REG-TTC-2425-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. C	3. C	4. D	5. B
6. A	7. B	8. B	9. B	10. D
11. C	12. A	13. C	14. B	15. B
16. C	17. B	18. B	19. C	20. D
21. D	22. C	23. D	24. D	25. D
26. A	27. D	28. A	29. A	30. A
31. D	32. B	33. C	34. B	35. B
36. D	37. B	38. C	39. C	40. A
41. B	42. A	43. B	44. C	45. A

1. $\boxed{\text{C}}$

$$\frac{\angle BAD}{\angle BDC} = \frac{\overset{\frown}{BD}}{\overset{\frown}{BC}}$$

$$\frac{\angle BAD}{20^\circ} = \frac{2}{1}$$

$$\angle BAD = 40^\circ$$

$$\angle ADB = 90^\circ$$

$$\angle ABD + \angle ADB + \angle BAD = 180^\circ$$

$$\angle ABD = 50^\circ$$

2. $\boxed{\text{C}}$

$\angle AOE = 2 \times 25^\circ = 50^\circ$.

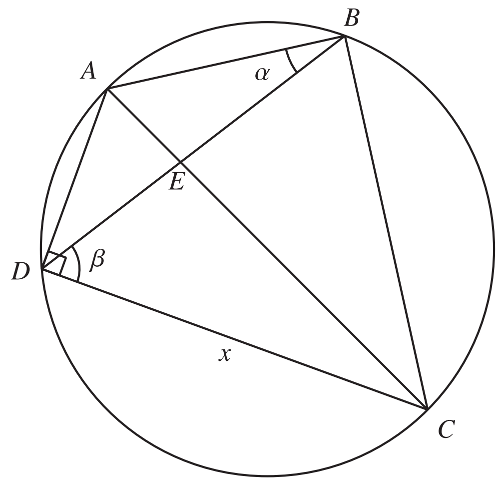
Since $OA = AE$, $\angle AEO = \angle AOE = 50^\circ$.

$\angle ABC = 180^\circ - \angle ADC = 155^\circ$.

$\angle ECB = \angle ABC - \angle BEC = 105^\circ$.

3. $\boxed{\text{C}}$

Draw circle $ABCD$.



Note that AC is a diameter of the circle and $\angle ABC = 90°$.

We have $\angle ACD = \angle ABD = \alpha$ and $\angle BAC = \angle BDC = \beta$.

$$\begin{aligned} BC &= AC \sin\beta \\ &= \left(\frac{x}{\cos\alpha}\right)\sin\beta \\ &= \frac{x\sin\beta}{\cos\alpha} \end{aligned}$$

4. $\boxed{\text{D}}$

$$\begin{aligned} \angle OCA &= 180° - \angle EFC - \angle CEF \\ &= 180° - (180° - \angle BFC) - (180° - \angle CED) \\ &= \angle BFC + \angle CED - 180° \\ &= 30° \end{aligned}$$

So, $\angle OAC = 30°$, $\angle AOC = 120°$ and $\angle COD = 60°$.

$$\frac{1+a}{b} = \frac{120°}{60°}$$

$$1 + a = 2b$$

$$2b - a = 1$$

5. $\boxed{\text{B}}$

$\angle DAO = \angle ADO = 46°$ and $\angle OCA = \angle OAC = 64° - 46° = 18°$.

$\angle AOC = 180° - 2 \times 18° = 144°$ and reflex $\angle AOC = 360° - 144° = 216°$.

$\angle ABC = \dfrac{1}{2} \times 216° = 108°$.

6. $\boxed{\text{A}}$

Let O be the centre of the circle.
Then $\angle AOC = 2\angle ABC = 56°$.

$$\frac{56°}{360°} = \frac{\overset{\frown}{AC}}{\text{circumference}}$$

$$\text{Circumference} = 45\,\text{cm}$$

7. $\boxed{\text{B}}$

$$\begin{aligned}\angle ADE &= \angle DAC + \angle ACD \\ &= 36° + 24° \\ &= 60°\end{aligned}$$

$$\begin{aligned}\angle ABE &= \angle ADE \\ &= 60°\end{aligned}$$

$$\begin{aligned}\angle ABD &= \angle ABE + \angle DBE \\ 90° &= 60° + \angle DBE \\ \angle DBE &= 30°\end{aligned}$$

8. $\boxed{\text{B}}$

Since $\overset{\frown}{PS} = \overset{\frown}{SR}$, we have $\angle POS = \angle ROS = \dfrac{136°}{2} = 68°$.
Since $OP = OS$, we have $\angle SPO = \angle PSO = \dfrac{180° - 68°}{2} = 56°$.

