Solution	Marks

REG-2324-MOCK-SET 9-MATH-EP(M2)
Suggested solutions

1. $f'\left(\frac{\pi}{2}\right) = \lim\limits_{h\to 0} \dfrac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$	1M
$= \lim\limits_{h\to 0} \dfrac{\left[\ln\left(\frac{\pi}{2}+h\right)\right][\sin(\pi+2h)] - \left(\ln\frac{\pi}{2}\right)(\sin\pi)}{h}$	
$= \lim\limits_{h\to 0} \dfrac{\left[\ln\left(\frac{\pi}{2}+h\right)\right](-\sin 2h)}{h}$	1M
$= -2\lim\limits_{h\to 0} \dfrac{\sin 2h}{2h} \ln\left(\dfrac{\pi}{2}+h\right)$	1M
$= -2(1)\ln\dfrac{\pi}{2}$	
$= -2\ln\dfrac{\pi}{2}$	1A
2. (a) $C_2^n(3)^2 = 252$	1M
$\dfrac{n(n-1)}{2} = 28$	1A
$\dfrac{n^2}{2} - \dfrac{n}{2} - 28 = 0$	
$n = 8$ or -7 (rejected)	1A
(b) $(1+3x)^n = (3x)^8 + C_1^8(3x)^7 + C_2^8(3x)^6 + \ldots$	1M
$= 6561x^8 + 17\,496x^7 + 20\,412x^6 + \ldots$	
$\left(1-\dfrac{2}{x}\right)^5 = 1 - 5\left(\dfrac{2}{x}\right) + 10\left(\dfrac{2}{x}\right)^2 + \ldots$	
$= 1 - \dfrac{10}{x} + \dfrac{40}{x^2} + \ldots$	
Required coefficient $= 1(20\,412) - 10(17\,496) + 40(6561)$	1M
$= 107\,892$	1A

Solution	Marks
3. (a) $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta + \cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta$	
$= (\sin\theta + 7\theta) + (\sin 3\theta + \sin 5\theta) + (\cos\theta + \cos 7\theta) + (\cos 3\theta + \cos 5\theta)$	
$= 2\sin 4\theta\cos 3\theta + 2\sin 4\theta\cos\theta + 2\cos 4\theta\cos 3\theta + 2\cos 4\theta\cos\theta$	1M
$= 2\sin 4\theta(\cos 3\theta + \cos\theta) + 2\cos 4\theta(\cos 3\theta + \cos\theta)$	
$= 2(\sin 4\theta + \cos 4\theta)(\cos 3\theta + \cos\theta)$	
$= 2(\sin 4\theta + \cos 4\theta)(2\cos 2\theta\cos\theta)$	1M
$= 4\cos\theta\cos 2\theta(\sin 4\theta + \cos 4\theta)$	1
(b) Using the result of (a),	
$4\cos\theta\cos 2\theta(\sin 4\theta + \cos 4\theta) = 0$	
We have $\cos\theta = 0$ or $\cos 2\theta = 0$ or $\sin 4\theta + \cos 4\theta = 0$.	1M
Solving, we have $\theta = \frac{\pi}{2}$ or $2\theta = \frac{\pi}{2}$ or $4\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$.	
Thus, $\theta = \frac{\pi}{2}$ or $\frac{\pi}{4}$ or $\frac{3\pi}{16}$ or $\frac{7\pi}{16}$.	1A+1A
4. (a) Area $= \frac{1}{2}(u)(5e^{u+2})$	
$= \frac{5}{2}ue^{u+2}$	1A
(b) Let A units be the area of $\triangle OPQ$.	
$A = \frac{5}{2}ue^{u+2}$	
$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{5}{2}(e^{u+2} + ue^{u+2})\frac{\mathrm{d}u}{\mathrm{d}t}$	1M
When $u = 5$ and $\frac{\mathrm{d}A}{\mathrm{d}t} = -15e^7$,	
$-15e^7 = \frac{5}{2}e^7(1+5)\frac{\mathrm{d}u}{\mathrm{d}t}$	1M
$\frac{\mathrm{d}u}{\mathrm{d}t} = -1$	1A
Rate of change of OQ is -1 unit per second.	
$PQ = 5e^{u+2}$	
$\frac{\mathrm{d}PQ}{\mathrm{d}t} = 5e^{u+2}\frac{\mathrm{d}u}{\mathrm{d}t}$	
When $u = 5$ and $\frac{\mathrm{d}u}{\mathrm{d}t} = -1$,	
$\frac{\mathrm{d}PQ}{\mathrm{d}t} = 5e^{5+2}(-1)$	
$= -5e^7$	1A
Rate of change of PQ is $-5e^7$ units per second.	

