Solution	Marks
REG-2425-MOCK-SET 1-MATH-EP(M2)	
Suggested solutions	
1. $f'(2)=\lim_{h\to 0}\frac{(2+h)\ln(2+h-1)-2\ln 1}{h}$	1M
$=\lim_{h\to 0}(2+h)\ln(1+h)^{\frac{1}{h}}$	1M
$=(2+0)(\ln e)$	1A
$=2$	1A
2. (a) $P(x)=(x-\lambda)^2\begin{vmatrix}4 & x-\lambda\\ (x-\lambda)^3 & 5\end{vmatrix}+\begin{vmatrix}3 & 2\\ 4 & x-\lambda\end{vmatrix}$	
$=20(x-\lambda)^2-(x-\lambda)^6+3(x-\lambda)-8$	1A
$-1195=20-C_2^6(-\lambda)^4$	1M
$\lambda^4=81$	
$\lambda=3$ or -3 (rejected)	1A
(b) $P'(x)=40(x-3)-6(x-3)^5+3$	1A
$P''(x)=40-30(x-3)^4$	
$P''(\lambda)=40$	1A
3. (a) $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$	
$=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$	1M
$=\frac{\cos 2\theta}{1}$	
$=\cos 2\theta$	1
(b) $\cos\frac{3\pi}{4}=\frac{1-\tan^2\frac{3\pi}{8}}{1+\tan^2\frac{3\pi}{8}}$	1M
$-\frac{1}{\sqrt{2}}=\frac{1-\tan^2\frac{3\pi}{8}}{1+\tan^2\frac{3\pi}{8}}$	1A
$-1-\tan^2\frac{3\pi}{8}=\sqrt{2}\left(1-\tan^2\frac{3\pi}{8}\right)$	
$\tan^2\frac{3\pi}{8}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$	1A
$=\frac{\sqrt{2}+1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$	1M
$=\frac{3+2\sqrt{2}}{2-1}$	
$=3+2\sqrt{2}$	1

Solution	Marks
4. (a) $A^2 - mA = nI$	
$A\left(\frac{1}{n}A - \frac{m}{n}I\right) = I$	1M
Thus, $A^{-1} = \frac{1}{n}A - \frac{m}{n}I$.	1A
(b) $A^3 = A^2A$	
$= (mA + nI)A$	1M
$= m(mA + nI) + nA$	
$= (m^2 + n)A + mnI$	1
(c) $A^3 = -A^{-1}$ when $m^2 + n = -\frac{1}{n}$ and $mn = \frac{m}{n}$.	1M
$mn = \frac{m}{n}$	
$0 = m\left(\frac{1}{n} - n\right)$	
$n = 1$ (rejected) or -1 or $m = 0$ (rejected)	1A
For $n = 1$, we have $m = \sqrt{2}$, $A^3 = A - \sqrt{2}I$ and $A^{-1} = -A + \sqrt{2}I = -A^3$.	1
5. (a) $W = \int (-10 - t)\,\mathrm{d}t$	1M
$= -10t - \frac{t^2}{2} + C$, where C is a constant	
Since $W = 540$ when $t = 0$, we have $C = 540$.	1M
So, we have $W = -10t - \frac{t^2}{2} + 540$.	
When $t = 24$, we have $W = 12 > 0$.	
Thus, the claim is incorrect.	1A
(b) When $t = 20$, $\frac{\mathrm{d}W}{\mathrm{d}t} = -10 - 20 = -30$ and $W = -10(20) - \frac{20^2}{2} + 540 = 140$.	
$140 = 5x^2 + 3x$	
$0 = 5x^2 + 3x - 140$	
$x = 5$ or $-\frac{28}{5}$ (rejected)	1A
$\frac{\mathrm{d}W}{\mathrm{d}t} = (10x + 3)\frac{\mathrm{d}x}{\mathrm{d}t}$	1M
$-10 - 20 = [10(5) + 3]\frac{\mathrm{d}x}{\mathrm{d}t}$	
$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{30}{53}$	1A

Solution	Marks
6. (a) When $n = 1$,	
L.H.S. $= \displaystyle\sum_{k=1}^{2} \frac{1}{(2k-1)(2k+1)} = \frac{1}{3} + \frac{1}{15} = \frac{2}{5}$ and R.H.S. $= \dfrac{1+1}{1(4+1)} = \dfrac{2}{5} =$ L.H.S.	
