

Solution	Marks
REG-2324-MOCK-SET 11-MATH-EP(M2)	
Suggested solutions	
1. $f'(1) = \lim_{h \rightarrow 0} \frac{\ln(1+h)^4 - \ln 1}{h}$ $= \lim_{h \rightarrow 0} \left[\frac{1}{h} \ln(1+h)^4 \right]$ $= \lim_{h \rightarrow 0} \ln \left[(1+h)^{\frac{1}{h}} \right]^4$ $= \ln e^4$ $= 4$	1M 1M 1M 1A
2. (a) $(1-2x)^n(2+3x)^2$ $= [1 + C_1^n(-2x) + C_2^n(-2x)^2 + \dots] (4 + 12x + 9x^2)$ $= [1 - 2nx + 2n(n-1)x^2 + \dots] (4 + 12x + 9x^2)$ Consider the coefficient of x^2. $(1)(9) + (-2n)(12) + 2n(n-1)(4) = 49$ $8n^2 - 32n - 40 = 0$ $n = 5 \quad \text{or} \quad -1 \text{ (rejected)}$ (b) $(1-2x)^2(2+3x)^2 = (1-10x+40x^2+\dots)(4+12x+9x^2)$. Required coefficient $= (1)(12) + (-10)(4)$ $= -28$	1M 1M+1A 1A 1A

Solution	Marks
3. (a) $\frac{1 - \cos 2\theta}{\sin 2\theta} = \frac{1 - (1 - 2 \sin^2 \theta)}{2 \sin \theta \cos \theta}$ $= \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta}$ $= \frac{\sin \theta}{\cos \theta}$ $= \tan \theta$ (b) $\sqrt{3} \cos 4x - \sqrt{3} \cos 4x \cos 8x = \sin 8x - \sin 8x \cos 4x$ $\sqrt{3} \cos 4x (1 - \cos 8x) = \sin 8x - \sin 8x \cos 4x$ $\sqrt{3} \cos 4x \tan 4x \sin 8x = \sin 8x (1 - \cos 4x)$ $\sin 8x (\sqrt{3} \sin 4x - 1 + \cos 4x) = 0$ $\sin 8x = 0 \quad \text{or} \quad \sqrt{3} \sin 4x - 1 + \cos 4x = 0$ $8x = 5\pi \text{ or } 6\pi \text{ or } 7\pi$ $x = \frac{5\pi}{8} \text{ or } \frac{3\pi}{4} \text{ or } \frac{7\pi}{8}$ $\sqrt{3} = \frac{1 - \cos 4x}{\sin 4x}$ $\tan 2x = \sqrt{3}$ $2x = \frac{4\pi}{3}$ $x = \frac{2\pi}{3}$ Thus, $x = \frac{5\pi}{8} \text{ or } \frac{2\pi}{3} \text{ or } \frac{3\pi}{4} \text{ or } \frac{7\pi}{8}$.	1M 1 1M 1M 1A+1A
4. (a) $\int (4 + k\sqrt{t})^2 dt = \int (16 + 8k\sqrt{t} + k^2 t) dt$ $= 16t + \frac{16kt^{\frac{3}{2}}}{3} + \frac{k^2 t^2}{2} + \text{constant}$ (b) We have $\Gamma: x = 4 \pm \sqrt{y}$. Volume $= \pi \int_0^{16} (4 + \sqrt{y})^2 dy - \pi \int_0^{16} (4 - \sqrt{y})^2 dy$ $= \pi \left[16y + \frac{16y^{\frac{3}{2}}}{3} + \frac{y^2}{2} \right]_0^{16} - \pi \left[16y - \frac{16y^{\frac{3}{2}}}{3} + \frac{y^2}{2} \right]_0^{16}$ $= \frac{2048\pi}{3}$	1M 1A 1M 1M+1A 1M 1A