Solution	Marks
5. (a) When $n = 1$,	
L.H.S. $= 1(3^1) = 3$ and R.H.S. $= \frac{1}{4}[3 + (2-1)3^2] = 3 =$ L.H.S.	
It is true for $n = 1$.	1
Assume $\sum_{k=1}^{p} k(3^k) = \frac{1}{4}[3 + (2p-1)3^{p+1}]$, where $p \in \mathbb{Z}^+$.	1
$\sum_{k=1}^{p+1} k(3^k) = \sum_{k=1}^{p} k(3^k) + (p+1)(3^{p+1})$	
$= \frac{1}{4}[3 + (2p-1)3^{p+1}] + (p+1)(3^{p+1})$	1M
$= \frac{1}{4}\left[3 + 3^{p+1}[(2p-1) + 4(p+1)]\right]$	
$= \frac{1}{4}\left[3 + (2p+1)3^{p+2}\right]$	
It is true for $n = p + 1$.	
By M.I., it is true $\forall n \in \mathbb{Z}^+$.	1
(b) $2(3) + 3(3^2) + 4(3^3) + \ldots + 51(3^{50})$	
$= \sum_{k=1}^{50} (k+1)(3^k)$	
$= \sum_{k=1}^{50} k(3^k) + \sum_{k=1}^{50} 3^k$	1M
$= \frac{1}{4}[3 + (2 \times 50 - 1)3^{50+1}] + \frac{3(3^{50}-1)}{3-1}$	1A
$= \frac{3}{4} + \frac{99}{4}(3^{51}) + \frac{3^{51}}{2} - \frac{3}{2}$	
$= \frac{101}{4}(3^{51}) - \frac{3}{4}$	1A

Solution	Marks
6. (a) Let $u = x^2 + 5$. Then $du = 2x\,dx$.	1M
$g(x) = \int \frac{3x}{\sqrt{x^2+5}}\,dx$	1M
$= \frac{3}{2}\int \frac{du}{\sqrt{u}}$	
$= 3u^{\frac{1}{2}} + C$	
$= 3(x^2+5)^{\frac{1}{2}} + C$	
G passes through the point $\left(\frac{1}{4}, \frac{31}{4}\right)$.	
$\frac{31}{4} = 3\left[\left(\frac{1}{4}\right)^2 + 5\right]^{\frac{1}{2}} + C$	1M
$C = 1$	
Required equation is $y = 3(x^2+5)^{\frac{1}{2}} + 1$.	1A
(b) (i) Let the coordinates of P be (a, b).	
$\frac{b-4}{a+1} = \frac{3a}{\sqrt{a^2+5}}$	1M
$\sqrt{a^2+5}\left[(3\sqrt{a^2+5}+1)-4\right] = 3a^2+3a$	1M
$3(a^2+5) - 3\sqrt{a^2+5} = 3a^2+3a$	
$-3\sqrt{a^2+5} = 3a-15$	
$9(a^2+5) = 9a^2-90a+225$	
$0 = -90a+180$	
$a = 2$	
Required coordinates are $(2, 10)$.	1A
(ii) Required equation is	
$\frac{y-10}{x-2} = \frac{3(2)}{\sqrt{2^2+5}}$	1M
$2x - y + 6 = 0$	1A