9. $\boxed{\text{B}}$

$$\angle ADB = 90°$$

$$\begin{aligned}(2\angle CBD) + (22° + 90°) &= 180° \\ \angle CBD &= 34°\end{aligned}$$

$$\begin{aligned}\angle BCD &= 180° - 22° - 34° \\ &= 124°\end{aligned}$$

10. $\boxed{\text{D}}$

$$\angle P + \angle R = 180°$$

$$\begin{aligned}\angle P &= 180° \times \frac{3}{3+5} \\ &= 67.5°\end{aligned}$$

$$\begin{aligned}\angle Q &= \angle P \times \frac{4}{3} \\ &= 90°\end{aligned}$$

$$\angle S = 180° - 90° = 90°$$

11. $\boxed{\text{C}}$

$\angle CAD = \angle CBD = 90^\circ$
$\angle BDC = \angle BAC = \alpha$
$BC = CD \sin\alpha$
$AD = CD \cos\beta$
$$\frac{BC}{AD} = \frac{CD\sin\alpha}{CD\cos\beta} = \frac{\sin\alpha}{\cos\beta}$$

12. $\boxed{\text{A}}$

$\angle BAD = 180^\circ - \angle DCE = 82^\circ$.
$\angle ABE = \angle BAD - \angle AEB = 47^\circ$.

13. $\boxed{\text{C}}$

$\angle ABD = 90^\circ$
We have $\angle ABO = 60^\circ$, $\angle OBD = 30^\circ$ and $\angle DBE = 15^\circ$.
$\angle DAE = \angle DBE = 15^\circ$
$DE = 6\sin 15^\circ \approx 1.55\,\text{cm}$
$\angle CED = \angle BAD = \angle OBA = 60^\circ$
$\angle CDE = \angle ABE = 105^\circ$
$\angle DCE = 180^\circ - 60^\circ - 105^\circ = 15^\circ = \angle DBE$
$CD = BD = 6\sin 60^\circ = 3\sqrt{3}\,\text{cm}$
Area of $\triangle CDE = \frac{1}{2}(CD)(DE)\sin 105^\circ$
$\approx 3.90\,\text{cm}^2$
$\angle DOE = 2 \times 15^\circ = 30^\circ$
Required area
$$= (\text{area of } \triangle CDE) - \left[3^2\pi \times \frac{30^\circ}{360^\circ} - \frac{1}{2}(3)^2\sin 30^\circ\right]$$
$\approx 3.79\,\text{cm}^2$

14. $\boxed{\text{B}}$

$\angle AOC = 2 \times 74^\circ = 148^\circ$.
$\angle COD = 148^\circ - 52^\circ = 96^\circ$.
$\angle ODC = \dfrac{180^\circ - 96^\circ}{2} = 42^\circ$.

15. $\boxed{\text{B}}$

$\angle BCD = 90^\circ$ and $\angle BDC = \angle DBC = \dfrac{180^\circ - 90^\circ}{2} = 45^\circ$.
$\angle BAC = \angle BDC = 45^\circ$ and $\angle ABE = \angle AEB = \dfrac{180^\circ - 45^\circ}{2} = 67.5^\circ$.
$\angle BAD = 90^\circ$ and $\angle BDA = 180^\circ - 90^\circ - 67.5^\circ = 22.5^\circ$.
$\angle ACB = \angle BDA = 22.5^\circ$.

16. $\boxed{\text{C}}$

$\angle CAD = \angle BAC \times \dfrac{16}{24} = 42°$

$\angle ACB = 180° - 90° - 63° = 27°$ and $\angle ADE = \angle ACB = 27°$

$\angle AED = 180° - 27° - 42° = 111°$

17. $\boxed{\text{B}}$

$\angle ABC = 140°$

$\angle AEC = \dfrac{360° - \angle ABC}{2}$

$= 110°$

$\angle DAE = \angle ADC - \angle AED$

$= 140° - 110°$

$= 30°$

18. $\boxed{\text{B}}$

$\angle BDC = \dfrac{\angle BOC}{2} = 35°$

$\angle ACD = 90°$

$\angle BEC = \angle ACD + \angle BDC$

$= 90° + 35°$

$= 125°$

19. $\boxed{\text{C}}$

Let M, N and T be mid-points of AB, CD and EF respectively.

$AB = CD = EF \quad \rightarrow \quad OT = OM = ON$

$\triangle OMQ \cong \triangle ONQ$ and $\triangle ONR \cong \triangle OTR$

Let $\angle OQN = a$ and $\angle ORN = b$.