The statement is true for $n = 1$.	1
Assume $\displaystyle\sum_{k=p}^{2p} \frac{1}{(2k-1)(2k+1)} = \frac{p+1}{(2p-1)(4p+1)}$ for some $p \in \mathbb{Z}^+$.	1
When $n = p + 1$,	
$\displaystyle\sum_{k=p+1}^{2p+2} \frac{1}{(2k-1)(2k+1)}$	
$\displaystyle= \sum_{k=p}^{2p} \frac{1}{(2k-1)(2k+1)} - \frac{1}{(2p-1)(2p+1)} + \frac{1}{(4p+1)(4p+3)} + \frac{1}{(4p+3)(4p+5)}$	1M
$\displaystyle= \frac{p+1}{(2p-1)(4p+1)} - \frac{1}{(2p-1)(2p+1)} + \frac{(4p+5)+(4p+1)}{(4p+1)(4p+3)(4p+5)}$	1
$\displaystyle= \frac{(p+1)(2p+1)-(4p+1)}{(2p-1)(2p+1)(4p+1)} + \frac{2}{(4p+1)(4p+5)}$	
$\displaystyle= \frac{p}{(2p+1)(4p+1)} + \frac{2}{(4p+1)(4p+5)}$	
$\displaystyle= \frac{p(4p+5)+2(2p+1)}{(2p+1)(4p+1)(4p+5)}$	
$\displaystyle= \frac{p+2}{(2p+1)(4p+5)}$	
The statement is true for $n = p + 1$.	
By mathematical induction, the statement is true $\forall n \in \mathbb{Z}^+$.	1
(b) $\displaystyle\sum_{k=505}^{2020} \frac{1}{(2k-1)(2k+1)} = \sum_{k=1010}^{2020} \frac{1}{(2k-1)(2k+1)} + \sum_{k=505}^{1010} \frac{1}{(2k-1)(2k+1)} - \frac{1}{(2019)(2021)}$	
$\displaystyle= \frac{1011}{(2019)(4041)} + \frac{506}{(1009)(2021)} - \frac{1}{(2019)(2021)}$	1M
$\approx 3.72 \times 10^{-4}$	1A

Solution	Marks
7. (a) $y = \int xe^{-2x}\,\mathrm{d}x$	1M
$= \frac{xe^{-2x}}{-2} + \frac{1}{2}\int e^{-2x}\,\mathrm{d}x$	1M
$= -\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + C$, where C is a constant.	
$\frac{1}{4} = 0 - \frac{1}{4} + C$	1M
$C = \frac{1}{2}$	
Equation of Γ is $y = -\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + \frac{1}{2}$.	1A
(b) $f''(x) = (1-2x)e^{-2x}$	
When $f''(x) = 0$, $x = \frac{1}{2}$.	
$\begin{array}{c\|c\|c} x & x < \frac{1}{2} & x > \frac{1}{2} \\ \hline f''(x) & + & - \end{array}$	1M
Γ has one point of inflexion at $x = \frac{1}{2}$.	
