REG-COT-2425-ASM-SET 1-MATH

Suggested solutions

Multiple Choice Questions

1. B	2. C	3. D	4. C	5. B
6. C	7. B	8. A	9. D	10. A
11. A	12. C	13. A	14. A	15. B
16. B	17. D	18. D	19. D	20. D
21. D	22. C	23. B	24. A	25. A
26. B	27. D	28. D	29. D	30. A

1. $\boxed{\text{B}}$

  I. ✗. Orthocentre is at B.

  II. ✓. Centroid always lies inside the triangle.

  III. ✗. Incentre always lies inside the triangle.

2. $\boxed{\text{C}}$

No steps required.

3. $\boxed{\text{D}}$

  I. ✓. AD is the perpendicular bisector of BC.

  II. ✓. AD is the altitude through A.

  III. ✓. AD is the median through A.

4. $\boxed{\text{C}}$

I is the incentre of $\triangle QRS$.

$\angle IRQ = \angle IRS = 12°$
$\angle SRQ = 12° + 12° = 24°$
$\angle QPS = \angle SQR$
In $\triangle PSR$,

$$70° + 2\angle QPS + 24° = 180°$$
$$\angle QPS = 43°$$
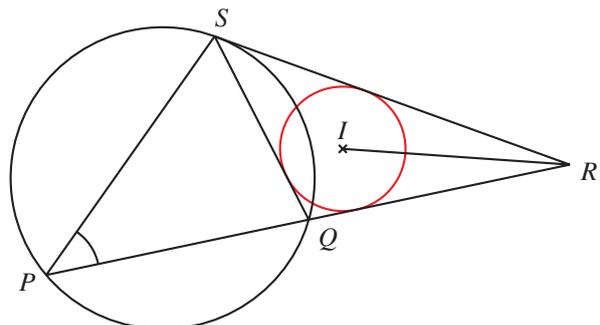
5. $\boxed{\text{B}}$

$BC = 2BL = 26\,\text{cm}$

$AB = 2BN = 10\,\text{cm}$

$AC = 2CM = 24\,\text{cm}$

Since $10^2 + 24^2 = 26^2$, $\angle BAC = 90°$.

Required area $= \dfrac{(10)(24)}{2}$

$ = 120\,\text{cm}^2$

6. $\boxed{\text{C}}$

Let the centre of circle be O.

BO is a median of $\triangle ABC$. So, BEO is a straight line.

$\angle BAC = \angle ABE = 27°$

$\angle CBD = \angle BAC = 27°$

$\angle ABC = 90°$

In $\triangle ABD$,

$$x + 27° + (90° + 27°) = 180°$$

$$x = 36°$$

7. $\boxed{\text{B}}$

Let $AC = 2$. Then $BC = 2$, $CD = 1$ and $AE = \dfrac{AB}{2} = \sqrt{2}$

Note that $CE \perp AB$, $\angle ACE = \sin^{-1} \dfrac{\sqrt{2}}{2} = 45°$

$\angle CAD = \tan^{-1} \dfrac{1}{2} \approx 26.6°$

$\theta = 180° - 45° - \tan^{-1} \dfrac{1}{2} \approx 108°$

$\sin \theta \approx 0.948683$.

By checking all options, we see that $\sin \theta = \dfrac{3\sqrt{10}}{10}$

8. $\boxed{\text{A}}$

   I. ✓. Let the radius of inscribed circle of $\triangle ABC$ be r.
Both angles are equal to $\tan^{-1} \dfrac{OV}{r}$.

  II. ✓. Let the radius of circumcircle ABC be R.
Then $OB = OC = R$ and both angles are equal to $\tan^{-1} \dfrac{OV}{R}$.

 III. ✗. If $\angle ABC = 90°$, then points O and B coincide.
Angle between plane VAB and plane ABC is $90°$, while angle between plane VAC and plane ABC is not $90°$.

9. $\boxed{\text{D}}$

y-coordinate of circumcentre $= \dfrac{(-2) + (8)}{2} = 3$

Let the coordinates of the circumcentre be $(h, \ 3)$.

$$\sqrt{(h-2)^2 + (3-8)^2} = \sqrt{(h-10)^2 + (14-3)^2}$$

$$h^2 - 4h + 29 = h^2 - 20h + 221$$

$$16h = 192$$

$$h = 12$$

x-coordinate of the circumcentre is 12.

