

REG-COT-2425-ASM-SET 2-MATH

Suggested solutions

Multiple Choice Questions

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. A | 2. C | 3. D | 4. A | 5. A |
| 6. B | 7. C | 8. B | 9. B | 10. A |
| 11. B | 12. C | 13. D | 14. D | 15. A |

1. **A**

Note that $\angle ABC = 90^\circ$.

Let the radius be r cm.

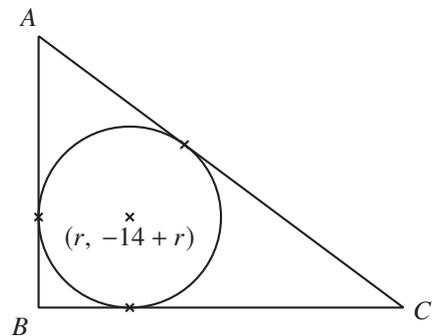
The coordinates of centre are $(r, -14 + r)$.

Consider the length of AC .

$$(24 - r) + (18 - r) = \sqrt{18^2 + 24^2}$$

$$r = 6$$

The coordinates of centre are $(6, -8)$.



2. **C**

The coordinates of A and B are $\left(\frac{k}{6}, 0\right)$ and $\left(0, -\frac{k}{3}\right)$ respectively.

Denote the mid-point of AB by M .

The coordinates of M are $\left(\frac{k}{12}, -\frac{k}{6}\right)$.

Let the coordinates of C be $(c, 0)$.

We have $CG : GM = 2 : 1$, where G is the centroid of $\triangle ABC$.

$$\frac{CG}{GM} = \frac{c - 0}{0 - \frac{k}{12}}$$

$$2 = -\frac{12c}{k}$$

$$c = -\frac{k}{6}$$

The coordinates of A and B are $\left(\frac{k}{6}, 0\right)$ and $\left(0, -\frac{k}{3}\right)$ respectively.

Let the coordinates of C be $(c, 0)$.

Note that the x -coordinate of the centroid of $\triangle ABC$ is 0.

$$\frac{\frac{k}{6} + 0 + c}{3} = 0$$

$$c = -\frac{k}{6}$$

3. D

G is the centre of circle ABC .

$$\text{Radius} = GB = \sqrt{(18 - 10)^2 + (3 + 3)^2} = 10$$

Since A is a point on the circle, the y -coordinate of A lies between $3 + 10$ and $3 - 10$ inclusively.

Thus, -8 is not a possible y -coordinate of A .

4. A

AB is the angle bisector of $\angle CAD$.

$$\angle BAC = \angle BAD = 48^\circ$$

$$\angle ACD = \angle BAD = 48^\circ$$

$$\angle ADC + \angle ACD + \angle CAD = 180^\circ$$

$$\angle ADC + 48^\circ + (48^\circ + 48^\circ) = 180^\circ$$

$$\angle ADC = 36^\circ$$

5. A

Note that $\angle POQ = 90^\circ$.

The orthocentre of $\triangle OPQ$ is O .

The circumcentre of $\triangle OPQ$ is the mid-point of PQ .

The coordinates of the circumcentre of $\triangle OPQ$ are $\left(\frac{p}{2}, \frac{q}{2}\right)$.

The straight line $2x - y = 3k$ passes through O and $\left(\frac{p}{2}, \frac{q}{2}\right)$.

$$\begin{cases} 2(0) - 0 = 3k \\ 2\left(\frac{p}{2}\right) - \frac{q}{2} = 3k \end{cases}$$

Solving, we have $k = 0$ and $2p = q$.

Thus, $p : q = 1 : 2$.

6. B

Denote the mid-point of AB by M .

Let G be the centroid of $\triangle ABP$.

Note that $MG : GP = 1 : 2$.

The y -coordinate of G is therefore constant.

The locus of centroid of $\triangle ABP$ is a straight line parallel to L .

7. C

I. ✓. y-coordinate of the mid-point of BC is 0.

y-coordinate of centroid of $\triangle ABC$

$$= \frac{1(6) + 2(0)}{1 + 2}$$

$$= 2$$

The centroid of $\triangle ABC$ lies on $y = 2$.