The claim is disagreed.	1A

Solution	Marks
8. (a) $g'(x) = e^x(\cos x - \sin x)$	1A
When $g'(x) = 0$,	
$\cos x - \sin x = 0$	
$\tan x = 1$	
$x = \frac{\pi}{4}$	
$\begin{array}{c\|c\|c} x & 0 < x < \frac{\pi}{4} & \frac{\pi}{4} < x < \pi \\ \hline g'(x) & + & - \end{array}$	
Thus, G has only one maximum point.	1
(b) $\int e^{2x} \cos 2x \, \mathrm{d}x = \frac{e^{2x} \cos 2x}{2} + \int e^{2x} \sin 2x \, \mathrm{d}x$	1M
$= \frac{e^{2x} \cos 2x}{2} + \frac{e^{2x} \sin 2x}{2} - \int e^{2x} \cos 2x \, \mathrm{d}x$	
$2 \int e^{2x} \cos 2x \, \mathrm{d}x = \frac{e^{2x}(\sin 2x + \cos 2x)}{2} + \text{constant}$	1M
$\int e^{2x} \cos 2x \, \mathrm{d}x = \frac{e^{2x}(\sin 2x + \cos 2x)}{4} + \text{constant}$	1A
(c) Volume $= \pi \int_0^{\frac{\pi}{4}} (e^x \cos x)^2 \, \mathrm{d}x$	1M
$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} e^{2x} \cos 2x \, \mathrm{d}x + \frac{\pi}{2} \int_0^{\frac{\pi}{4}} e^{2x} \, \mathrm{d}x$	1M
$= \frac{\pi}{2} \left[\frac{e^{2x}(\sin 2x + \cos 2x)}{4} \right]_0^{\frac{\pi}{4}} + \frac{\pi}{2} \left[\frac{e^{2x}}{2} \right]_0^{\frac{\pi}{4}}$	
$= \frac{3\pi(e^{\frac{\pi}{2}} - 1)}{8}$	1A

<table>
<tr><th>Solution</th><th>Marks</th></tr>
<tr><td>9. (a) Vertical asymptote is $x = -\frac{1}{2}$.</td><td>1A</td></tr>
<tr><td>$f(x) = \frac{x^2 - 4x}{2x + 1}$</td><td></td></tr>
<tr><td>$= \frac{x}{2} - \frac{9}{4} + \frac{9}{4(2x+1)}$</td><td>1M</td></tr>
<tr><td>Oblique asymptote is $y = \frac{x}{2} - \frac{9}{4}$.</td><td>1A</td></tr>
<tr><td>(b) $f'(x) = \frac{1}{2} - \frac{9}{4}(2x+1)^{-2}(2)$</td><td>1M</td></tr>
<tr><td>$= \frac{1}{2} - \frac{9}{2(2x+1)^2}$</td><td>1A</td></tr>
<tr><td>(c) $\frac{1}{2} - \frac{9}{2(2x+1)^2} = 0$</td><td>1M</td></tr>
<tr><td>$(2x+1)^2 = 9$</td><td></td></tr>
<tr><td>$x = -2 \quad \text{or} \quad 1$</td><td></td></tr>
<tr><td><table>
<tr><td>x</td><td>$x < -2$</td><td>$-2 < x < -\frac{1}{2}$</td><td>$-\frac{1}{2} < x < 1$</td><td>$x > 1$</td></tr>
<tr><td>$f'(x)$</td><td>+</td><td>−</td><td>−</td><td>+</td></tr>
</table></td><td>1M</td></tr>
<tr><td>Maximum point is $(-2, -4)$ and minimum point is $(1, -1)$.</td><td>1A+1A</td></tr>
<tr><td>(d) $0 = \frac{x^2 - 4x}{2x + 1}$</td><td></td></tr>
<tr><td>$x = 0 \quad \text{or} \quad 4$</td><td></td></tr>
<tr><td>Area $= \int_0^4 [-f(x)]\,dx$</td><td>1M+1A</td></tr>
<tr><td>$= \int_0^4 \left(\frac{9}{4} - \frac{x}{2} - \frac{9}{4(2x+1)}\right) dx$</td><td></td></tr>
<tr><td>$= \left[\frac{9x}{4} - \frac{x^2}{4} - \frac{9}{8}\ln|2x+1|\right]_0^4$</td><td>1M</td></tr>
<tr><td>$= 5 - \frac{9}{8}\ln 9$</td><td>1A</td></tr>
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Solution	Marks
10. (a) $\frac{A}{u^2+1}+\frac{B}{u^2+4}=\frac{A(u^2+4)+B(u^2+1)}{(u^2+1)(u^2+4)}$	
$=\frac{(A+B)u^2+(4A+B)}{(u^2+1)(u^2+4)}$	
Comparing like terms, we have $A+B=0$ and $4A+B=1$.	1M
Solving, we have $A=\frac{1}{3}$ and $B=-\frac{1}{3}$.	