Solution	Marks
5. Let (a, b) be the point of contact. $4x + y^2 = 16$ $4 + 2y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{2}{y}$ The tangent passes through (a, b) and $(6, 0)$. $\frac{b - 0}{a - 6} = -\frac{2}{b}$ $b^2 = 12 - 2a$ $(16 - 4a) = 12 - 2a$ $a = 2$ The points of contact are $(2, 2\sqrt{2})$ and $(2, -2\sqrt{2})$. The equation of the tangent to C at $(2, 2\sqrt{2})$ is $\frac{y - 2\sqrt{2}}{x - 2} = -\frac{2}{2\sqrt{2}}$ $x + \sqrt{2}y - 6 = 0$ The equation of the tangent to C at $(2, -2\sqrt{2})$ is $\frac{y + 2\sqrt{2}}{x - 2} = -\frac{2}{-2\sqrt{2}}$ $x - \sqrt{2}y - 6 = 0$	1M 1M 1M 1A 1M 1A 1A

Solution		Marks						
6. (a) $y = \int x \ln x^4 \, dx$ $= 4 \int x \ln x \, dx$ $= 2x^2 \ln x - 2 \int x \, dx$ $= 2x^2 \ln x - x^2 + C$ Γ passes through the point $P(e^2, 3e^4 + 7)$. $3e^4 + 7 = 2(e^2)^2 \ln e^2 - (e^2)^2 + C$ $C = 7$ Required equation is $y = 2x^2 \ln x - x^2 + 7$.		1M 1A						
(b) $f'(x) = 4x \ln x$ $f''(x) = 4(\ln x + 1)$ When $f''(x) = 0$, $x = \frac{1}{e}$.		1A						
<table border="1"> <tr> <td>x</td><td>$0 < x < \frac{1}{e}$</td><td>$x > \frac{1}{e}$</td></tr> <tr> <td>$f''(x)$</td><td>–</td><td>+</td></tr> </table>	x	$0 < x < \frac{1}{e}$	$x > \frac{1}{e}$	$f''(x)$	–	+		1M
x	$0 < x < \frac{1}{e}$	$x > \frac{1}{e}$						
$f''(x)$	–	+						
The point of inflexion of Γ is $\left(\frac{1}{e}, -3e^{-2} + 7\right)$.		1A						

Solution	Marks
7. (a) $\int \ln(x + \sqrt{x^2 - 1}) \, dx$ $= x \ln(x + \sqrt{x^2 - 1}) - \int \frac{x}{x + \sqrt{x^2 - 1}} \left(1 + \frac{1}{2}(x^2 - 1)^{-\frac{1}{2}}(2x)\right) dx$ $= x \ln(x + \sqrt{x^2 - 1}) - \int \frac{x(x^2 - 1)^{-\frac{1}{2}}(\sqrt{x^2 - 1} + x)}{x + \sqrt{x^2 - 1}} dx$ $= x \ln(x + \sqrt{x^2 - 1}) - \int \frac{x}{\sqrt{x^2 - 1}} dx$ Let $u = x^2 - 1$. Then $du = 2x \, dx$. $\int \ln(x + \sqrt{x^2 - 1}) \, dx$ $= x \ln(x + \sqrt{x^2 - 1}) - \frac{1}{2} \int u^{-\frac{1}{2}} du$ $= x \ln(x + \sqrt{x^2 - 1}) - u^{\frac{1}{2}} + \text{constant}$ $= x \ln(x + \sqrt{x^2 - 1}) - \sqrt{x^2 - 1} + \text{constant}$ (b) $\ln(x + \sqrt{x^2 - 1}) = 0$ $x + \sqrt{x^2 - 1} = 1$ $x^2 - 1 = (1 - x)^2$ $x = 1$ Required area $= \int_1^{\frac{e+e^{-1}}{2}} \ln(x + \sqrt{x^2 - 1}) \, dx$ $= \left[x \ln(x + \sqrt{x^2 - 1}) - \sqrt{x^2 - 1} \right]_1^{\frac{e+e^{-1}}{2}}$ $= \frac{1}{e}$	1M+1M 1M 1A 1M 1M 1A

