Solution	Marks
ELITE-2425-MOCK-SET 8-MATH-CP 1	
Suggested solutions	
1. $\dfrac{(m^2n^{-3})^{-7}}{m^{-3}n^{20}} = \dfrac{m^{-14}n^{21}}{m^{-3}n^{20}}$	1M
$= \dfrac{n^{21-20}}{m^{-3+14}}$	1M
$= \dfrac{n}{m^{11}}$	1A
2. Mean $= \dfrac{20+23+\ldots+63}{25} = 45$	1A
Lower quartile $= 35$	1A
Standard deviation ≈ 12.2	1A
3. Let the number of boys and girls in the school be b and g respectively.	
$\begin{cases} b = g(1+15\%) \\ b - 85 = g(90\%) \end{cases}$	1A
$1.15g - 85 = 0.9g$	1M
$g = 340$	
When $g = 340$, $b = g(1+15\%) = 391$. The total number of boys and girls is $340 + 391 = 731$	1A
4. (a) $x^3y - 4xy^3 = xy(x^2 - 4y^2)$	
$= xy(x+2y)(x-2y)$	1A
(b) $2x^2 - 3xy - 2y^2 = (x-2y)(2x+y)$	1A
(c) $x^3y - 4xy^3 - 2x^2 + 3xy + 2y^2 = xy(x+2y)(x-2y) - (x-2y)(2x+y)$	1M
$= (x-2y)[xy(x+2y) - (2x+y)]$	1A
$= (x-2y)(x^2y + 2xy^2 - 2x - y)$	
5. (a) Cost per kg $= \dfrac{2(27)+1(66)}{2+1}$	1M
$= \$40/\text{kg}$	1A
(b) Required marked price $= \dfrac{40(1+20\%)}{1-20\%}$	1M
$= \$60$	1A

Solution	Marks
6. (a) $f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)\left[\frac{2}{2^2}+5\right]-k\left(\frac{1}{2}\right)^2-7=-3$	1M
$k=6$	1A
(b) $f(x)=2x(2x^2+5)-6x^2-7=4x^3-6x^2+10x-7$	
$$\begin{array}{r} 4x^2+\ \ 2x+14 \\ x-2\,\big)\ \overline{4x^3-6x^2+10x-\ \ 7} \\ \underline{4x^3-8x^2\qquad\qquad} \\ 2x^2+10x\qquad \\ \underline{2x^2-\ \ 4x\qquad} \\ 14x-\ \ 7 \\ \underline{14x-28} \\ 21 \end{array}$$	1M
Required quotient is $4x^2+2x+14$.	1A
7. (a) $\frac{\sin(90°+\theta)}{\cos(270°-\theta)}=\frac{\cos\theta}{-\sin\theta}$	1A
$=-\frac{1}{\tan\theta}$	
(b) $\frac{\sin(90°+\theta)}{\cos(270°-\theta)}=1$	
$-\frac{1}{\tan\theta}=1$	1M
$\tan\theta=-1$	
$\theta=135°$ or $315°$	1A+1A
8. (a) $204°-24°=180°$. So, P, O and Q are collinear.	1M
$114°-24°=90°$. So, $PQ\perp OR$.	1A
(b) Perimeter $=\sqrt{15^2+36^2}+\sqrt{27^2+36^2}+(15+27)$	1M+1M
$=126$	1A

Solution	Marks
9. (a) $\angle QPR = \angle SPT$ *(given)*	
$PR = PS$ *(given)*	
$\angle PTS = \angle PQR$ *(equal chords, equal $\angle$s)*	
$\triangle PQR \cong \triangle PTS$ *(AAS)*	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

Solution	Marks
(b) $\triangle PQR \cong \triangle PTS$	
$QR = TS$	1M
$\angle QTR = \angle SRT$	1M
So, $QT // RS$. The claim is agreed.	1A
10. (a) Let $g(x) = ax + bx^2$, where a and b are non-zero constants.	1A
$\begin{cases} -93 = -3a + 9b \\ 2 = 2a + 4b \end{cases}$	1M
Solving, $a = 13$ and $b = -6$	1A
(b) $13x - 6x^2 = x + k$	
$-6x^2 + 12x - k = 0$	
$\Delta = 12^2 - 4(-6)(-k) \geq 0$	1M
$k \leq 6$	1A
11. (a) Median = 26.5 min	1A
Range = 42.3 − 16.8 = 25.5 min	1A
(b) Let t minutes be the time taken by the new student.	
$28 \times 35 + t = (28 - 0.3) \times 36$	
$t = 17.2$	1A
Range = 42.3 − 16.8 = 25.5	1M
The claim is not correct.	1A