II. ✓. Note that the y-axis passes through A and is perpendicular to BC .

Thus, the orthocentre of $\triangle ABC$ lies on the y-axis.

8. B

Coordinates of three vertices are $(0, 0)$, $(6, 0)$ and $(0, 8)$.

Let the radius of the inscribed circle be r .

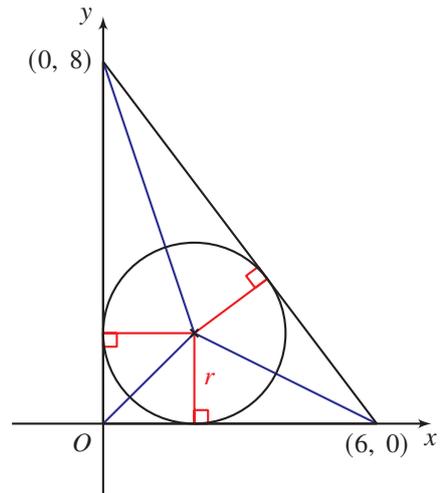
Length of hypotenuse = $\sqrt{6^2 + 8^2} = 10$

Consider the area of triangle.

$$\frac{(6)(8)}{2} = \frac{(6)(r)}{2} + \frac{(8)(r)}{2} + \frac{(10)(r)}{2}$$

$$r = 2$$

Coordinates of in-centre are $(2, 2)$.



9. B

$$\angle BAC + 2\angle CBD + 2\angle BCD = 180^\circ$$

$$\angle CBD + \angle BCD = 55^\circ$$

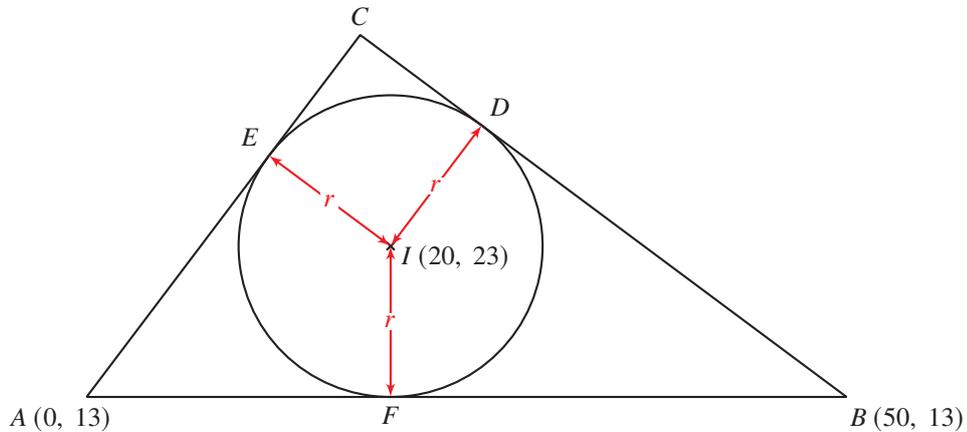
$$\angle BDC + \angle CBD + \angle BCD = 180^\circ$$

$$\angle BDC = 125^\circ$$

10. A

Since C is the orthocentre of $\triangle ABC$, $\angle ACB = 90^\circ$.

Let D, E and F be the points of contact as shown below. Denote the incentre of $\triangle ABC$ by I .



Consider the vertical distance from $(20, 23)$ to AB .

$$\begin{aligned} r &= 23 - 13 \\ &= 10 \end{aligned}$$

Note that $CDIE$ is a square.

We have $AC = AE + r = AF + r = 20 + 10 = 30$.

Consider the horizontal distance between A and C .

Horizontal distance = $AC \cos \angle BAC$

$$\begin{aligned} &= 30 \cos \angle BAC \\ &= 30 \left(\frac{AC}{AB} \right) \quad (\text{Note that } \angle ABC \text{ is a right-angled triangle.}) \\ &= 30 \times \frac{30}{50} \\ &= 18 \end{aligned}$$

The x -coordinate of C is $0 + 18 = 18$.

11. B

I. \checkmark . Since G is the orthocentre of $\triangle ABC$, $\angle AQG = \angle ARG = 90^\circ$.

$AQGR$ is a cyclic quadrilateral (*opp. \angle s supp.*).

