Solution	Marks
REG-2324-MOCK-SET 10-MATH-EP(M2)	
Suggested solutions	
1. $\dfrac{\mathrm{d}}{\mathrm{d}x}(\tan 2x) = \lim\limits_{h\to 0}\dfrac{\tan(2x+2h)-\tan 2x}{h}$	1M
$= \lim\limits_{h\to 0}\dfrac{\tan[(2x+2h)-2x][1+\tan(2x+2h)\tan 2x]}{h}$	1M
$= \lim\limits_{h\to 0}\dfrac{\sin 2h}{2h}\times\dfrac{2(1+\tan(2x+2h)\tan 2x)}{\cos 2h}$	1M
$= 1\times\dfrac{2(1+\tan^2 2x)}{1}$	
$= 2\sec^2 2x$	1A
2. (a) $\tan 3\theta = \tan(2\theta+\theta)$	
$= \dfrac{\tan 2\theta + \tan\theta}{1-\tan 2\theta\tan\theta}$	1M
$= \dfrac{\frac{2\tan\theta}{1-\tan^2\theta}+\tan\theta}{1-\frac{2\tan\theta}{1-\tan\theta}\times\tan\theta}$	
$= \dfrac{2\tan\theta+\tan\left(1-\tan^2\theta\right)}{(1-\tan^2\theta)+2\tan^2\theta}$	
$= \dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$	1
(b) $\sqrt{3}\tan\theta+3 = \cot^2\theta+3\sqrt{3}\cot\theta$	
$\sqrt{3}\tan^3\theta+3\tan^2\theta = 1+3\sqrt{3}\tan\theta$	
$-\sqrt{3}(3\tan\theta-\tan^3\theta) = 1-3\tan^2\theta$	
$\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = -\dfrac{1}{\sqrt{3}}$	
$\tan 3\theta = -\dfrac{1}{\sqrt{3}}$	1M
$3\theta = \pm\dfrac{7\pi}{6}$ or $\pm\dfrac{\pi}{6}$	
$\theta = \pm\dfrac{7\pi}{18}$ or $\pm\dfrac{\pi}{18}$	1A+1A

Solution	Marks
3. (a) $y = \int \frac{x+1}{x+3}\,\mathrm{d}x$	1M
$= \int \frac{x+3-2}{x+3}\,\mathrm{d}x$	
$= \int \left(1 - \frac{2}{x+3}\right)\mathrm{d}x$	
$= x - 2\ln\lvert x+3\rvert + C$	1A
Γ passes through $(e-3,\ 2e-5)$.	
$2e - 5 = (e-3) - 2\ln e + C$	
$C = e$	
Required equation is $x - 2\ln\lvert x+3\rvert + e$.	1A
(b) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x}\left(1 - \frac{2}{x+3}\right)$	
$= \frac{2}{(x+3)^2}$	1A
> 0	
Γ does not have a point of inflexion.	1A
4. (a) General term $= C_r^9 (x)^{9-r}\left(\frac{k}{x^2}\right)^r$	1M
$= C_r^9 k^r x^{9-3r}$	
Put $9 - 3r = 3$, we have $r = 2$.	
$C_2^9 k^2 = 576$	1M
$k^2 = 16$	
$k = 4$ or -4 (rejected)	1A
(b) $\left(x + \frac{k}{x^2}\right)^9 (2+x)^3$	
$= \left[x^9 + C_1^9 (x)^8\left(\frac{4}{x^2}\right) + C_2^9 (x)^7\left(\frac{4}{x^2}\right)^2 + \ldots + \left(\frac{4}{x^2}\right)^9\right](8 + 12x + 6x^2 + x^3)$	1A
Constant term	
$= C_3^9 (4)^3 \times 8 + C_4^9 (4)^4 \times 1$	1M
$= 75\,264$	1A