10. $\boxed{\text{A}}$

Point B can be obtained by rotating A through $90°$ anticlockwise about the origin. So, $\angle AOB = 90°$.

Circumcentre is the mid-point of AB, required y-coordinate $= \dfrac{8-2}{2} = 3$

11. $\boxed{\text{A}}$

Since AB is parallel to the y-axis, altitude through O is parallel to the x-axis.

Let the coordinates of orthocentre H be $(x, \ 0)$. Since $AH \perp OB$,

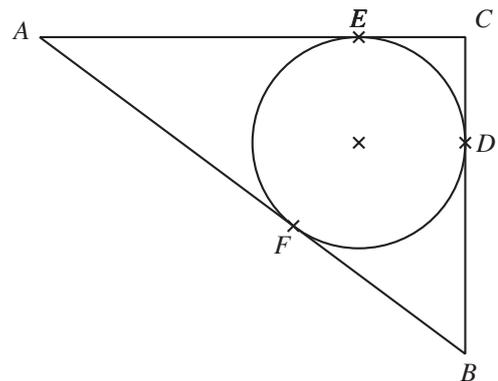$$\frac{12-0}{16-x} \times \frac{-12-0}{16-0} = -1$$

$$x = 7$$

12. $\boxed{\text{C}}$

Let D, E, F be the point of contact of the inscribed circle and BC, AC, AB respectively.

Let $CE = x$. Then $AE = 12 - x$, $CD = x$ and
$BD = 5 - x$.

By tangent properties, $AF = 12 - x$ and $BF = 5 - x$.

Since $AB = \sqrt{5^2 + 12^2} = 13$, we have $x = 2$.

Thus, the coordinates of incentre are $(10, \ 3)$.

13. $\boxed{\text{A}}$

Circumcentre lies on perpendicular bisector of OA.

y-coordinate of the circumcentre $= \dfrac{0 + 12}{2} = 6$

14. A

Altitude through Q is horizontal.

y-coordinate of the orthocentre = 48

Let the coordinates of the orthocentre be $(x, 48)$.

$$\frac{48 - 0}{x - 0} \times \frac{60 - 48}{0 - 96} = -1$$
$$x = 6$$

15. B

Observe that $\angle BAC = 90°$.

   I. ✗. Circumcentre is at the mid-point of BC.

  II. ✓. x-coordinate of centroid is 1 by symmetry.

 III. ✓. Orthocentre is at A (because it is a right-angled triangle).

16. B

The coordinates of the vertices of the triangle are $(0, 0)$, $(6, 0)$ and $(0, 8)$.

Let the radius of inscribed circle be r.

By considering the area of the triangle,

$$\frac{(6)(8)}{2} = \frac{(6)(r)}{2} + \frac{(8)(r)}{2} + \frac{(\sqrt{6^2 + 8^2})(r)}{2}$$
$$r = 2$$

The coordinates of the incentre are $(2, 2)$.

17. D

Denote the incentre by X.

Let D, E and F be points on OA, OB and AB respectively such that $XD \perp OA$, $XE \perp OB$ and $XF \perp AB$.
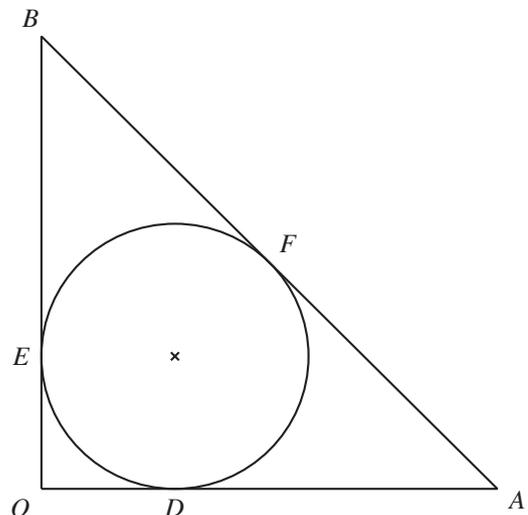
Let the radius be r.

$OD = OE = r$.

$AD = AF = 6 - r$ and $BE = BF = 6 - r$.

$$(6 - r) + (6 - r) = \sqrt{6^2 + 6^2}$$
$$r = 6 - 3\sqrt{2}$$

18. $\boxed{\text{D}}$

Altitude through *B* is vertical.

The *x*-coordinate of the orthocentre is 24.