Solution	Marks
7. (a) $\int (\ln x)^2\,\mathrm{d}x = x(\ln x)^2 - 2\int \ln x\,\mathrm{d}x$	1M
$= x(\ln x)^2 - 2x\ln x + 2\int \mathrm{d}x$	1M
$= x(\ln x)^2 - 2x\ln x + 2x + \text{constant}$	1A
(b) Note that the x-intercepts of the graphs of $y = \ln(1+x)$ and $y = \ln 2x$ are 0 and $\frac{1}{2}$ respectively.	
$\ln(1+x) = \ln 2x$	
$1 + x = 2x$	
$x = 1$	1A
Let $u = x + 1$ and $w = 2x$. Then $\mathrm{d}u = \mathrm{d}x$ and $\mathrm{d}w = 2\,\mathrm{d}x$.	
Volume $= \pi\int_0^1 [\ln(1+x)]^2\,\mathrm{d}x - \pi\int_{\frac{1}{2}}^1 (\ln 2x)^2\,\mathrm{d}x$	1M
$= \pi\int_1^2 (\ln u)^2\,\mathrm{d}u - \frac{1}{2}\pi\int_1^2 (\ln w)^2\,\mathrm{d}w$	
$= \frac{\pi}{2}\int_1^2 (\ln u)^2\,\mathrm{d}u$	
$= \frac{\pi}{2}\Big[u(\ln u)^2 - 2u\ln u + 2u\Big]_1^2$	1M
$= \pi[(\ln 2)^2 - 2\ln 2 + 1]$	1A

Solution	Marks
8. (a) $\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} a & 2 \\ b & c \end{pmatrix} = \begin{pmatrix} a & 2 \\ b & c \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$	
$\begin{pmatrix} a+2b & 2+2c \\ -a & -2 \end{pmatrix} = \begin{pmatrix} a-2 & 2a \\ b-c & 2b \end{pmatrix}$	
We have $a+2b = a-2$, $2+2c = 2a$, $-a = b-c$ and $-2 = 2b$.	1M
Solving, we have $b=-1$ and $c = a-1$.	
Thus, $B = \begin{pmatrix} a & 2 \\ -1 & a-1 \end{pmatrix}$.	1A
(b) $\det B = a(a-1)+2$	1M
$= a^2 - a + 2$	
$= \left(a - \frac{1}{2}\right)^2 + \frac{7}{4}$	
$\geq \frac{7}{4}$	
> 0	
Thus, B cannot be a singular matrix.	1
(c) $B^{-1} = \frac{1}{a^2-a+2}\begin{pmatrix} a-1 & -2 \\ 1 & a \end{pmatrix}$	1M
$(B^T)^{-1} = (B^{-1})^T$	
$= \frac{1}{a^2-a+2}\begin{pmatrix} a-1 & 1 \\ -2 & a \end{pmatrix}$	1A