Solution	Marks
5. (a) $\int u(4^u)\,\mathrm{d}u = \dfrac{u(4^u)}{\ln 4} - \dfrac{1}{\ln 4}\int 4^u\,\mathrm{d}u$	1M
$= \dfrac{u(4^u)}{\ln 4} - \dfrac{4^u}{(\ln 4)^2} + \text{constant}$	1A
$= \dfrac{u(4^u)}{2\ln 2} - \dfrac{4^u}{4(\ln 2)^2} + \text{constant}$	
(b) Required area	
$= \int_{-1}^{0} x^2(4^x)\,\mathrm{d}x$	1A
$= \left[\dfrac{x^2(4^x)}{\ln 4}\right]_{-1}^{0} - \dfrac{2}{\ln 4}\int_{-1}^{0} x(4^x)\,\mathrm{d}x$	1M
$= \dfrac{1}{8\ln 2} - \dfrac{1}{\ln 2}\left[\dfrac{x(4^x)}{\ln 4} - \dfrac{4^x}{(\ln 4)^2}\right]_{-1}^{0}$	1M
$= \dfrac{3 - 2\ln 2 - 2(\ln 2)^2}{16(\ln 2)^3}$	1A
6. (a) $y = 2e^{\frac{x}{2}}$	
$\dfrac{\mathrm{d}y}{\mathrm{d}x} = e^{\frac{x}{2}}$	1M
Let the coordinates of R be $(r,\ 0)$.	
$\dfrac{0 - 2e^{\frac{u}{2}}}{r - u} = e^{\frac{u}{2}}$	1M
$-2 = r - u$	
$r = u - 2$	1A
The coordinates of R are $(u - 2,\ 0)$.	
(b) Let A be the area of $\triangle PQR$.	
$A = \dfrac{(2e^{\frac{u}{2}} - \ln(2u))(2)}{2}$	1M
$= 2e^{\frac{u}{2}} - \ln(2u)$	
$\dfrac{\mathrm{d}A}{\mathrm{d}t} = \left(e^{\frac{u}{2}} - \dfrac{1}{u}\right)\dfrac{\mathrm{d}u}{\mathrm{d}t}$	1M
When Q lies on the x-axis, $u = \dfrac{1}{2}$.	1A
$0.5 = \left(e^{\frac{1}{4}} - 2\right)\dfrac{\mathrm{d}u}{\mathrm{d}t}$	1M
$\dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{1}{2(e^{\frac{1}{4}} - 2)}$	1A
Required rate is $\dfrac{1}{2(e^{\frac{1}{4}} - 2)}$ unit per second.	

Solution	Marks
7. (a) (i) $\begin{vmatrix} 0 & 1 & \lambda+1 \\ \lambda & 2 & 2 \\ 1 & -1 & -4\lambda \end{vmatrix} = -\lambda \begin{vmatrix} 1 & \lambda+1 \\ -1 & -4\lambda \end{vmatrix} + \begin{vmatrix} 1 & \lambda+1 \\ 2 & 2 \end{vmatrix}$	1M
$= 3\lambda^2 - 3\lambda$	
$= 3\lambda(\lambda - 1)$	
If (E) has a unique solution, then $3\lambda(\lambda - 1) \neq 0$.	1M
Solving, we have $\lambda \neq 0$ and $\lambda \neq 1$.	1A
Thus, $\lambda < 0$ or $0 < \lambda < 1$ or $\lambda > 1$.	
(ii) $y = \dfrac{\begin{vmatrix} 0 & 0 & \lambda+1 \\ \lambda & \mu & 2 \\ 1 & \lambda & -4\lambda \end{vmatrix}}{3\lambda(\lambda-1)}$	1M
$= \dfrac{\mu(\lambda^2 - 1)}{3\lambda(\lambda - 1)}$	
$= \dfrac{\mu(\lambda + 1)}{3\lambda}$	1A
(b) The augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & \mu \\ 1 & -1 & 0 & \mu \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -1 & 0 & \mu \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & \mu \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -1 & 0 & \mu \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & \mu \end{array}\right)$	1M
We have $\mu = 0$.	1A
The solution set is $\{(-t,\ -t,\ t) : t \in \mathbb{R}\}$.	1A
8. (a) When $n = 1$,	
L.H.S. $= \sin x \cos 2x =$ R.H.S.	
It is true for $n = 1$.	1
Assume $\sin x \sum_{k=1}^{p} \cos 2kx = \sin px \cos[(p+1)x]$, where $p \in \mathbf{Z}^+$.	1
$\sin x \sum_{k=1}^{p+1} \cos 2kx$	
$= \sin x \sum_{k=1}^{p} \cos 2kx + \sin x \cos[(2p+2)x]$	
$= \sin px \cos[(p+1)x] + \sin x \cos[(2p+2)]$	1M
$= \frac{1}{2}[\sin[(2p+1)x] + \sin(-x)] + \frac{1}{2}[\sin[(2p+3)x] + \sin[(-2p-1)x]]$	1M
$= \frac{1}{2}[\sin[(2p+3)x - \sin x]$	
$= \sin[(p+1)x]\cos[(p+2)x]$	
It is true for $n = p + 1$.	
By M.I., it is true $\forall n \in \mathbf{Z}^+$.	1