Let the coordinates of the orthocentre be $(24, \ y)$.

$$\frac{y - 0}{24 - 0} \times \frac{18 - 0}{24 - 48} = -1$$
$$y = 32$$

19. $\boxed{\text{D}}$

Since OA is vertical, orthocentre lies on the horizontal line through *B*

Thus, *y*-coordinate of orthocentre $= -12$.

Let the coordinates of orthocentre *H* be $(x, \ -12)$. As $AH \perp BO$,

$$\frac{36 + 12}{0 - x} \times \frac{12}{16} = -1$$
$$x = 36$$

20. $\boxed{\text{D}}$

Let $H \ (h, \ k)$ be the orthocentre.

$$AH \perp BC \quad \text{and} \qquad\qquad BH \perp AC$$

$$\frac{k + 19}{h + 38} \times \frac{9 + 1}{-10 + 2} = -1 \qquad \frac{k - 9}{h + 10} \times \frac{-1 + 19}{-2 + 38} = -1$$
$$\frac{k + 19}{h + 38} = \frac{4}{5} \qquad\qquad\qquad \frac{k - 9}{h + 10} = -2$$

Solving, we have $h = -8$ and $k = 5$.

21. $\boxed{\text{D}}$

x-coordinate of vertex $= \dfrac{-k}{2(1)} = -\dfrac{k}{2}$

mid-point of $PR = \left(-\dfrac{k}{2}, \ 0\right)$

Consider the *x*-coordinate of centroid,

$$-2 = \frac{0(1) + \left(-\frac{k}{2}\right)(2)}{1 + 2}$$
$$k = 6$$

22. $\boxed{C}$

Let the coordinates of Q be (p, q). Denote the orthocentre of $\triangle OPQ$ by H.

$OP \perp QH$ and $PH \perp OQ$

$\dfrac{-18}{26} \times \dfrac{q - 21}{p + 3} = -1$ $\dfrac{-18 + 3}{26 - 21} \times \dfrac{q}{p} = -1$

$\qquad 13p - 9q = 300$ $p - 3q = 0$

Solving, we have $p = 30$ and $q = 10$.

Thus, y-coordinate of Q is 10.

23. $\boxed{B}$

$\triangle OAB$ is a right-angled triangle. So, orthocentre of $\triangle OAB$ is at point O.

Area of $\triangle OAB = \dfrac{1}{2}(12)(16) = 96$.

$AB = \sqrt{12^2 + 16^2} = 20$. Let the required distance be d. By considering the area of $\triangle OAB$,

$\qquad \dfrac{1}{2}(AB)(d) = 96$

$\qquad\qquad d = 9.6$

24. $\boxed{A}$

The straight line $x - 2y + 10 = 0$ is perpendicular to the straight line $2x + y + a = 0$.

The triangle formed is a right-angled triangle. The orthocentre lies at the right-angled vertex.

When $x = -6$,

$\qquad (-6) - 2y + 10 = 0$

$\qquad\qquad\qquad y = 2$

Substitute $(-6, 2)$ into $2x + y + a = 0$,

$\qquad 2(-6) + (2) + a = 0$

$\qquad\qquad\qquad a = 10$

25. $\boxed{A}$

$P\left(\dfrac{k}{2}, 0\right)$, $Q(0, -k)$ and $R(0, k)$.

Circumcentre lies on perpendicular bisector of QR, i.e., the x-axis.

Circumcentre is at $(-3, 0)$.

$\qquad \dfrac{k}{2} - (-3) = \sqrt{3^2 + k^2}$

$\qquad\qquad 0 = \dfrac{3k^2}{4} - 3k$

$\qquad\qquad k = 4$ or 0

y-coordinate of $R = 4$

26. B

$A = \left(\dfrac{k}{3},\, 0 \right)$ and $B = \left(0,\, \dfrac{k}{4} \right)$

Circumcentre lies on perpendicular bisector of AC. So $h = \dfrac{k + \frac{k}{3}}{2} = \dfrac{2k}{3}$.

Circumcentre is equidistant from B and A,

$$\sqrt{\left(\dfrac{2k}{3} - k \right)^2 + 38^2} = \sqrt{\left(\dfrac{2k}{3} \right)^2 + \left(38 - \dfrac{k}{4} \right)^2}$$

$$-\dfrac{4k^2}{3} + k^2 = -19k + \dfrac{k^2}{16}$$

$$k = 48 \quad \text{or} \quad 0 \text{ (rejected)}$$

So, $h = \dfrac{2}{3} \times 48 = 32$.

