

(a)  $\angle PRQ = 180^\circ - 56^\circ - 74^\circ = 50^\circ$

$$\frac{PQ}{\sin 50^\circ} = \frac{30}{\sin 74^\circ}$$

$$PQ \approx 23.9 \text{ cm}$$

(b) (i) Let  $S$  be a point on  $PQ$  such that  $TS \perp PQ$ .

Required angle is  $\angle TSV$ .

In  $\triangle PRT$ ,  $\angle TPR = \frac{56^\circ}{2} = 28^\circ$ ,  $\angle TRP = \frac{50^\circ}{2} = 25^\circ$  and  
 $\angle PTR = 180^\circ - 28^\circ - 25^\circ = 127^\circ$ .

$$\frac{RT}{\sin 28^\circ} = \frac{30}{\sin 127^\circ} \quad \text{and} \quad \frac{PT}{\sin 25^\circ} = \frac{30}{\sin 127^\circ}$$

$$RT \approx 17.6 \text{ cm}$$

$$PT \approx 15.9 \text{ cm}$$

Note that  $\angle VRT = 40^\circ$ .

$$VT = RT \tan 40^\circ \approx 14.8 \text{ cm}$$

$$TS = PT \sin 28^\circ \approx 7.45 \text{ cm}$$

Consider  $\triangle TSV$ .

$$\tan \angle TSV = \frac{VT}{TS}$$

$$\angle TSV \approx 63.3^\circ$$

(ii) Let  $U$  be a point on  $PQ$  such that  $TU \perp PQ$ .

Required angle is  $\angle TUV$ .

Since  $T$  is the circumcentre of  $\triangle PQR$ , we have  $\angle PTR = 2\angle PQR = 148^\circ$ .

$$\sin \frac{148^\circ}{2} = \frac{\left(\frac{30}{2}\right)}{RT}$$

$$RT \approx 15.6 \text{ cm}$$

$$VT = RT \tan 40^\circ \approx 13.1 \text{ cm}$$

Note that  $U$  is the mid-point of  $PQ$ .

$$\angle PTU = \frac{1}{2}\angle PTQ = \frac{1}{2}(2\angle PRQ) = 50^\circ$$

$$PT = RT \approx 15.6 \text{ cm}$$

$$TU = PT \cos 50^\circ \approx 10.0 \text{ cm}$$

Consider  $\triangle TUV$ .

$$\tan \angle TUV = \frac{VT}{TU}$$

$$\angle TUV \approx 52.5^\circ$$

(iii) Let  $W$  be a point on  $PQ$  such that  $TW \perp PQ$ .

Required angle is  $\angle TWV$ .

$$\angle TPQ = 180^\circ - 90^\circ - 74^\circ = 16^\circ$$

$$\angle TPR = 56^\circ - 16^\circ = 40^\circ$$

$$\angle PRW = 180^\circ - 90^\circ - 56^\circ = 34^\circ$$

$$\angle TRQ = 50^\circ - 34^\circ = 16^\circ$$

$$\angle PTR = 180^\circ - 34^\circ - 40^\circ = 106^\circ$$

Consider  $\triangle PTR$ .

$$\frac{RT}{\sin 40^\circ} = \frac{30}{\sin 106^\circ}$$
$$RT \approx 20.1 \text{ cm}$$

$$VT = RT \tan 40^\circ \approx 16.8 \text{ cm}$$

$$TW = RW - RT$$

$$= 30 \sin 56^\circ - RT$$

$$\approx 4.81 \text{ cm}$$

Consider  $\triangle TWV$ .

$$\tan \angle TWV = \frac{VT}{TW}$$

$$\angle TWV \approx 74.1^\circ$$