Solution	Marks

9. (a) The vertical asymptote is $x = -1$. **1A**

$$f(x) = \frac{(x-2)^3}{(x+1)^2}$$

$$= x - 8 + \frac{27x}{(x+1)^2}$$ **1M**

The oblique asymptote is $y = x - 8$. **1A**

(b) $f(x) = x - 8 + \dfrac{27x}{(x+1)^2}$

$$f'(x) = 1 + \frac{27(x+1)^2 - 27x(2)(x+1)}{(x+1)^4}$$ **1M**

$$= \frac{(x+7)(x-2)^2}{(x+1)^3}$$

When $f'(x) = 0$, $x = 2$ or -7.

x	$x < -7$	$-7 < x < -1$	$-1 < x < 2$	$x > 2$
$f'(x)$	$+$	$-$	$+$	$+$

1M

G has exactly one turning point, which is a maximum point at $x = -7$. **1**

(c) $$f''(x) = \frac{(x+1)^3[(x-2)^2 + 2(x+7)(x-2)] - (x+7)(x-2)^2(3)(x+1)^2}{(x+1)^6}$$

$$= \frac{54(x-2)}{(x+1)^4}$$ **1A**

When $f''(x) = 0$, $x = 2$.

x	$x < -1$	$-1 < x < 2$	$x > 2$
$f''(x)$	$-$	$-$	$+$

1M

The point of inflexion is $(2,\ 0)$. **1A**

(d) Let $u = x + 1$. Then $\mathrm{d}u = \mathrm{d}x$.

Volume

$$= \pi \int_0^2 \left[\frac{(x-2)^3}{(x+1)^2}\right]^2 \mathrm{d}x$$ **1M**

$$= \pi \int_1^3 \frac{(u-3)^6}{u^4}\,\mathrm{d}u$$

$$= \pi \int_1^3 \frac{u^6 - 18u^5 + 135u^4 - 540u^3 + 1215u^2 - 1458u + 729}{u^4}\,\mathrm{d}u$$ **1M**

$$= \pi \int_1^3 (u^2 - 18u + 135 - 540u^{-1} + 1215u^{-2} - 1458u^{-3} + 729u^{-4})\,\mathrm{d}u$$

$$= \pi \left[\frac{u^3}{3} - 9u^2 + 135u - 540\ln|u| - 1215u^{-1} + \frac{729}{u^2} - \frac{243}{u^3}\right]_1^3$$

$$= \pi\left(\frac{1808}{3} - 540\ln 3\right)$$ **1A**

Solution	Marks
10. (a) $\overrightarrow{OR} = \frac{1}{2}\overrightarrow{OP} + \frac{1}{2}\overrightarrow{OQ}$	
$= 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$	
$\overrightarrow{OP} \times \overrightarrow{OR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \frac{25}{4} & 123 \quad 4 \quad 12 \end{vmatrix}$	1M
$= 27\mathbf{i} + 36\mathbf{j} - \frac{75}{4}\mathbf{k}$	1A
(b) Area $= \left\lvert \overrightarrow{OP} \times \overrightarrow{OR} \right\rvert$	1M
$= \sqrt{27^2 + 36^2 + \left(\frac{75}{4}\right)^2}$	
$= \frac{195}{4}$	1A
(c) (i) $\overrightarrow{PQ} = 6\mathbf{i} - \frac{9}{2}\mathbf{j}$	
$\overrightarrow{NR} = \overrightarrow{OR} - \overrightarrow{ON} = (3 - 27\alpha)\mathbf{i} + (4 - 36\alpha)\mathbf{j} + \left(12 + \frac{75\alpha}{4}\right)\mathbf{k}$	
$\overrightarrow{NR} \cdot \overrightarrow{PQ}$	
$= 6(3 - 27\alpha) - \frac{9}{2}(4 - 36\alpha)$	1M
$= 0$	
$\overrightarrow{NR}$ and $\overrightarrow{PQ}$ are orthogonal vectors.	1A
(ii) (1) $\overrightarrow{NQ} = \overrightarrow{OQ} - \overrightarrow{ON} = (6 - 27\alpha)\mathbf{i} + \left(\frac{7}{4} - 36\alpha\right)\mathbf{j} + \left(12 + \frac{75\alpha}{4}\right)\mathbf{k}$	
We have $\frac{6 - 27\alpha}{-12} = \frac{\frac{7}{4} - 36\alpha}{-41} = \frac{12 + \frac{75\alpha}{4}}{\beta}$.	1M
$\frac{6 - 27\alpha}{-12} = \frac{\frac{7}{4} - 36\alpha}{-41}$ and $\frac{6 - 27\alpha}{-12} = \frac{12 + \frac{75\alpha}{4}}{\beta}$	
$\alpha = \frac{1}{3}$ $\qquad$ $\beta = 73$	
We have $\alpha = \frac{1}{3}$ and $\beta = 73$.	1A+1A
(2) Note that ON is perpendicular to the plane OPQ and $NR \perp PQ$.	
We have $OR \perp PQ$ and $\theta = \angle ORN$.	1M
$\tan\theta = \frac{ON}{OR}$	1M
$= \frac{\frac{1}{3}\left(\frac{195}{4}\right)}{\sqrt{3^2 + 4^2 + 12^2}}$	
$= \frac{5}{4}$	1A

