{2}, -\dfrac{1}{2}\right)$.

II. ✓.

$AB = \sqrt{(2-1)^2 + (1+2)^2} = \sqrt{10}$

$\text{Radius} = \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2 + \dfrac{13}{2}} = 3 < \sqrt{10}$

III. ✓.

Denote the centre by G.

Slope of $AB = \dfrac{1+2}{2-1} = 3$

Slope of $BG = \dfrac{1+\frac{1}{2}}{2-\frac{3}{2}} = 3$

A, B and G are collinear.

Thus, G lies on the straight line passing through A and B.

28. B

$$\text{Required probability} = \frac{3+5}{6^2} = \frac{2}{9}$$

29. B

50% of the data lies between lower quartile and upper quartile.

30. D

We have $m = n = 5$.

I. ✓.

II. ✓. $\text{Mean} = \dfrac{1+2+5+12+\ldots+5}{10} = 5.2$

III. ✓. $\text{Range} = 12 - 1 = 11$

31. B

$$1 - \frac{ab}{a^2-b^2} - \frac{b}{b-a} = \frac{(a^2-b^2) - ab + b(a+b)}{(a+b)(a-b)} = \frac{a^2}{a^2-b^2}$$

32. $\boxed{\text{C}}$

$\boxed{(0,\ -1)}$

$$\log_7 x = 0 \quad \text{and} \quad \log_7 y = -1$$
$$x = 1 \qquad\qquad y = 7^{-1}$$

We have $7^{-1} = a(1)^b$, and $a = \frac{1}{7}$.

$\boxed{(2,\ 0)}$

$$\log_7 x = 2 \quad \text{and} \quad \log_7 y = 0$$
$$x = 49 \qquad\qquad y = 1$$

We have $1 = \frac{1}{7}(49)^b$, and $b = \frac{1}{2}$.

33. $\boxed{\text{D}}$

Each digit in hexadecimal number corresponds to four digits in its binary representation.

$F6_{16} = 11110110_2$ and $14_{16} = 10100_2$

Thus, $14F6_{16} = 1010011110110_2$.

34. $\boxed{\text{C}}$

Put $k = 1$.

$$\frac{5k + 10i}{1 - 2i} = \frac{5 + 10i}{1 - 2i} = -3 + 4i$$

Imaginary part is 4.

Check the value of each option when $k = 1$.

A. 5

B. −3

C. 4

D. 0

35. D

α is a root of the equation.

$$2\alpha^2 + 4\alpha - 1 = 0$$
$$\alpha^2 = -2\alpha + \frac{1}{2}$$

$$\begin{aligned}\alpha^2 - 2\beta &= \left(-2\alpha + \frac{1}{2}\right) - 2\beta \\ &= -2(\alpha + \beta) + \frac{1}{2} \\ &= -2(-2) + \frac{1}{2} \\ &= \frac{9}{2}\end{aligned}$$

36. A

$$\begin{aligned}\text{General term} &= (2^{n+1} - 2) - (2^n - 2) \\ &= 2^n(2 - 1) \\ &= 2^n\end{aligned}$$

I. ✓. $\dfrac{T(n+1)}{T(n)} = \dfrac{2^{n+1}}{2^n} = 2 = \text{constant.}$

II. ✗. Second term $= 2^2 = 4 \neq 6$.

III. ✗.

37. C

Let K be a point on VB such that $AK \perp VB$. We have $CK \perp VB$ also.
Required angle is $\angle AKC$.
$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2}\,\text{cm}$
$VA = VB = VC = \sqrt{8^2 + \left(\frac{12\sqrt{2}}{2}\right)^2} = \sqrt{136}\,\text{cm}$
In $\triangle VAB$,

$$\cos\angle ABV = \frac{\left(\frac{12}{2}\right)}{VB}$$
$$\angle ABV \approx 59.0°$$

$AK = CK = 12\sin\angle ABV \approx 10.3\,\text{cm}$
In $\triangle AKC$,

$$AC^2 = AK^2 + CK^2 - 2(AK)(CK)\cos\angle AKC$$
$$\angle AKC \approx 111°$$

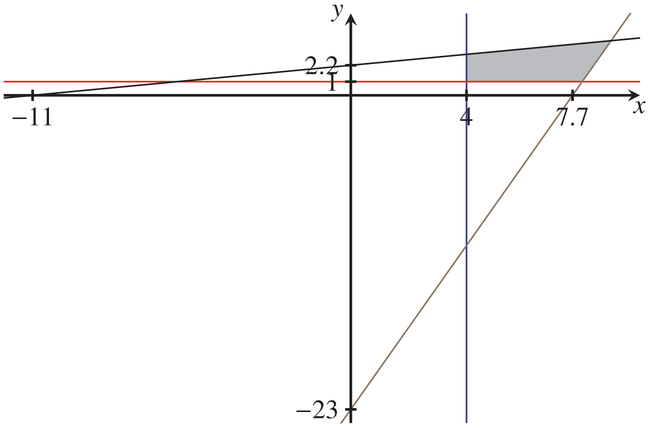
38. C

$g(x) = -\frac{1}{2}f(x)$
The graph of $y = f(x)$ is reduced along the y-axis to $\frac{1}{2}$ times the original and then is reflected about the x-axis to the graph of $y = g(x)$.
The answer is C.

39. B

Line	x-intercept	y-intercept
$x = 4$	4	
$y = 1$		1
$3x - y = 23$	7.7	-23
$x - 5y + 11 = 0$	-11	2.2

Sketch the solution region using the intercepts.



The value of $y - 4x + 20$ is larger when x is smaller and y is larger.
$y - 4x + 20$ attains its maximum value at the top left corner, which is $(4,\ 3)$ or $(9,\ 4)$.

$(x,\ y)$	$(4,\ 3)$	$(9,\ 4)$
$y - 4x + 20$	7	-12

The greatest value is 7.

40. B

$\angle AEF = 180° - 125° = 55°$

$\angle EAF = \angle EFB - \angle AEF$

$= 120° - 55°$

$= 65°$

Since $BA = BC$, we have $\angle BCA = \angle BAC = 65°$.

$\angle ABC = 180° - \angle BAC - \angle BCA$

$= 180° - 65° - 65°$

$= 50°$

$\angle DFB = 180° - \angle DFB - \angle FBD$

$\angle CDE = 180° - 120° - 50°$

$= 10°$

41. D

x-coordinate of vertex $= \frac{-k}{2(1)} = -\frac{k}{2}$

mid-point of $PR = \left(-\frac{k}{2},\ 0\right)$

Consider the x-coordinate of centroid,

$$-2 = \frac{0(1) + \left(-\frac{k}{2}\right)(2)}{1+2}$$

$$k = 6$$

42. A

Required number $= C_5^{25} - C_5^{15} - C_5^{10}$

$= 49\,875$

43. A

Solve the system $\begin{cases} 2x + y - 5 = 0 \\ x^2 + y^2 - kx + 6y - 10 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
2	2	+

Required range does not contain 2 and 2 is not a boundary value of the range.

The answer is A.

44. A

$$\sin\theta = 5\tan\theta$$
$$\sin\theta = \frac{5\sin\theta}{\cos\theta}$$
$$\sin\theta\left(1 - \frac{5}{\cos\theta}\right) = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \cos\theta = 5 \text{ (rejected)}$$
$$\theta = 0° \quad \text{or} \quad 180°$$

45. D

$$\text{Required probability} = \frac{\frac{4}{5}\times\frac{2}{3}}{\frac{4}{5}\times\frac{2}{3}+\frac{1}{5}\times\frac{1}{3}}$$
$$= \frac{8}{9}$$