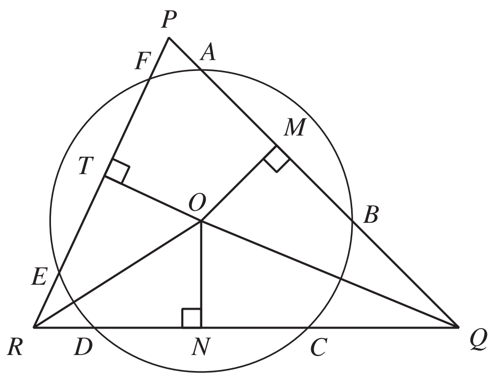
We have $\angle OQM = a$ and $ORT = b$.

$2a + 2b + 42° = 180°$

$a + b = 69°$

$\angle QOR = 180° - (a + b)$

$= 111°$



20. $\boxed{\text{D}}$

Note that $\triangle EBC \sim \triangle EDA$.

$$\frac{CE}{AE} = \frac{BE}{DE}$$

$$\frac{CE}{4+8} = \frac{8}{10+CE}$$

$$(CE)^2 + 10(CE) - 96 = 0$$

$$CE = 6\text{ cm} \quad \text{or} \quad -16\text{ cm (rejected)}$$

21. $\boxed{\text{D}}$

$$\text{Radius} = \sqrt{7^2 + \left(\frac{48}{2}\right)^2}$$

$$= 25\text{ cm}$$

Let θ be the angle of the sector corresponding to the shaded segment.

$$\cos\frac{\theta}{2} = \frac{7}{25}$$

$$\theta \approx 147^\circ$$

$$\text{Required area} = 25^2\pi \times \frac{\theta}{360^\circ} - \frac{(48)(7)}{2}$$

$$\approx 636\text{ cm}^2$$

22. $\boxed{\text{C}}$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$2(44^\circ) + \angle AOB = 180^\circ$$

$$\angle AOB = 92^\circ$$

$$\angle BOD = 2\angle BCD$$

$$92^\circ + \angle AOD = 2(62^\circ)$$

$$\angle AOD = 32^\circ$$

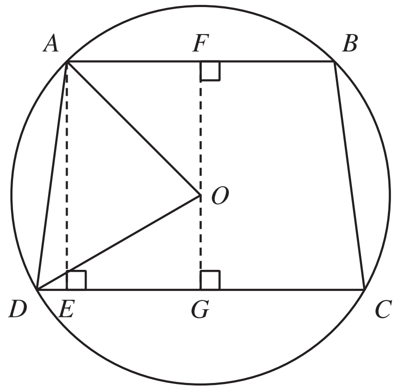
23. $\boxed{\text{D}}$

Note that $\angle ADC = \angle BCD = 80^\circ$ and $AB // CD$.

Let E be a point on CD such that $AE \perp CD$.

Let F and G be the mid-points of AB and CD respectively.

Denote the centre of the circle by O.



Consider $\triangle ADE$.

$$\tan \angle ADE = \frac{AE}{DE}$$

$$\tan 80^\circ = \frac{AE}{\left(\frac{8-6}{2}\right)}$$

$$AE = \tan 80^\circ \text{ cm}$$

Let $OG = x$ cm. Consider $\triangle ODG$ and $\triangle OAF$.

$$r^2 = 3^2 + (AE - x)^2 = 4^2 + x^2$$

$$-2(AE)x = 7 - AE^2$$

$$x \approx 2.22$$

$$\begin{aligned} \text{Required area} &= r^2\pi \\ &= (4^2 + x^2)\pi \\ &\approx 65.7 \text{ cm}^2 \end{aligned}$$

24. $\boxed{\text{D}}$

$\angle CAD = \angle CAB = \dfrac{50^\circ}{2} = 25^\circ$

$\angle ABC = 90^\circ$

$$\angle ABC + \angle BCA + \angle BAC = 180^\circ$$

$$\angle BCA = 65^\circ$$

$\angle AEB = \angle BCA = 65^\circ$

25. $\boxed{\text{D}}$

Reflex $\angle AOC = 2x$. So, $y = 360^\circ - 2x$.

26. $\boxed{\text{A}}$

Centre of small circle lies on RT, then $\angle SRT = \angle TRU$.

$\angle RTU = \angle RUQ = 40°$ and $\angle RTS = 90°$

$\angle SRU = 180° - (40° + 90°) = 50°$

$\angle TRU = \dfrac{50°}{2} = 25°$ and $\angle PUT = \angle TRU = 25°$

27. $\boxed{\text{D}}$

$\angle BAC = 180° - 63° - 59° = 58°$.

$\angle BOC = 58° \times 2 = 116°$ and $\angle BCO = \dfrac{180° - 116°}{2} = 32°$.