Solution	Marks
11. (a) $f'(u)=(m+1)(1+u)^m(1-u)^n-n(1+u)^{m+1}(1-u)^{n-1}$	1M
$(m+1)\int_{-1}^{1}(1+u)^m(1-u)^n\,\mathrm{d}u=\int_{-1}^{1}f'(u)\,\mathrm{d}u+n\int_{-1}^{1}(1+u)^{m+1}(1-u)^{n-1}\,\mathrm{d}u$	1M
$(m+1)\int_{-1}^{1}(1+u)^m(1-u)^n\,\mathrm{d}u=\Big[(1+u)^{m+1}(1-u)^n\Big]_{-1}^{1}+n\int_{-1}^{1}(1+u)^{m+1}(1-u)^{n-1}\,\mathrm{d}u$	1M
$\int_{-1}^{1}(1+u)^n(1-u)^n\,\mathrm{d}u=\dfrac{n}{m+1}\int_{-1}^{1}(1+u)^{m+1}(1-u)^{n-1}\,\mathrm{d}u$	1
(b) Let $t=\tan x$. Then $\mathrm{d}t=\sec^2 x\,\mathrm{d}x$.	
$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\dfrac{(1+\tan x)^2(\cos 4x+1)}{\cos^6 x}\,\mathrm{d}x$	
$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec^6 x(1+\tan x)^2\left[(2\cos^2 2x-1)+1\right]\mathrm{d}x$	1M
$=2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec^6 x(1+\tan x)^2(2\cos^2 x-1)^2\,\mathrm{d}x$	
$=2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec^2 x(1+\tan x)^2(2-\sec^2 x)^2\,\mathrm{d}x$	
$=2\int_{-1}^{1}(1+t)^2[2-(1+t^2)]^2\,\mathrm{d}t$	1M
$=2\int_{-1}^{1}(1+t)^2[(1+t)(1-t)]^2\,\mathrm{d}t$	
$=2\int_{-1}^{1}(1+t)^4(1-t)^2\,\mathrm{d}t$	
$=\dfrac{4}{5}\int_{-1}^{1}(1+t)^5(1-t)\,\mathrm{d}t$	1M
$=\dfrac{4}{5}\times\dfrac{1}{6}\int_{-1}^{1}(1+t)^6\,\mathrm{d}t$	1M
$=\dfrac{2}{15}\left[\dfrac{(1+t)^7}{7}\right]_{-1}^{1}$	
$=\dfrac{256}{105}$	1A
(c) Let $f(x)=\dfrac{\tan x(\cos 4x+1)}{\cos^6 x}$ and $g(x)=\dfrac{(1+\tan^2 x)(\cos 4x+1)}{\cos^6 x}$.	
$f(-x)=\dfrac{\tan(-x)(\cos(-4x)+1)}{\cos^6(-x)}=-\dfrac{\tan x(\cos 4x+1)}{\cos^6 x}=-f(x)$	1M
$g(-x)=\dfrac{(1+\tan^2(-x))(\cos(-4x)+1)}{\cos^6(-x)}=\dfrac{(1+\tan^2 x)(\cos 4x+1)}{\cos^6 x}=g(x)$	