Thus, $\frac{1}{(u^2+1)(u^2+4)}=\frac{1}{3(u^2+1)}-\frac{1}{3(u^2+4)}$.	1A
(b) Let $u=-x$. Then $\mathrm{d}u=-\,\mathrm{d}x$.	1M
$\int_{-a}^{a} f(x)\,\mathrm{d}x=\int_{-a}^{0} f(x)\,\mathrm{d}x+\int_{0}^{a} f(x)\,\mathrm{d}x$	
$=-\int_{a}^{0} f(-u)\,\mathrm{d}u+\int_{0}^{a} f(x)\,\mathrm{d}x$	1M
$=\int_{0}^{a} f(u)\,\mathrm{d}u+\int_{0}^{a} f(x)\,\mathrm{d}x$	
$=2\int_{0}^{a} f(x)\,\mathrm{d}x$	1
(c) (i) Use the result of (b).	
$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{\mathrm{d}u}{(u^2+1)(u^2+4)}=\int_{-\sqrt{3}}^{\sqrt{3}}\left(\frac{1}{3(u^2+1)}-\frac{1}{3(u^2+4)}\right)\mathrm{d}u$	1M
Let $u=\tan\theta$. Then $\mathrm{d}u=\sec^2\theta\,\mathrm{d}\theta$.	1M
Let $u=2\tan\phi$. Then $\mathrm{d}u=2\sec^2\phi\,\mathrm{d}\phi$.	
$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{\mathrm{d}u}{(u^2+1)(u^2+4)}=\frac{1}{3}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{\sec^2\theta}{\tan^2\theta+1}\,\mathrm{d}\theta-\frac{1}{3}\int_{-\tan^{-1}\frac{\sqrt{3}}{2}}^{\tan^{-1}\frac{\sqrt{3}}{2}}\frac{2\sec^2\phi}{4\tan^2\phi+4}\,\mathrm{d}\phi$	1A
$=\frac{1}{3}\Big[\theta\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}-\frac{1}{6}\Big[\phi\Big]_{-\tan^{-1}\frac{\sqrt{3}}{2}}^{\tan^{-1}\frac{\sqrt{3}}{2}}$	
$=\frac{2\pi}{9}-\frac{1}{3}\tan^{-1}\frac{\sqrt{3}}{2}$	1A
(ii) Note that $\frac{1}{\tan^2(-x)+4}=\frac{1}{\tan^2 x+4}$.	
Let $u=\tan x$. Then $\mathrm{d}u=\sec^2\theta\,\mathrm{d}x=(u^2+1)\,\mathrm{d}x$.	
$\int_{0}^{\frac{\pi}{3}}\frac{\mathrm{d}x}{\tan^2 x+4}=\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{\mathrm{d}x}{\tan^2 x+4}$	1M
$=\frac{1}{2}\int_{-\sqrt{3}}^{\sqrt{3}}\frac{\mathrm{d}u}{(u^2+1)(u^2+4)}$	1M
$=\frac{\pi}{9}-\frac{1}{6}\tan^{-1}\frac{\sqrt{3}}{2}$	1A

Solution	Marks
11. (a) (i) $\begin{vmatrix} 1 & 1 & 1 \\ -1 & a & 1 \\ 4 & 1 & a \end{vmatrix} = a^2 - 3a + 2$	1A
$= (a-1)(a-2)$	
Since (E) has a unique solution, $a^2 - 3a + 2 \neq 0$.	