II. \checkmark . Since G is the orthocentre of $\triangle ABC$, $\angle CQB = \angle CRB = 90^\circ$.

$BCQR$ is a cyclic quadrilateral (*converse of \angle s in the same segment*).

III. \times . Note that the circle passing through C, Q and R is circle $BCQR$.

Since P is a point on the line segment BC , P lies inside the circle $BCQR$.

$CPRQ$ cannot be a cyclic quadrilateral.

12. C

The coordinates of P are $\left(-\frac{k}{3}, 0\right)$.

$$\text{Solve } \begin{cases} 3x - 4y + k = 0 \\ 4x + 3y - k = 0 \end{cases}$$

The coordinates of Q are $\left(\frac{k}{25}, \frac{7k}{25}\right)$.

Since the in-centre of $\triangle PQR$ lies on the x -axis, PI is the angle bisector of $\angle RPQ$.

$$\text{Slope of } PR = -1 \times (\text{slope of } L_1) = -\frac{3}{4}$$

The equation of PR is

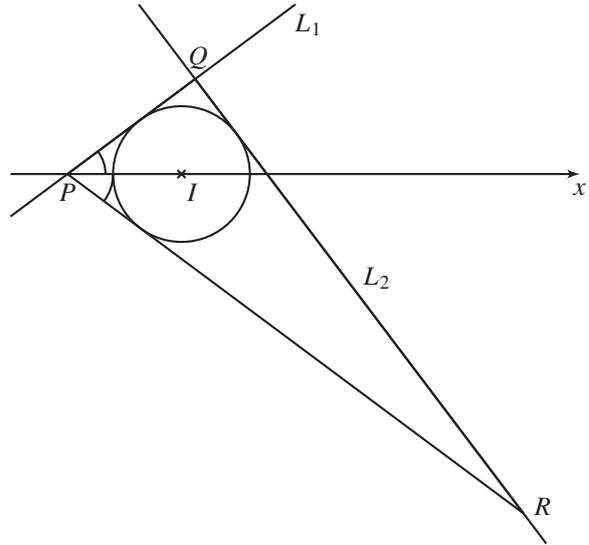
$$y - 0 = -\frac{3}{4} \left(x + \frac{k}{3}\right)$$

$$3x + 4y + k = 0$$

$$\text{Solve } \begin{cases} 3x + 4y + k = 0 \\ 4x + 3y - k = 0 \end{cases}$$

The coordinates of R are $(k, -k)$.

The x -coordinate of R is k .



Program method

Let $k = 300$.

The coordinates of P and Q are $(-100, 0)$ and $(12, 84)$ respectively.

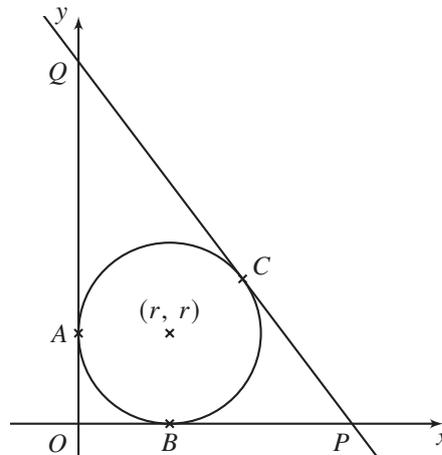
Note that R lies on L_2 : $4x + 3y - 300 = 0$, we can obtain the coordinates of R using the value in each option, and the coordinates of the in-centre of $\triangle PQR$ through the calculator program.

Option	Coordinates of R	Coordinates of in-centre
A.	$(-2100, 2900)$	$(-84.1, 97.7)$
B.	$(-300, 500)$	$(-73.0, 96.1)$
C.	$(300, -300)$	$(0, 0)$
D.	$(2100, -2700)$	$(-1.72, -12.0)$

The in-centre should lie on the x -axis. The answer is C.

13. D

The coordinates of P and Q are $(6, 0)$ and $(0, 8)$ respectively.



Let the radius of the inscribed circle be r .

Consider the area of $\triangle OPQ$.

$$\frac{(6)(8)}{2} = \frac{(6)(r)}{2} + \frac{(8)(r)}{2} + \frac{(\sqrt{6^2 + 8^2})(r)}{2}$$

$$r = 2$$

Required coordinates are $(2, 2)$.