Solution	Marks
Thus, $f(x)$ is an odd function and $g(x)$ is an even function.	
$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+\tan x)^2(\cos 4x+1)}{\cos^6 x}\,\mathrm{d}x = \frac{256}{105}$$	
$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+\tan^2 x)(\cos 4x+1)}{\cos^6 x}\,\mathrm{d}x + 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan x(\cos 4x+1)}{\cos^6 x}\,\mathrm{d}x = \frac{256}{105}$$	1M
$$2\int_{0}^{\frac{\pi}{4}} \frac{(1+\tan^2 x)(\cos 4x+1)}{\cos^6 x}\,\mathrm{d}x = \frac{256}{105}$$	1M
$$2\int_{0}^{\frac{\pi}{4}} \sec^8 x(\cos 4x+1)\,\mathrm{d}x = \frac{256}{105}$$	
$$\int_{0}^{\frac{\pi}{4}} \sec^8 x(\cos 4x+1)\,\mathrm{d}x = \frac{128}{105}$$	1A

Solution	Marks
12. (a) (i) $QQ^TP = \begin{pmatrix} a \\ b \\ c \end{pmatrix}\begin{pmatrix} a & b & c \end{pmatrix}P$	
$= \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}\begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}$	1M
$= \begin{pmatrix} abc-abc & -a^2c+a^2c & a^2b-a^2b \\ b^2c-b^2c & -abc+abc & ab^2-ab^2 \\ bc^2-bc^2 & -ac^2+ac^2 & abc-abc \end{pmatrix}$	
$= \mathbf{0}$	
$PQQ^T = \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}\begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$	
$= \mathbf{0}$	1
(ii) $P^2 = \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}\begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}$	1M
$= \begin{pmatrix} -b^2-c^2 & ab & ac \\ ab & -a^2-c^2 & bc \\ ac & bc & -a^2-b^2 \end{pmatrix}$	
$= \begin{pmatrix} a^2-1 & ab & ac \\ ab & b^2-1 & bc \\ ac & bc & c^2-1 \end{pmatrix}$	1M
$I+P^2 = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix} = QQ^T$	1
$\left[\frac{1}{2}(I-P+QQ^T)\right](I+P)$	1M
$= \frac{1}{2}[I+P-P-P^2+QQ^T+QQ^TP]$	
$= \frac{1}{2}[I-P^2+QQ^T+\mathbf{0}]$	1M
$= \frac{1}{2}[I-P^2+(I+P^2)]$	1M
$= \frac{1}{2}(2I)$	
$= I$	

Solution	Marks
$(I+P)\left[\frac{1}{2}(I-P+QQ^T)\right]$	
$=\frac{1}{2}\left[I-P+P-P^2+QQ^T+PQQ^T\right]$	
$=\frac{1}{2}\left[I-P^2+(I+P^2)+\mathbf{0}\right]$	
$=\frac{1}{2}(2I)$	
$=I$	
Thus, $(I+P)^{-1}=\frac{1}{2}(I-P+QQ^T)$.	1
(b) Take $a=\frac{1}{\sqrt{6}}$, $b=\frac{\sqrt{3}}{\sqrt{6}}$ and $c=\frac{\sqrt{2}}{\sqrt{6}}$, such that $a^2+b^2+c^2=1$.	1M
We have $M=\sqrt{6}\begin{pmatrix}0 & -c & b\\ c & 0 & -a\\ -b & a & 0\end{pmatrix}=\sqrt{6}P$.	
$(I+P^2)P=QQ^TP$	1M
$P+P^3=\mathbf{0}$	
$M+\frac{1}{6}M^3+\frac{1}{6^2}M^5+\ldots+\frac{1}{6^{1048}}M^{2097}$	
$=\sqrt{6}P+\frac{1}{6}(\sqrt{6}P)^3+\frac{1}{6^2}(\sqrt{6}P)^5+\ldots+\frac{1}{6^{1048}}(\sqrt{6}P)^{2097}$	
$=\sqrt{6}(P+P^3+P^5+\ldots+P^{2097})$	
$=\sqrt{6}[P+P^2(P+P^3)+\ldots+P^{2094}(P+P^3)]$	1M
$=\sqrt{6}P$	
$=\begin{pmatrix}0 & -\sqrt{2} & \sqrt{3}\\ \sqrt{2} & 0 & -1\\ -\sqrt{3} & 1 & 0\end{pmatrix}$	1A