Solution	Marks
12. (a) Slope of $L_1 = \frac{7}{6}$	1A
Equation of L_2 is	
$y + 12 = \frac{-6}{7}(x - 14)$	1M
$6x + 7y = 0$	1A
(b) Solve $\begin{cases} 6x + 7y = 0 \\ 7x - 6y - 22 = 0 \end{cases}$, the coordinates of M are $\left(\frac{154}{85}, -\frac{132}{85}\right)$.	1A
Coordinates of H and K are $\left(\frac{22}{7}, 0\right)$ and $\left(0, -\frac{11}{3}\right)$.	1A
Since H, M and K are collinear,	
Required ratio $= HM : KM$	
$= \left(\frac{22}{7} - \frac{154}{85}\right) : \frac{154}{85}$	1M
$= 36 : 49$	1A
13. (a) $BC = \sqrt{40^2 - 24^2} = 32\text{ cm}$	1A
Height of the pyramid $= \sqrt{29^2 - \left(\frac{40}{2}\right)^2} = 21\text{ cm}$	
Volume of the pyramid $= \frac{1}{3}(24)(32)(21)$	1M
$= 5376\text{ cm}^3$	1A
(b) (i) Let r cm and h cm be the radius and the height of the cylinder.	
$4\pi r^2 = 2\pi r^2 + 2\pi rh$	1M
$r = h$	
$\frac{4}{3}\pi r^3 + \pi r^2(r) = 5376$	1M
$\pi r^3 = 2304$	
Volume of the sphere $= \frac{4}{3}\pi r^3 = 3072\text{ cm}^3$	1A
(ii) Original total surface area	
$= (24)(32) + \frac{(24)\sqrt{21^2 + 16^2}}{2} \times 2 + \frac{(32)\sqrt{21^2 + 12^2}}{2} \times 2$	1M
$\approx 2180\text{ cm}^2$	
New total surface area	
$= 4\pi r^2 \times 2$	
$= 8\pi\left(\frac{2304}{\pi}\right)^{\frac{2}{3}}$	
$\approx 2040\text{ cm}^2 < 2180\text{ cm}^2$	1A
The claim is disagreed.	1A

Solution	Marks
14. (a) $\sqrt{(x-6)^2+(y+4)^2}=\sqrt{(12-6)^2+(4+4)^2}$	1M
$(x-6)^2+(y+4)^2=100$	1A
(b) When $(x,\ y)=(-2,\ -10)$, $(x-6)^2+(y+4)^2=(-2-6)^2+(-10+4)^2=100$.	1M
Thus, $B\,(-2,\ -10)$ lies on Γ.	1
(c) Let M be the mid-point of AB. The coordinates of M are $(5,\ -3)$.	1A
Q lies on the perpendicular bisector of AB. So, $QM \perp AB$.	
$QM = GA - GM = 10 - \sqrt{(6-5)^2+(-3+4)^2} = 10-\sqrt{2}$	1A
$AM=\sqrt{(12-5)^2+(4+3)^2}=\sqrt{98}$	
$\tan\angle QAB = \dfrac{10-\sqrt{2}}{\sqrt{98}}$	1M
$\angle QAB \approx 40.9^\circ > 40^\circ$	1A
The claim is correct.	1A
15. Substitute $(0,\ 1)$ and $(1,\ 6)$,	
$\begin{cases}1=m-2n\\6=mn-2n\end{cases}$	
$6=(2n+1)n-2n$	1M
$0=2n^2-n-6$	
$n=2 \quad \text{or} \quad -\dfrac{3}{2}$ (rejected)	1A
When $n=2$, $m=2(2)+1=5$.	
$y=5\times 2^x-4$	
$2^x=\dfrac{y+4}{5}$	
$x=\log_2\dfrac{y+4}{5}$	1A
16. (a) $z=\dfrac{5(a+i)}{a-i}\times\dfrac{a+i}{a+i}$	1M
$=\dfrac{5(a^2-1)+10ai)}{a^2+1}$	
$=\dfrac{5(a^2-1)}{a^2+1}+\dfrac{10a}{a^2+1}i$	1A
(b) $\dfrac{5(a^2-1)}{a^2+1}-\dfrac{10a}{a^2+1}=1$	1M
$5a^2-10a-5=a^2+1$	
$4a^2-10a-6=0$	
$a=3 \quad \text{or} -0.5$ (rejected)	1A
$z=4+3i$, and the two roots of the equation are $4\pm 3i$.	
$s=(4+3i)(4-3i)=25$	1A