Solution	Marks
8. (a) When $n = 1$, $\text{L.H.S.} = \frac{1}{(2+2)(2+3)} + \frac{1}{(3+2)(3+3)} = \frac{1}{12}$ $\text{R.H.S.} = \frac{1}{6(1+1)} = \frac{1}{12} = \text{L.H.S.}$ It is true for $n = 1$. Assume $\sum_{k=2p}^{3p} \frac{1}{(k+2)(k+3)} = \frac{1}{6(p+1)}$, where $p \in \mathbb{Z}^+$. $\begin{aligned} & \sum_{k=2p+2}^{3p+3} \frac{1}{(k+2)(k+3)} \\ &= \sum_{k=2p}^{3p} \frac{1}{(k+2)(k+3)} - \frac{1}{(2p+2)(2p+3)} - \frac{1}{(2p+3)(2p+4)} + \frac{1}{(3p+3)(3p+4)} \\ & \quad + \frac{1}{(3p+4)(3p+5)} + \frac{1}{(3p+5)(3p+6)} \\ &= \frac{1}{6(p+1)} - \frac{1}{(2p+2)(2p+3)} - \frac{1}{(2p+3)(2p+4)} + \frac{1}{(3p+3)(3p+4)} \\ & \quad + \frac{1}{(3p+4)(3p+5)} + \frac{1}{(3p+5)(3p+6)} \\ &= \frac{1}{6(p+1)} - \left(\frac{1}{2p+2} - \frac{1}{2p+3} \right) - \left(\frac{1}{2p+3} - \frac{1}{2p+4} \right) + \left(\frac{1}{3p+3} - \frac{1}{3p+4} \right) \\ & \quad + \left(\frac{1}{3p+4} - \frac{1}{3p+5} \right) + \left(\frac{1}{3p+5} - \frac{1}{3p+6} \right) \\ &= \frac{1}{6(p+1)} - \frac{1}{2p+2} + \frac{1}{2p+4} + \frac{1}{3p+3} - \frac{1}{3p+6} \\ &= \frac{1}{p+1} \left(\frac{1}{6} - \frac{1}{2} + \frac{1}{3} \right) + \frac{1}{p+2} \left(\frac{1}{2} - \frac{1}{3} \right) \\ &= \frac{1}{6(p+2)} \end{aligned}$ It is true for $n = p + 1$. By M.I., it is true $\forall n \in \mathbb{Z}^+$. (b) $\sum_{k=40}^{90} \frac{1}{(k+2)(k+3)}$ $\begin{aligned} &= \sum_{k=40}^{60} \frac{1}{(k+2)(k+3)} + \sum_{k=60}^{90} \frac{1}{(k+2)(k+3)} - \frac{1}{(62)(63)} \\ &= \frac{1}{6(20+1)} + \frac{1}{6(30+1)} - \frac{1}{3906} \\ &= \frac{17}{1302} \end{aligned}$	1 1 1M 1M 1 1M 1A