Solution	Marks
9. (a) (i) $\dfrac{d}{dx}\left(\dfrac{x^2}{\sqrt{4-x^2}}\right)=\dfrac{2x\sqrt{4-x^2}-x^2\left(\frac{1}{2}\right)(4-x^2)^{-\frac{1}{2}}(-2x)}{4-x^2}$	
$=\dfrac{2x(4-x^2)+x^3}{(4-x^2)^{\frac{3}{2}}}$	
$=\dfrac{x(8-x^2)}{(4-x^2)^{\frac{3}{2}}}$	1A
(ii) Let $x=2\sin\theta$. Then $dx=2\cos\theta\,d\theta$.	1M
$\displaystyle\int\frac{x^2}{\sqrt{4-x^2}}\,dx=\int\frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}}\cdot 2\cos\theta\,d\theta$	
$\displaystyle=4\int\sin^2\theta\,d\theta$	
$\displaystyle=2\int(1-\cos 2\theta)\,d\theta$	1M
$=2\theta-\sin 2\theta+\text{constant}$	
$=2\theta-2\sin\theta\cos\theta+\text{constant}$	
$=2\sin^{-1}\dfrac{x}{2}-2\left(\dfrac{x}{2}\right)\left(\dfrac{\sqrt{2^2-x^2}}{2}\right)+\text{constant}$	
$=2\sin^{-1}\dfrac{x}{2}-\dfrac{x\sqrt{4-x^2}}{2}+\text{constant}$	1A
(b) Let $u=-x$. Then $du=-dx$.	
$\displaystyle\int_{-1}^{1}p(x)q(x)\,dx=\int_{-1}^{0}p(x)q(x)\,dx+\int_0^1 p(x)q(x)\,dx$	
$\displaystyle=-\int_{1}^{0}p(-u)q(-u)\,du+\int_0^1 p(x)q(x)\,dx$	1M
$\displaystyle=\int_0^1[1-p(u)]q(u)\,du+\int_0^1 p(x)q(x)\,dx$	1M
$\displaystyle=\int_0^1[1-p(x)]q(x)\,dx+\int_0^1 p(x)q(x)\,dx$	
$\displaystyle=\int_0^1[1-p(x)+p(x)]q(x)\,dx$	
$\displaystyle=\int_0^1 q(x)\,dx$	1
(c) $\displaystyle\int_{-1}^{1}\frac{x(8-x^2)}{(4-x^2)^{\frac{3}{2}}}\ln(e^x+1)\,dx$	
$\displaystyle=\left[\frac{x^2}{\sqrt{4-x^2}}\ln(e^x+1)\right]_{-1}^{1}-\int_{-1}^{1}\frac{x^2}{\sqrt{4-x^2}}\left(\frac{e^x}{e^x+1}\right)dx$	1M+1M
$\displaystyle=\frac{1}{\sqrt{3}}\ln\frac{e+1}{e^{-1}+1}-\int_{-1}^{1}\left(\frac{x^2}{\sqrt{4-x^2}}\right)\left(\frac{e^x}{e^x+1}\right)dx$	
Put $p(x)=\dfrac{e^x}{e^x+1}$ and $q(x)=\dfrac{x^2}{\sqrt{4-x^2}}$.	
$p(x)+p(-x)=\dfrac{e^x}{e^x+1}+\dfrac{e^{-x}}{e^{-x}+1}=\dfrac{e^x}{e^x+1}+\dfrac{1}{e^x(e^{-x}+1)}=1$	

Solution	Marks
$q(-x)=\dfrac{(-x)^2}{\sqrt{4-(-x)^2}}=\dfrac{x^2}{\sqrt{4-x^2}}=q(x)$	
Thus, $q(x)$ is an even function.	
Using the result of (b),	
$\displaystyle\int_{-1}^{1}\frac{x(8-x^2)}{(4-x^2)^{\frac{3}{2}}}\ln(e^x+1)\,\mathrm{d}x$	
$\displaystyle=\frac{1}{\sqrt{3}}\ln\frac{e+1}{e^{-1}+1}-\int_0^1\frac{x^2}{\sqrt{4-x^2}}\,\mathrm{d}x$	1M
$\displaystyle=\frac{1}{\sqrt{3}}\ln\frac{e+1}{e^{-1}(1+e)}-\left[2\sin^{-1}\frac{x}{2}-\frac{x\sqrt{4-x^2}}{2}\right]_0^1$	1M
$\displaystyle=\frac{1}{\sqrt{3}}-\left(2\times\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)$	
$\displaystyle=\frac{5\sqrt{3}}{6}-\frac{\pi}{3}$	1A

Solution	Marks
10. (a) Vertical asymptote is $x = 1$.	1A
$f(x) = \dfrac{x^2(x+5)}{(x-1)^2}$	
$= x + 7 + \dfrac{13x-7}{(x-1)^2}$	1M
Oblique asymptote is $y = x + 7$.	1A
(b) $f(x) = x + 7 + \dfrac{13x-7}{(x-1)^2}$	
$f'(x) = 1 + \dfrac{13(x-1)^2 - (13x-7)(2)(x-1)}{(x-1)^4}$	
$= 1 + \dfrac{-13x+1}{(x-1)^3}$	1A
$f''(x) = \dfrac{-13(x-1)^3 - (-13x+1)(3)(x-1)^2}{(x-1)^6}$	
$= \dfrac{2(13x+5)}{(x-1)^4}$	1A
(c) Note that $f'(x) = \dfrac{x(x-5)(x+2)}{(x-1)^3}$.	
When $f'(x) = 0$, $x = 5$ or 0 or -2.	1A

x	$x < -2$	$-2 < x < 0$	$0 < x < 1$	$1 < x < 5$	$x > 5$
$f'(x)$	$+$	$-$	$+$	$-$	$+$