Refer to the figure.

$$OA = OB = r$$

$$BP = CP = 6 - r$$

$$AQ = CQ = 8 - r$$

Consider the length of PQ .

$$(6 - r) + (8 - r) = \sqrt{6^2 + 8^2}$$

$$r = 2$$

Required coordinates are $(2, 2)$.

14. D

Note that $AB = AD = AE$.

A is the centre of the circumcircle of BDE .

Since BE is a diameter of circle BDE , we have $\angle BDE = 90^\circ$.

$$\angle ADE = \angle DEA = 35^\circ$$

$$\angle ADB = 90^\circ - 35^\circ = 55^\circ$$

$$\angle CBD = \angle ADB = 55^\circ$$

15. A

Note that $\angle AOB = 90^\circ$ and $\triangle OAB$ is a right-angled triangle.

Consider the circumcircle OAB , AB is a diameter.

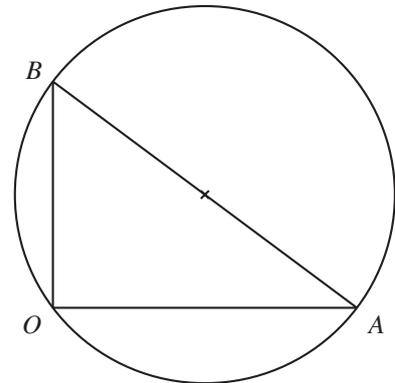
Circumcentre of $\triangle OAB$ is the mid-point of AB .

Coordinates of mid-point of AB are $\left(\frac{a}{2}, \frac{b}{2}\right)$.

$$4\left(\frac{a}{2}\right) + 16\left(\frac{b}{2}\right) = 17a$$

$$8b = 15a$$

$$a : b = 15 : 8$$



Conventional Questions

16. (a) Slope of $L_2 = -\frac{1}{2}$. Slope of $L_1 = 2$. 1M
The equation of L_1 is

$$y - 4 = 2(x - 3)$$

$$y = 2x - 2$$

1A

- (b) $B(8, 0)$ and $C(0, 4)$. 1A

Since AC is parallel to the x -axis, x -coordinate of the orthocentre = 8. 1A

Let the coordinates of orthocentre be $(8, k)$.

$$\frac{k - 4}{8 - 0} \times \frac{4 - 0}{3 - 8} = -1$$

1M

$$k = 14$$

Required coordinates are $(8, 14)$. 1A

17. (a) Slope of $AB = \frac{4 - 0}{3 - 2} = 4$ 1M

The equation of L is $y = -\frac{x}{4}$. 1A

- (b) Orthocentre lies on the altitude through B , i.e., $x = 3$.

Put $x = 3$ into equation of L , $y = -\frac{3}{4}$. 1M

The coordinates of orthocentre are $\left(3, -\frac{3}{4}\right)$. 1A

18. (a) $CE \perp AB$ *(property of orthocentre)*

$BD \perp AC$ *(property of orthocentre)*

$$\angle BEC = \angle BDC = 90^\circ$$

Thus, $BCDE$ is a cyclic quadrilateral. *(converse of \angle s in the same segment)*

Marking Scheme	
Case 1	Any correct proof with correct reasons. 2
Case 2	Any correct proof without reasons. 1

- (b) (i) Coordinates of centre = $\left(\frac{-6 + 14}{2}, \frac{-6 - 6}{2}\right) = (4, -6)$ 1A

The equation of the circle is

$$(x - 4)^2 + (y + 6)^2 = (0 - 4)^2 + (8 + 6)^2$$

1M

$$(x - 4)^2 + (y + 6)^2 = 100$$

1A

- (ii) Distance between A and centre = $\sqrt{4^2 + (-6 - 8)^2} = \sqrt{212}$

Radius of circle = 10

$$\text{Angle between two tangents} = 2 \times \sin^{-1} \frac{10}{\sqrt{212}} \approx 86.8^\circ \neq 90^\circ$$