27. D

Circumcentre is equidistant from the two vertices.

$$\sqrt{(k+4)^2 + (-4+8)^2} = \sqrt{(k-6)^2 + (-4-2)^2}$$

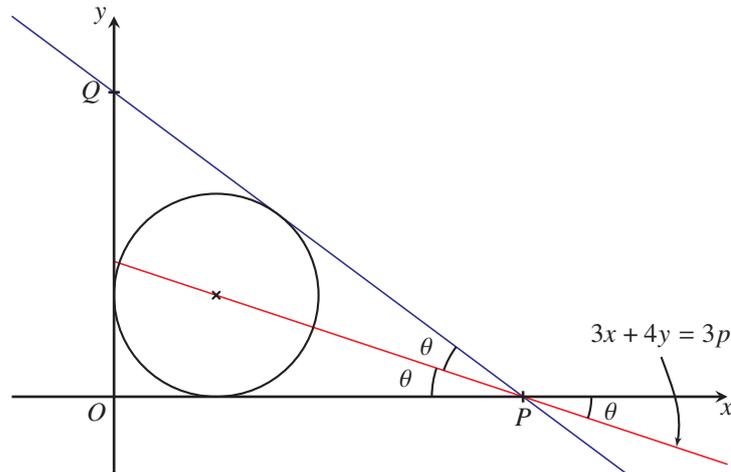$$k^2 + 8k + 32 = k^2 - 12k + 72$$

$$k = 2$$

28. $\boxed{\text{D}}$

Note that $(p,\ 0)$ satisfies the equation $3x + 4y = 3p$.

The straight line $3x + 4y = 3p$ passes through $P\ (p,\ 0)$ and the in-centre of $\triangle OPQ$.

Therefore, it is the angle bisector of $\angle OPQ$.

Let the acute angle between the straight line and the x-axis be θ.

$$\text{Slope of the line} = -\frac{3}{4} = -\tan\theta \quad \text{and} \quad \frac{OQ}{OP} = \tan 2\theta$$

$$\theta = \tan^{-1}\frac{3}{4} \qquad\qquad \frac{q}{p} = \frac{24}{7}$$

$$p : q = 7 : 24$$

29. $\boxed{\text{D}}$

The line $5x + 4y = 4b$ passes through $B\ (0,\ b)$.

Therefore, $5x + 4y = 4b$ passes through the mid-point of $OA\ \left(\dfrac{a}{2},\ 0\right)$.

$$5\left(\frac{a}{2}\right) + 0 = 4b$$

$$\frac{a}{b} = \frac{8}{5}$$

30. $\boxed{\text{A}}$

    I. ✓. G lies inside $\triangle OAB$, which is in the second quadrant. The x- and y-coordinates are not equal (one positive and one negative).

    II. ✓. Let the radius of inscribed circle be r. Then the coordinates of G are $(-r,\ r)$.

$$4r + (-r) = 3kb$$
$$r = kb$$

Using tangent properties, OB is divided into two segments with lengths $b - r$ and r.
OA is divided into two segments with lengths $10 - r$ and r.

$$(10 - r) + (b - r) = \sqrt{10^2 + b^2}$$
$$[10 + b(1 - 2k)]^2 = b^2 + 100$$
$$100 + 20b(1 - 2k) + b^2(1 - 2k)^2 = b^2 + 100$$
$$b^2(4k^2 - 4k) + 20b(1 - 2k) = 0$$
$$b = -\frac{20(1 - 2k)}{4k^2 - 4k}$$
$$= \frac{5(1 - 2k)}{k(1 - k)}$$

Required distance $= r = kb = \dfrac{5(1 - 2k)}{1 - k}$

    III. ✗. When $k = \dfrac{1}{6}$, $r = \dfrac{5(1 - 2k)}{1 - k} = 4$.

Equation of inscribed circle is $(x + 4)^2 + (y - 4)^2 = 4^2$.

$$(x + 4)^2 + (5 - 3x - 4)^2 = 16$$
$$10x^2 + 2x + 1 = 0$$

$\Delta = 2^2 - 4(10)(1) = -36 < 0.$

The straight line $3x + y = 5$ does not cut the inscribed circle of $\triangle OAB$ and hence is not a tangent.