28. $\boxed{\text{A}}$

BD is diameter $\Rightarrow \angle DAB = 90°$

Let $\angle CAE = \theta$.

Consider $\triangle CAE$, $\angle ACD = \angle CAE + 14° = \theta + 14°$.

Since AT is tangent, $\angle DAT = \angle ACD = \theta + 14°$.

Consider $\triangle TAE$,

$$\begin{aligned} 47° + (90° + \theta + 14°) + 14° &= 180° \\ \theta &= 15° \end{aligned}$$

29. $\boxed{\text{A}}$

BC is a diameter, $\angle BDC = 90°$.

Since $\triangle BDC \sim \triangle CDA$, $\dfrac{BD}{BC} = \dfrac{CD}{AC}$

$$\frac{BD}{CD} = \frac{3}{4}$$

$\angle BCD = \tan^{-1}\dfrac{3}{4}$ and $BD = 9\sin\angle BCD = 5.4\,\text{cm}$.

30. $\boxed{\text{A}}$

$\angle DAC = \angle ABC$

$\angle BAC = 90°$

$$\begin{aligned} \angle ADE + \angle BAD + \angle ABC + \angle BED &= 360° \\ 54° + 90° + 2\angle ABC + 90° &= 360° \\ \angle ABC &= 63° \end{aligned}$$

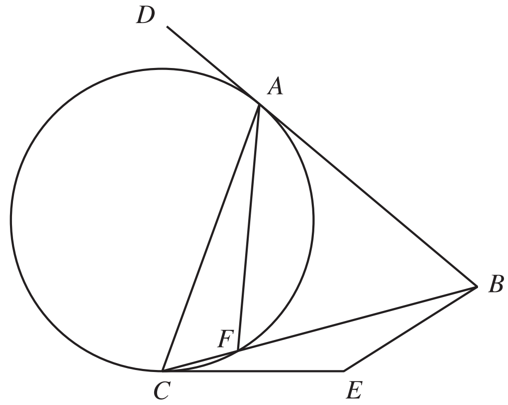
$$\begin{aligned} \angle ABC + 90° + \angle BCA &= 180° \\ \angle BCA &= 27° \end{aligned}$$

31. $\boxed{\text{D}}$

Denote the intersections of BC and the circle by C and F.

$\angle CAF = \angle ECB = 30°$

$\angle BAF = \angle FCA$

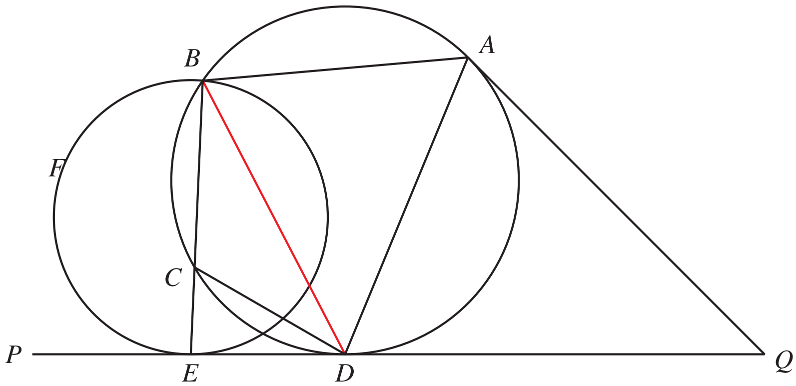


$$\angle FCA + \angle BAF + 30° + 40° = 180°$$
$$2\angle BAF = 110°$$
$$\angle BAF = 55°$$

$\angle CAD = 180° - 30° - 55° = 95°$

32. $\boxed{\text{B}}$

Since $AQ = DQ$, we have $\angle ADQ = \dfrac{180° - 50°}{2} = 65°$.



$\angle ABD = \angle ADQ = 65°$

$\angle CBD = 100° - 65° = 35°$

$\angle CDE = \angle CBD = 35°$

$\angle CDB = \angle CBD = 35°$

$\angle PEB = \angle EBD + \angle BDE = 35° + (35° + 35°) = 105°$

33. $\boxed{\text{C}}$

$$\angle AOC + \angle OAC + \angle OCA = 180°$$
$$\angle AOC + 2(27°) = 180°$$
$$\angle AOC = 126°$$

$\angle BOC = \angle AOC \times \dfrac{2}{1+2} = 84°$

$\angle BAC = \dfrac{\angle BOC}{2} = 42°$

$\angle BCD = \angle BAC = 42°$

34. $\boxed{\text{B}}$

$\angle DCA = \angle YAD = 31^\circ$
$AX = AB \quad \Rightarrow \quad \angle XAB = \dfrac{180^\circ - 64^\circ}{2} = 58^\circ$
$\angle ACB = \angle XAB = 58^\circ$
$\angle BCD = 31^\circ + 58^\circ = 89^\circ$

