DAYCP-UTRE-2425-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a) Let $S = an + bn^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 7920 = 12a + 144b \\ 12\,160 = 16a + 256b \end{cases}$$ 1M

Solving, we have $a = 360$ and $b = 25$. 1A

Required income $= 360(24) + 25(24)^2$

$= \$23\,040$ 1A

(b) $25n^2 + 360n = 17\,200$ 1M

$25n^2 + 360n - 17\,200 = 0$

$n = 20$ or -34.4 (rejected) 1A

The number of handbags she sells in that month is 20.

2. (a) Let $C = ma^2 + na^3$, where m and n are non-zero constants. 1A

$$\begin{cases} 0.28 = 4m + 8n \\ 2.5 = 25m + 125n \end{cases}$$ 1M

Solving, we have $m = \frac{1}{20}$ and $n = \frac{1}{100}$. 1A

Required cost

$= \frac{1}{20}(4)^2 + \frac{1}{100}(4^3)$

$= \$1.44$ 1A

(b) $1.5z = \frac{1}{20}z^2 + \frac{1}{100}z^3$ 1M

$0 = z\left(\frac{z^2}{100} + \frac{z}{20} - \frac{3}{2}\right)$

$z = 0$ (rejected) or 10 or -15 (rejected) 1A

3. (a) Let $C = a + b\sqrt[3]{n}$, where a and b are non-zero constants. 1A

$$\begin{cases} 20\,000 = a + b\sqrt[3]{8000} \\ 23\,000 = a + b\sqrt[3]{27\,000} \end{cases}$$ 1M

Solving, we have $a = 14\,000$ and $b = 300$. 1A

Required cost $= 14\,000 + 300\sqrt[3]{125\,000}$

$= \$29\,000$ 1A

(b) Total cost of producing a batch of 729 000 components

$= 14\,000 + 300\sqrt[3]{729000}$ 1M

$= \$41\,000$

Total cost of producing three batches of 125 000 components

$= 29\,000 \times 3$

$= \$87\,000$

$> 2 \times \$41\,000$

The claim is agreed. 1A

4. (a) Let $C = ax + by$, where a and b are non-zero constants. 1A

$4550 = 10a + 3b$ 1M

$6760 = 16a + 4b$

Solving, we have $a = 260$ and $b = 650$. 1A

Thus, $C = 260x + 650y$.

(b) $4030 = 260x + 650y$ 1M

$x = \dfrac{31 - 5y}{2}$

Since x and y are positive integers, y is an odd number.

Thus, we have $(x,\ y) = (13,\ 1)$ or $(8,\ 3)$ or $(3,\ 5)$. 1A+1A

5. (a) Let $F = a + bv^2$, where a and b are non-zero constants. 1A

$60 = a + b(20)^2$ 1M

$85 = a + b(30)^2$

Solving, we have $a = 40$ and $b = \dfrac{1}{20}$.

Required resistance $= 40 + \dfrac{1}{20}(50)^2$

$= 165$ units 1A

(b) Let u km/h be the original speed of the car.

$40 + \dfrac{1}{20}(2v)^2 = 2 \times \left[40 + \dfrac{1}{20}v^2\right]$ 1M+1A

$\dfrac{v^2}{10} = 40$

$v = 20$ or -20 (rejected) 1A

6. (a) Let $P = a + bV$, where a and b are non-zero constants. 1A

$$90 = a + 30b$$ 1M

$$114 = a + 50b$$

Solving, we have $a = 54$ and $b = \frac{6}{5}$. 1A

Required volume $= \dfrac{144 - 54}{\left(\frac{6}{5}\right)}$

$= 75\ \text{m}^3$ 1A

(b) Ratio of the volume of X to the volume of Y

$= \left(\sqrt{\dfrac{1}{16}}\right)^3$ 1M

$= \dfrac{1}{64}$

Volume of $Y = 75(64)$

$= 4800\ \text{m}^3$

Required cost $= 54 + \dfrac{6}{5}(4800)$

$= \$5814$ 1A

7. (a) Let $f(x) = k_1 + k_2(x - 3)^2$, where k_1 and k_2 are non-zero constants. 1A

$$-6 = k_1 + k_2(4 - 3)^2$$ 1M

$$42 = k_1 + k_2(-2 - 3)^2$$

Solving, we have $k_1 = -8$ and $k_2 = 2$.

$f(0) = -8 + 2(0 - 3)^2 = 10$ 1A

(b) The coordinates of D are $(3,\ -8)$. 1M

$$2(x - 3)^2 - 8 = 0$$

$$(x - 3)^2 = 4$$

$$x = 1 \quad \text{or} \quad 5$$

We have $b = 1$ and $c = 5$. 1M

Required area

$= \dfrac{1}{2}(5 - 1)(10 - 0) + \dfrac{1}{2}(5 - 1)(0 + 8)$ 1M

$= 36$ 1A

8. (a) Let $f(x) = a + bx$, where a and b are non-zero constants. 1A

$$\begin{cases} 22 = a + 7b \\ 4 = a - 2b \end{cases}$$ 1M

Solving, we have $a = 8$ and $b = 2$. 1A

Thus, $f(x) = 8 + 2x$.

(b) (i) The coordinates of B and C are $(-4,\ 0)$ and $(0,\ 8)$ respectively. 1M

$BD = \sqrt{(-4-0)^2 + (0-3)^2} = 5$

$CD = 8 - 3 = 5$

Thus, $BD = CD$. 1

(ii) Slope of $L_1 \times$ slope of $L_2 = 2 \times \left(-\frac{1}{2}\right) = -1$

We have $L_1 \perp L_2$. 1M

Since $BD = CD$, we have $BM = CM$.

Since $CF : CM : BM = 2 : 1 : 1$, we have $FM : BM = 3 : 1$. 1M

Required ratio

$= FM : BM$

$= 3 : 1$ 1A

9. (a) The equation of L_3:

$$\frac{y - 80}{x - 240} = \frac{1}{3}$$

$$x - 3y = 0$$ 1A

The equation of L_4:

$$\frac{y - 80}{x - 240} = \frac{80 - 10}{240 - 380}$$

$$x + 2y - 400 = 0$$ 1A

Required system of inequalities is $\begin{cases} x - 3y \geq 0 \\ x + 2y - 400 \leq 0 \\ x \geq 60 \\ y \geq 10 \end{cases}$. 1A

(b) The coordinates of the vertices of the shaded region are $(240,\ 80)$, $(380,\ 10)$, $(60,\ 10)$ and $(60,\ 20)$.

$(x,\ y)$	$(240,\ 80)$	$(380,\ 10)$	$(60,\ 10)$	$(60,\ 20)$
$4x + 15y$	2160	1670	390	540

1M+1A

The maximum value of P is 2160. 1A

10. (a) Slope of $L_1 = \frac{6-0}{6-0} = 1$

The equation of L_1 is $y = x$. 1A

Required system of inequalities is $\begin{cases} x - y \le 0 \\ 2x + 3y \le 30 \\ x \ge 0 \end{cases}$. 1A+1A

(b) Let x and y be the numbers of apples and oranges Mary has bought respectively.

We have $\begin{cases} x - y \ge 0 \\ 2x + 3y \le 30 \\ x \text{ and } y \text{ are non-negative integers} \end{cases}$.

Denote the total number of fruit be P.

We have $P = x + y$. 1A

$(x,\ y)$	$(0,\ 0)$	$(0,\ 10)$	$(6,\ 6)$
P	0	10	12

1M

The greatest possible total number of fruit is 12.

The claim is disagreed. 1A

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