1M+1A

The claim is not agreed. 1A

19. (a) $\frac{y-2}{x-1} \times \frac{y-8}{x-9} = -1$ 1M+1A
- $(y-2)(y-8) + (x-1)(x-9) = 0$
- $x^2 + y^2 - 10x - 10y + 25 = 0$ 1A
- (b) (i) $8^2 + 1^2 - 10(8) - 10(1) + 25 = 0$
- Thus, C lies on S . 1
- (ii) $(5, 5)$ 1A
- (iii) Since AB is a diameter of the circle, $\angle ACB = 90^\circ$.
- So, the orthocentre H is at point $C(8, 1)$. 1M
- Circumcentre J is the mid-point of AB . The line joining J and H is a median of $\triangle ABC$.
- Since centroid G lies on median of $\triangle ABC$, G, J and H are collinear.
- The claim is agreed. 1A

20. (a) Let $x = \angle BAI$ and $y = \angle ABI$.

$$\begin{aligned} \angle PAC &= \angle BAI = x && \text{(property of incentre)} \\ PB &= PC && \text{(equal } \angle\text{s, equal chords)} \\ \angle PIB &= \angle ABI + \angle BAI && \text{(ext. } \angle \text{ of } \triangle) \\ &= x + y \\ \angle IBC &= \angle ABI = y && \text{(property of incentre)} \\ \angle PBC &= \angle PAC = x && \text{(} \angle\text{s in the same segment)} \\ \angle IBP &= x + y \\ &= \angle PIB \\ PB &= PI && \text{(sides opp. equal } \angle\text{s)} \end{aligned}$$

Thus, $PB = PI = PC$.

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

- (b) $\angle IAY = \angle PSC$ ($\angle\text{s in the same segment}$)
- $\angle AYI = 90^\circ$ (given)
- $\angle SCP = 90^\circ$ (\angle in semicircle)
- $= \angle AYI$
- $\triangle IAY \sim \triangle PSC$ (AA)

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(c) $\frac{IY}{PC} = \frac{IA}{PS}$ 1M
 $\frac{r}{IP} = \frac{AI}{2R}$

$AI \cdot IP = 2Rr$

The claim is agreed. 1A

(d) Let the equation of circle BPC be $x^2 + y^2 + Dx + Ey + F = 0$, where D , E and F are constants.

$$\begin{cases} (-16)^2 - 16D + F = 0 \\ (-8)^2 - 8E + F = 0 \\ 16^2 + 16D + F = 0 \end{cases} \quad 1M$$

Solving, we have $D = 0$, $E = -24$ and $F = -256$.

Radius of the circumcircle = $\sqrt{0^2 + 12^2 + 256} = 20$ 1A

$BP = \sqrt{16^2 + 8^2} = 8\sqrt{5}$ 1A

By (c), $AI = \frac{2Rr}{IP} = \frac{2(20)(9)}{8\sqrt{5}} = 9\sqrt{5}$ 1A

21. (a) $\angle OMQ = 90^\circ = \angle ONQ$ (given)

$AB = CD$ (given)

$OM = ON$ (equal chords, equidistant from centre)

$OQ = OQ$ (common side)

$\triangle QNO \cong \triangle QMO$ (RHS)

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $OT = ON = 130$

$OQ = \sqrt{312^2 + 130^2} = 338$ and $OT : OQ = 130 : 338 = 5 : 13$

x -coordinate of $T = 312 \times \left(-\frac{5}{13}\right) = -120$ 1M

y -coordinate of $T = -130 \times \left(-\frac{5}{13}\right) = 50$

The coordinates of T are $(-120, 50)$. 1A

Let the coordinates of R be $(h, -130)$ such that $QR \perp ON$.

$$\frac{-130 - 50}{h + 120} \times \frac{50 - 0}{-120 - 0} = -1 \quad 1M$$

$h = -195$

Let the coordinates of P be (a, b) . Note that T is the mid-point of PR .

$$\frac{a + (-195)}{2} = -120 \quad \text{and} \quad \frac{b + (-130)}{2} = 50 \quad 1\text{M}$$

$$a = -45 \qquad b = 230$$

The coordinates of P are $(-45, 230)$. 1A

(ii) $OP = \sqrt{45^2 + 230^2} = \sqrt{54925} \neq OQ$ 1M

Thus, O is not the circumcentre of $\triangle PQR$.

The claim is disagreed. 1A