Thus, we have $a \neq 1$ and $a \neq 2$.	1A
(ii) When $a = 2$, the augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ -1 & 2 & 1 & 3 \\ 4 & 1 & 2 & b \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 2 & 7 \\ 0 & -3 & -2 & b-16 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 2 & 7 \\ 0 & 0 & 0 & b-9 \end{array}\right)$	1M
When (E) is consistent, $b = 9$.	1A
The solution set of (E) is $\left\{\left(\frac{5-t}{3}, \frac{7-2t}{3}, t\right) : t \in \mathbb{R}\right\}$.	1A
(b) The solution set is $\left\{\left(\frac{5-t}{3}, \frac{7-2t}{3}, t\right) : t \in \mathbb{R}\right\}$.	
$p\left(\frac{5-t}{3}\right) = \left(\frac{7-2t}{3}\right)t$	1M
$0 = -\frac{2t^2}{3} + \left(\frac{7+p}{3}\right)t - \frac{5p}{3}$	
$\Delta = \left(\frac{7+p}{3}\right)^2 - 4\left(-\frac{2}{3}\right)\left(-\frac{5p}{3}\right) \geq 0$	1M+1A
$\frac{p^2}{9} - \frac{26p}{9} + \frac{49}{9} \geq 0$	
$p \leq 13 - 2\sqrt{30}$ or $p \geq 13 + 2\sqrt{30}$	1A
(c) $\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ -1 & 1 & 1 & 3 \\ 8 & 2 & 2 & 11 \\ 3 & 5 & 5 & 19 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 2 & 7 \\ 0 & -6 & -6 & -21 \\ 0 & 2 & 2 & 7 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 2 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$	1M
So, (F) is consistent.	1A
The solution set is $\left\{\left(\frac{1}{2}, \frac{7-2s}{2}, s\right) : s \in \mathbb{R}\right\}$.	1A

Solution	Marks
12. (a) $\overrightarrow{AB}=\overrightarrow{DC}$	
$(16-a)\mathbf{i}+(b-6)\mathbf{j}-6\mathbf{k}=10\mathbf{i}-3\mathbf{j}+(c+3)\mathbf{k}$	1M
We have $a=6$, $b=3$ and $c=-9$.	1A+1A
(b) $\overrightarrow{OA}\cdot\overrightarrow{AD}=6-6+0=0$	1A
(c) $\overrightarrow{AB}\times\overrightarrow{AD}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 10 & -3 & -6\\ 1 & -1 & 0\end{vmatrix}$	
$=-6\mathbf{i}-6\mathbf{j}-7\mathbf{k}$	1A
(d) (i) Let $\mathbf{n}=-\overrightarrow{AB}\times\overrightarrow{AD}=6\mathbf{i}+6\mathbf{j}+7\mathbf{k}$.	
$\overrightarrow{OE}=\dfrac{\overrightarrow{OA}\cdot\mathbf{n}}{\mathbf{n}\cdot\mathbf{n}}\mathbf{n}$	1M
$=\dfrac{36+36-21}{36+36+49}(6\mathbf{i}+6\mathbf{j}+7\mathbf{k})$	
$=\dfrac{51}{121}(6\mathbf{i}+6\mathbf{j}+7\mathbf{k})$	1A
Required distance $=\dfrac{51}{121}\sqrt{6^2+6^2+7^2}$	1M
$=\dfrac{51}{11}$	1A
(ii) $OA=\sqrt{6^2+6^2+3^2}=9$	
Let θ be the angle between OA and the plane $ABCD$.	
$\sin\theta=\dfrac{\left(\frac{51}{11}\right)}{9}$	1M
$\theta=\sin^{-1}\dfrac{51}{99}$	1A
(iii) Since $\overrightarrow{OA}\cdot\overrightarrow{AD}=0$ and E is the projection of O on $ABCD$, $\angle OAD=\angle EAD=90^\circ$.	1M
The angle between $\triangle OAD$ and the plane $ABCD$ is $\angle OAE$.	
The claim is agreed.	1A

END OF PAPER