1M

Maximum point is $\left(-2, \dfrac{4}{3}\right)$. 1A

(d) When $f''(x) = 0$, $x = \dfrac{-5}{13}$.

x	$x < -\dfrac{5}{13}$	$-\dfrac{5}{13} < x < 1$	$x > 1$
$f''(x)$	$-$	$+$	$+$

1M

G has one point of inflexion at $x = -\dfrac{5}{13}$.

The claim is disagreed. 1A

(e) Let $u = x - 1$. Then $\mathrm{d}u = \mathrm{d}x$.

Solution	Marks
$\text{Area} = \int_{-5}^{0} f(x)\,\mathrm{d}x$	1M
$= \int_{-5}^{0} \left(x + 7 + \dfrac{13x - 7}{(x-1)^2} \right) \mathrm{d}x$	
$= \left[\dfrac{x^2}{2} + 7x \right]_{-5}^{0} + \int_{-6}^{-1} \dfrac{13(u+1) - 7}{u^2}\,\mathrm{d}u$	
$= \dfrac{45}{2} + \int_{-6}^{-1} (13u^{-1} + 6u^{-2})\,\mathrm{d}u$	
$= \dfrac{45}{2} + \left[13\ln\lvert u\rvert - 6u^{-1} \right]_{-6}^{-1}$	1M
$= \dfrac{55}{2} - 13\ln 6$	1A

Solution	Marks
11. (a) $\overrightarrow{AB} = 7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k}$	
$\overrightarrow{AC} = 4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$	
$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 2 & -10 \\ 4 & 5 & 2 \end{vmatrix}$	1M
$= 54\mathbf{i} - 54\mathbf{j} + 27\mathbf{k}$	
A required vector $= \dfrac{1}{\sqrt{54^2 + 54^2 + 27^2}}(54\mathbf{i} - 54\mathbf{j} + 27\mathbf{k})$	1M
$= \dfrac{2}{3}\mathbf{i} - \dfrac{2}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$	1A
(b) (i) $\overrightarrow{DA} = -7\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$	
Let $\mathbf{n} = \dfrac{2}{3}\mathbf{i} - \dfrac{2}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$ such that $\mathbf{n}$ is perpendicular to Π.	
$\overrightarrow{DE} = \left(\dfrac{\overrightarrow{DA} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}}\right)\mathbf{n}$	1M
$= \dfrac{-\frac{14}{3} - \frac{8}{3} - \frac{5}{3}}{1}\left(\dfrac{2}{3}\mathbf{i} - \dfrac{2}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}\right)$	
$= -6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	
$\overrightarrow{AE} = \overrightarrow{DE} - \overrightarrow{DA}$	1M
$= \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	1A
(ii) $\overrightarrow{BC} = -3\mathbf{i} + 3\mathbf{j} + 12\mathbf{k}$	
$\overrightarrow{BF} = \overrightarrow{AF} - \overrightarrow{AB}$	
$= \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) - (7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k})$	1M
$= (\lambda - 7)\mathbf{i} + (2\lambda - 2)\mathbf{j} + (2\lambda + 10)\mathbf{k}$	
Note that B, C and F are collinear.	
$\dfrac{\lambda - 7}{-3} = \dfrac{2\lambda - 2}{3} = \dfrac{2\lambda + 10}{12}$	1M
Solving, we have $\lambda = 3$.	1A
(iii) Required angle is $\angle DFE$.	1M
$\overrightarrow{EF} = \overrightarrow{AF} - \overrightarrow{AE} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$	
$EF = \sqrt{2^2 + 4^2 + 4^2} = 6$	
$DE = \sqrt{6^2 + 6^2 + 3^2} = 9$	
$\tan\theta = \dfrac{DE}{EF}$	1M
$= \dfrac{3}{2}$	1A