35. $\boxed{\text{B}}$

Let $\angle CAP = x$. Then $\angle ABC = \angle CAP = x$.

$$\angle DBA + \angle BAP = 180^\circ$$
$$40^\circ + x + x + 90^\circ = 180^\circ$$
$$x = 25^\circ$$

36. $\boxed{\text{D}}$

$\angle EOD = 2\angle CBE = 30^\circ$ and $\angle EDO = \angle EOD = 30^\circ$
$\angle ODC = \dfrac{180^\circ - 30^\circ}{2} = 75^\circ$ and $\angle CDB = 75^\circ - 30^\circ = 45^\circ$
$\angle ABC = \angle CDB = 45^\circ$ and $\angle BAC = 180^\circ - 2 \times 45^\circ = 90^\circ$

37. $\boxed{\text{B}}$

Let O be the centre of the semi-circle $ABCD$.
We have $OC \perp EF$ and $OB \perp BE$.
Thus, $\angle BOC = 360^\circ - 90^\circ - 90^\circ - 90^\circ = 90^\circ$.
$\angle AOC = \angle AFE + 90^\circ = 132^\circ$
$\angle AOB = 132^\circ - 90^\circ = 42^\circ$
$\angle ACB = \dfrac{\angle AOB}{2} = 21^\circ$

38. $\boxed{\text{C}}$

$\angle ADC = 180^\circ - 100^\circ = 80^\circ$
$\angle ADB = 80^\circ \times \dfrac{2}{2+3} = 32^\circ$
$\angle BCA = \angle ADB = 32^\circ$
$\angle DAT = \angle ADC - \angle ATD = 50^\circ$
$\angle ACT = \angle DAT = 50^\circ$
$\angle BCT = 32^\circ + 50^\circ = 82^\circ$

39. $\boxed{\text{C}}$

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ$$
$$\angle BOC = 180^\circ - 2 \times 55^\circ$$
$$= 70^\circ$$
$\angle OCT = 90^\circ$
$$\angle OTC + \angle OCT + \angle COT = 180^\circ$$
$$\angle OTC = 180^\circ - 70^\circ - 90^\circ$$
$$\angle ATC = 20^\circ$$

40. $\boxed{\text{A}}$

$\angle AGE = 2 \times 66^\circ = 132^\circ$
$\angle CEA = 132^\circ \times \dfrac{6}{5+6} = 72^\circ$
$\angle CAT = \angle CEA = 72^\circ$ and $\angle ATC = 180^\circ - 2 \times 72^\circ = 36^\circ$

41. $\boxed{\text{B}}$

Let O be the centre of circle ABD.
$\angle EAO = \dfrac{60^\circ}{2} = 30^\circ$ and $AO = 9 \tan 30^\circ = 3\sqrt{3}$ cm.
$BC = AC - AB = 9 \tan 60^\circ - 2 \times (3\sqrt{3}) \approx 5.20$ cm.

42. $\boxed{\text{A}}$

$\angle AEB = \angle SAB = 60^\circ$
$\angle BCD = \angle AEB = 60^\circ$
$\angle BCE : \angle ECD = 15^\circ : 60^\circ - 15^\circ = 1 : 3$

43. $\boxed{\text{B}}$

$EA = ED$
$\angle EDA = \dfrac{180^\circ - 86^\circ}{2} = 47^\circ$
$\angle ACD = \angle EDA = 47^\circ$
$\angle FGH = 180^\circ - \angle FEH = 94^\circ$
$\angle GBC = \dfrac{180^\circ - 94^\circ}{2} = 43^\circ$
$\angle BDC = \angle GBC = 43^\circ$
$\angle AID = \angle BDC + \angle ACD = 90^\circ$

44. $\boxed{\text{C}}$

RT is the angle bisector of $\angle SRP$. So, $\angle SRP = 24^\circ$.
$\angle QPS = \angle QSR$.

$$\angle QPS + (\angle QSR + 70^\circ) + 24^\circ = 180^\circ$$
$$\angle QPS = 43^\circ$$

45. $\boxed{\text{A}}$

$\angle CBP = \angle BCP = \angle BAC = 56^\circ$
$\angle BPC = 180^\circ - 56^\circ - 56^\circ = 68^\circ$
$\angle PBD = 90^\circ$
$\angle BQP = 180^\circ - 90^\circ - 68^\circ$
$= 22^\circ$

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