Solution	Marks
17. (a) $f(x)=\frac{-1}{3}x^2-\frac{k}{2}x+k-1$	
$=\frac{-1}{3}\left(x^2+\frac{3k}{2}+\frac{9k^2}{16}\right)+\frac{3k^2}{16}+k-1$	1M
$=\frac{-1}{3}\left(x+\frac{3k}{4}\right)^2+\frac{3k^2}{16}+k-1$	
The coordinates of S are $\left(-\frac{3k}{4},\ \frac{3k^2}{16}+k-1\right)$.	1A
(b) (i) $g(x)=\frac{3}{2}f(-x)$	
The coordinates of T are $\left(\frac{3k}{4},\ \frac{9k^2}{32}+\frac{3k}{2}-\frac{3}{2}\right)$.	1A
(ii) Since S is the orthocentre of $\triangle OST$, $\angle OST=90°$.	
$\frac{\frac{3k^2}{16}+k-1}{-\frac{3k}{4}}\times\frac{\frac{9k^2}{32}+\frac{3k}{2}-\frac{3}{2}-\frac{3k^2}{16}-k+1}{\frac{3k}{4}+\frac{3k}{4}}=-1$	1M
$\frac{1}{2}\left(\frac{3k^2}{16}+k-1\right)^2=\frac{9k^2}{8}$	
$\frac{3k^2}{16}+k-1=\pm\frac{3k}{2}$	
If $\frac{3k^2}{16}+k-1=\frac{3k}{2}$,	
$\frac{3k^2}{16}-\frac{k}{2}-1=0$	
$k=4$ or $-\frac{4}{3}$ (rejected)	1A
If $\frac{3k^2}{16}+k-1=-\frac{3k}{2}$,	
$\frac{3k^2}{16}+\frac{5k}{2}-1=0$	
$k\approx-13.7$ (rejected) or 0.389 (rejected)	
Coordinates of T are $(3,\ 9)$, and the circumcentre of $\triangle OST$ is at the mid-point of OT.	
Required coordinates are $\left(\frac{3}{2},\ \frac{9}{2}\right)$.	1A

Solution	Marks
18. (a) Required probability $= \frac{3}{7} \times \frac{3}{8} + \frac{4}{7} \times \frac{6}{8}$	1M
$= \frac{33}{56}$	1A
(b) Required probability $= \frac{33}{56} + \left(\frac{23}{56}\right)^2 \frac{33}{56} + \left(\frac{23}{56}\right)^4 \frac{33}{56} + \dots$	1M
$= \frac{\frac{33}{56}}{1 - \frac{529}{3136}}$	1M
$= \frac{56}{79}$	1A
(c) Required probability $= \frac{\left(\frac{23}{56}\right) \frac{33}{56} + \left(\frac{23}{56}\right)^3 \frac{33}{56}}{1 - \frac{56}{79}}$	1M+1A
≈ 0.972	1A

Solution	Marks
19. (a) $BD^2 = 20^2 + 25^2 - 2(20)(25)\cos 120^\circ$	1M
$BD = 5\sqrt{61}$ cm	
$20^2 = 25^2 + BD^2 - 2(25)(BD)\cos\angle BDC$	
$\angle BDC \approx 26.3^\circ$	1A
$\dfrac{AD}{BD} = \dfrac{BD}{25}$	1M
$AD = 61$ cm	1A
(b) (i) Let E be a point on DC produced such that $BE \perp CD$.	
Let F be a point on AD such that $EF \perp CD$. Required angle is $\angle BEF$.	1A
$BE = 20\sin(180^\circ - 120^\circ) = 10\sqrt{3}$ cm	
$\dfrac{25}{\sin\angle DAC} = \dfrac{61}{\sin 140^\circ}$	1M
$\angle DAC \approx 15.3^\circ$	
$\angle ADC = 180^\circ - 140^\circ - \angle DAC \approx 24.7^\circ$	
$DE = CD + 20\cos(180^\circ - 120^\circ) = 35$ cm	
$EF = DE\tan\angle ADC \approx 16.1$ cm	1A
$DF = \dfrac{DE}{\cos\angle ADC} \approx 38.5$ cm	
$BF^2 = BD^2 + DF^2 - 2(BD)(DF)\cos\angle BDA$	
$\quad = BD^2 + DF^2 - 2(BD)(DF)\cos\angle BDC$	
$BF \approx 17.7$ cm	
$BF^2 = BE^2 + EF^2 - 2(BE)(EF)\cos\angle BEF$	
$\angle BEF \approx 63.7^\circ$	1A
Required angle is 63.7°.	
(ii) Let G be a point on the ground such that BG is perpendicular to the plane ACD.	
Required angle is $\angle BDG$.	1A
$BG = BE\sin\angle BEF \approx 15.5$ cm	1A
$\sin\angle BDG = \dfrac{BG}{BD}$	
$\angle BDG \approx 23.4^\circ$	1A
Required angle is 23.4°.	
(iii) Let the required distance be h cm.	
Area of $\triangle BCD = \dfrac{1}{2}(20)(25)(\sin 120^\circ) = 125\sqrt{3}\text{ cm}^2$.	
Area of $\triangle ACD = \dfrac{1}{2}(AD)(25)(\sin\angle ADC) \approx 319\text{ cm}^2$	
By considering the volume of the tetrahedron $ABCD$,	
$\dfrac{1}{3}(125\sqrt{3})(h) = \dfrac{1}{3}(\text{area of }\triangle ACD)(BG)$	1M
$h \approx 22.9\text{ cm} < 23\text{ cm}$	
The claim is disagreed.	1A

END OF PAPER