Solution	Marks
12. (a) (i) (1) $\Delta = \begin{vmatrix} 1 & -2 & h \\ 1 & 3 & -1 \\ h & 7 & 1 \end{vmatrix}$	
$= -3h^2 + 9h + 12$	1A
$= -3(h+1)(h-4)$	
Since (E) has a unique solution, we have $\Delta \neq 0$.	1M
Thus, $h \neq -1$ and $h \neq 4$.	1A
(2) $x = \dfrac{\begin{vmatrix} 16 & -2 & h \\ k & 3 & -1 \\ 4 & 7 & 1 \end{vmatrix}}{-3(h+1)(h-4)}$	1M
$= \dfrac{-7hk + 12h - 2k - 168}{3(h+1)(h-4)}$	1A
$y = \dfrac{\begin{vmatrix} 1 & 16 & h \\ 1 & k & -1 \\ h & 4 & 1 \end{vmatrix}}{-3(h+1)(h-4)}$	
$= \dfrac{hk - k + 12}{3(h-4)}$	
$z = \dfrac{\begin{vmatrix} 1 & -2 & 16 \\ 1 & 3 & k \\ h & 7 & 4 \end{vmatrix}}{-3(h+1)(h-4)}$	1A
(ii) (1) When $h = 4$, the augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 1 & -2 & 4 & 16 \\ 1 & 3 & -1 & k \\ 4 & 7 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -2 & 4 & 16 \\ 0 & 5 & -5 & k-16 \\ 0 & 15 & -15 & -60 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -2 & 4 & 16 \\ 0 & 5 & -5 & k-16 \\ 0 & 0 & 0 & -12-3k \end{array}\right)$	1M
Since (E) is consistent, we have $-12 - 3k = 0$.	
Thus, $k = -4$.	1
(2) When $h = 4$ and $k = -4$, the augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 1 & -2 & 4 & 16 \\ 0 & 5 & -5 & -20 \\ 0 & 0 & 0 & 0 \end{array}\right)$	
The solution set is $\{(8-2t,\ t-4,\ t) : t \in \mathbb{R}\}$.	1A
(b) When $h = 4$ and $k = -4$, the solution set to the first three equations is $\{(8-2t,\ t-4,\ t) : t \in \mathbb{R}\}$.	
$x^2 + y^2 - 5z^2 = 0$	
$(8-2t)^2 + (t-4)^2 - 5t^2 = 0$	
$80 - 40t = 0$	
$t = 2$	
Thus, (F) is consistent when $h = 4$.	1M

Solution	Marks
When $h = -1$ and $k = -4$, the augmented matrix of the first three equations is	
$\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 16 \\ 1 & 3 & -1 & -4 \\ -1 & 7 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -2 & 1 & 16 \\ 0 & 5 & 0 & -20 \\ 0 & 5 & 0 & 20 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -2 & 1 & 16 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 40 \end{array}\right)$	
The system is inconsistent when $h = -1$.	1M
When $h \neq -1$ and $h \neq -4$ and $k = -4$,	
we have $x = \dfrac{40}{3(h+1)}$, $y = \dfrac{-4}{3}$ and $z = \dfrac{40}{3(h+1)}$.	
$x^2 + y^2 - 5z^2 = 0$	
$\left[\dfrac{40}{3(h+1)}\right]^2 + \left(\dfrac{-4}{3}\right)^2 - 5\left[\dfrac{40}{3(h+1)}\right]^2 = 0$	
$\dfrac{16}{9} = 4\left[\dfrac{40}{3(h+1)}\right]^2$	
$(h+1)^2 = 400$	
$h = 19 \quad \text{or} \quad -21$	
(F) is consistent when $h = 19$ or $h = -21$.	1M
Thus, (F) is consistent when $h = -21$ or 4 or 19.	
The claim is agreed.	1A

END OF PAPER