Solution	Marks
9. (a) $PQ = \sqrt{u^2 + 64} + \sqrt{(u-9)^2 + 16}$ $\frac{dPQ}{du} = \frac{u}{\sqrt{u^2 + 64}} + \frac{u-9}{\sqrt{(u-9)^2 + 16}}$ $= \frac{u\sqrt{(u-9)^2 + 16} + (u-9)\sqrt{u^2 + 64}}{\sqrt{u^2 + 64}\sqrt{(u-9)^2 + 16}}$ $= \frac{u^2[(u-9)^2 + 16] - (u-9)^2(u^2 + 64)}{\sqrt{u^2 + 64}\sqrt{(u-9)^2 + 16}(u\sqrt{(u-9)^2 + 16} - (u-9)\sqrt{u^2 + 64})}$ $= \frac{-48(u-6)(u-18)}{\sqrt{u^2 + 64}\sqrt{(u-9)^2 + 16}(u\sqrt{(u-9)^2 + 16} - (u-9)\sqrt{u^2 + 64})}$ For $\frac{dPQ}{du} = 0$, we have $u = 6$. Thus, we have $a = 6$.	1M 1M 1M 1A
(b) (i) Let A square units be the area of the rectangle $PQSR$. $A = uPQ$ $\frac{dA}{du} = PQ + u \frac{dPQ}{du}$ When $u = 6$, $\frac{dA}{du} = \sqrt{6^2 + 64} + \sqrt{(6-9)^2 + 16} + 6(0)$ $= 10 \neq 0$ Hence, A does not attain its minimum value when $u = 6$. The claim is disagreed.	1M 1M 1M 1A
(ii) $OP = \sqrt{u^2 + (u^2 + 64)}$ $= \sqrt{2u^2 + 64}$ $\frac{dOP}{dt} = \frac{2u}{\sqrt{2u^2 + 64}} \frac{du}{dt}$ When $u = 6$, $12 = \frac{2(6)}{\sqrt{2(6)^2 + 64}} \left(\frac{du}{dt} \Big _{u=6} \right)$ $\frac{du}{dt} \Big _{u=6} = 2\sqrt{34}$ Let w units be the perimeter of the rectangle $PQSR$. $w = 2(u + PQ)$ $\frac{dw}{dt} = 2 \frac{du}{dt} + 2 \frac{dPQ}{du} \cdot \frac{du}{dt}$ When $u = 6$, $\frac{dw}{dt} \Big _{u=6} = 2(2\sqrt{34}) + 2(0)(2\sqrt{34})$ $= 4\sqrt{34}$	1M 1M 1A
Required rate is $4\sqrt{34}$ units per second.	

Solution		Marks
10. (a) (i) (1)	$\begin{vmatrix} 1 & a+2 & 1 \\ 1 & -a & 6b-1 \\ 2 & a+3 & 2b+1 \end{vmatrix} = \begin{vmatrix} 1 & -a \\ 2 & a+3 \end{vmatrix} - (6b-1) \begin{vmatrix} 1 & a+2 \\ 2 & a+3 \end{vmatrix} + (2b+1) \begin{vmatrix} 1 & a+2 \\ 1 & -a \end{vmatrix}$ $= (3a+3) - (6b-1)(-a-1) + (2b+1)(-2a-2)$ $= 2b(a+1)$ Since (E) has unique solution, we have $2b(a+1) \neq 0$. Thus, $a \neq -1$ and $b \neq 0$.	1M
(2)	$x = \frac{\begin{vmatrix} 2 & a+2 & 1 \\ 1 & -a & 6b-1 \\ 3 & a+3 & 2b+1 \end{vmatrix}}{2b(a+1)}$ $= \frac{-4b+1}{2b(a+1)}$ $y = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6b-1 \\ 2 & 3 & 2b+1 \end{vmatrix}}{2b(a+1)}$ $= \frac{4b-1}{2b(a+1)}$ $z = \frac{\begin{vmatrix} 1 & a+2 & 2 \\ 1 & -a & 1 \\ 2 & a+3 & 3 \end{vmatrix}}{2b(a+1)}$ $= \frac{a+1}{2b(a+1)}$ $= \frac{1}{2b}$	1M 1A
(ii) (1)	The augmented matrix of (E) is $\left(\begin{array}{ccc c} 1 & 1 & 1 & 2 \\ 1 & 1 & 6b-1 & 1 \\ 2 & 2 & 2b+1 & 3 \end{array} \right) \sim \left(\begin{array}{ccc c} 1 & 1 & 1 & 2 \\ 0 & 0 & 6b-2 & -1 \\ 0 & 0 & 2b-1 & -1 \end{array} \right) \sim \left(\begin{array}{ccc c} 1 & 1 & 1 & 2 \\ 0 & 0 & 6b-2 & -1 \\ 0 & 0 & -\frac{1}{3} & -\frac{2}{3} \end{array} \right)$ We have $z = 2$. $(6b-2)(2) = -1$ $b = \frac{1}{4}$	1M
(2)	The augmented matrix of (E) is $\left(\begin{array}{ccc c} 1 & 1 & 1 & 2 \\ 0 & 0 & -\frac{1}{2} & -1 \\ 0 & 0 & -\frac{1}{3} & -\frac{2}{3} \end{array} \right) \sim \left(\begin{array}{ccc c} 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right)$ The solution set is $\{(-t, t, 2) : t \in \mathbb{R}\}$.	1A
(b)	When $a = -1$ and $b = \frac{1}{4}$, the system (F) is equivalent to (E).	

Solution	Marks
The solution set of (F) is $\{(-t, t, 2) : t \in \mathbb{R}\}$. $4(-t) + t^2 + 2 < -2$ $t^2 - 4t + 4 < 0$ $(t - 2)^2 < 0$ Note that $(t - 2)^2 \geq 0$ for all real values of t. There is no real solutions of (F) satisfying $4x + y^2 + z < -2$.	1M 1M 1A

Solution		Marks
11. (a)	$\int_0^1 x e^{ax} dx = \left[\frac{x e^{ax}}{a} \right]_0^1 - \frac{1}{a} \int_0^1 e^{ax} dx$ $= \frac{e^a}{a} - \left[\frac{e^{ax}}{a^2} \right]_0^1$ $= \frac{a e^a - e^a + 1}{a^2}$	1M+1M
(b)	Let $u = \ln(x+1)$. Then $du = \frac{1}{x+1} dx$. $\int_0^{e-1} \frac{\ln(x+1)}{(1+x)^r} dx = \int_0^1 \frac{u}{(e^u)^{r-1}} du$ $= \int_0^1 u e^{(1-r)u} du$ $= \frac{(1-r)e^{1-r} - e^{1-r} + 1}{(1-r)^2}$ $= \frac{1 - r e^{1-r}}{(1-r)^2}$	1 1M
(c)	Let $v = (e-1)\tan x$. Then $dv = (e-1)\sec^2 x dx$. $\int_0^{\frac{\pi}{4}} \frac{\ln[1+(e-1)\tan x]}{(1+e\tan x - \tan x)^3} dx + \int_0^{\frac{\pi}{4}} \frac{[\ln[1+(e-1)\tan x]] \tan^2 x}{(1+e\tan x - \tan x)^3} dx$ $= \int_0^{\frac{\pi}{4}} \frac{\ln[1+(e-1)\tan x] \sec^2 x}{[1+(e-1)\tan x]^3} dx$ $= \frac{1}{e-1} \int_0^{e-1} \frac{\ln(1+v)}{(1+v)^3} dv$ $= \frac{1}{e-1} \times \frac{1-3e^{1-3}}{(1-3)^2}$ $= \frac{e^2-3}{4e^2(e-1)}$	1M 1A
(d)	Let $w = \frac{\pi}{2} - 2x$. Then $dw = -2 dx$. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{[\ln[1+(e-1)\cot 2x]] \csc^2 2x}{(1+e\cot 2x - \cot 2x)^3} dx$ $= -\frac{1}{2} \int_{\frac{\pi}{4}}^0 \frac{\ln[1+(e-1)\cot(\frac{\pi}{2}-w)] \csc^2(\frac{\pi}{2}-w)}{[1+e\cot(\frac{\pi}{2}-w) - \cot(\frac{\pi}{2}-w)]^3} dw$ $= \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{\ln[1+(e-1)\tan w] \sec^2 w}{(1+e\tan w - \tan w)^3} dw$ $= \frac{1}{2} \times \frac{e^2-3}{4e^2(e-1)}$ $= \frac{e^2-3}{8e^2(e-1)}$	1M 1A

Solution		Marks
12. (a) (i)	Note that $AK \perp BC$, $BK \perp AC$ and $HA = HB = HC$. $ \begin{aligned} & (\overrightarrow{HK} - \overrightarrow{HA} - \overrightarrow{HB} - \overrightarrow{BC}) \cdot \overrightarrow{BC} \\ &= \overrightarrow{AK} \cdot \overrightarrow{BC} - (\overrightarrow{HB} + \overrightarrow{HC}) \cdot (\overrightarrow{HC} - \overrightarrow{HB}) \\ &= 0 - (HC^2 - HB^2) \\ &= 0 \\ & (\overrightarrow{HK} - \overrightarrow{HA} - \overrightarrow{HB} - \overrightarrow{HC}) \cdot \overrightarrow{AC} \\ &= \overrightarrow{BK} \cdot \overrightarrow{AC} - (\overrightarrow{HA} + \overrightarrow{HC}) \cdot (\overrightarrow{HC} - \overrightarrow{HA}) \\ &= 0 - (HC^2 - HA^2) \\ &= 0 \end{aligned} $	1M 1M 1
(ii)	Using the result of (a)(i), we have $\overrightarrow{HK} - \overrightarrow{HA} - \overrightarrow{HB} - \overrightarrow{HC} = \mathbf{0}$. Thus, we have $\overrightarrow{HK} = \overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC}$.	1M 1
(b) (i)	We have $\overrightarrow{AB} = -4\mathbf{i}$ and $\overrightarrow{AC} = -2\sqrt{2}\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$. $ \begin{aligned} \overrightarrow{AB} \times \overrightarrow{AC} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 0 & 0 \\ -2\sqrt{2} & -2 & -2 \end{vmatrix} \\ &= -8\mathbf{j} + 8\mathbf{k} \\ S &= \frac{1}{2} \overrightarrow{AB} \times \overrightarrow{AC} \\ &= \frac{1}{2} \sqrt{8^2 + 8^2} \\ &= 4\sqrt{2} \end{aligned} $	1M 1A
(ii)	Let r be the radius of the circumcircle ABC. $ \begin{aligned} r &= \frac{abc}{4S} \\ &= \frac{\sqrt{32 - 16\sqrt{2}}(4)(4)}{4(4\sqrt{2})} \\ &= \sqrt{16 - 8\sqrt{2}} \end{aligned} $ Note that $AB = AC$. Denote the mid-point of BC by M. $ \begin{aligned} \overrightarrow{MA} &= \frac{1}{2}\overrightarrow{BA} + \frac{1}{2}\overrightarrow{CA} \\ &= (2 + \sqrt{2})\mathbf{i} + \mathbf{j} + \mathbf{k} \\ \overrightarrow{HA} &= \frac{r}{MA} \overrightarrow{MA} \\ &= \frac{\sqrt{16 - 8\sqrt{2}}}{\sqrt{8 + 4\sqrt{2}}} [(2 + \sqrt{2})\mathbf{i} + \mathbf{j} + \mathbf{k}] \\ &= \sqrt{\frac{8(2 - \sqrt{2})^2}{4(2 + \sqrt{2})(2 - \sqrt{2})}} [(2 + \sqrt{2})\mathbf{i} + \mathbf{j} + \mathbf{k}] \\ &= (2 - \sqrt{2})[(2 + \sqrt{2})\mathbf{i} + \mathbf{j} + \mathbf{k}] \end{aligned} $	1M 1A

Solution	Marks
(iii) $\overrightarrow{HK} = \overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC}$ $= \overrightarrow{HA} + (\overrightarrow{HA} + \overrightarrow{AB}) + (\overrightarrow{HA} + \overrightarrow{AC})$ $= (2 - 2\sqrt{2})\mathbf{i} + (4 - 3\sqrt{2})\mathbf{j} + (4 - 3\sqrt{2})\mathbf{k}$	1M 1A