REG-2425-MOCK-SET 1-MATH-CP 2

Answers:

1. D	2. D	3. C	4. B	5. D	6. C	7. B	8. D	9. B	10. C
11. D	12. A	13. B	14. B	15. C	16. D	17. A	18. C	19. C	20. A
21. A	22. B	23. C	24. A	25. B	26. D	27. A	28. C	29. B	30. A
31. B	32. C	33. C	34. D	35. C	36. D	37. B	38. C	39. D	40. B
41. A	42. A	43. B	44. A	45. D					

Suggested Solutions:

1. D

ab^2 term in the expansion $= (-b)(ab) = -ab^2$

2. D

$$\frac{(-3)^{2n}}{9^{n+1}} = \frac{9^n}{9^{n+1}} = \frac{1}{9}$$

3. C

Solve $\begin{cases} 2x + 3y = 2 \\ 5x + 4y = -16 \end{cases}$, we have $x = -8$ and $y = 6$.

$x^2 + y^2 = 8^2 + 6^2 = 100$

4. B

Put $x = 1$ and $x = -1$ respectively,

$0 - b(2) = 3 - 5$ and $-2a - 0 = 3 + 5$

$b = 1$ $\quad a = -4$

5. D

$$\begin{aligned} p &= \frac{1 - 2q}{1 + 2q} \\ p + 2pq &= 1 - 2q \\ q(2p + 2) &= 1 - p \\ q &= \frac{1 - p}{2(1 + p)} \end{aligned}$$

6. $\boxed{\text{C}}$

$$n < \frac{3.5 + 0.25}{0.080 - 0.0005}$$

$n < 47.2$

Greatest value of n is 47.

7. $\boxed{\text{B}}$

$$3 > 4 - x \quad \text{and} \quad 1 - \frac{6 - x}{2} > -1$$

$x > 1$ $\qquad\qquad\qquad\qquad$ $x > 2$

Thus, $x > 2$.

8. $\boxed{\text{D}}$

$$-(-2)^2 - k(-2) + 3k = 26$$

$$k = 6$$

$f(1) = -1 - 6 + 18 = 11$

9. $\boxed{\text{B}}$

$$4^3 + k(4)^2 + 2k(4) + 8 = 0$$

$$k = -3$$

$P(x) = x^3 - 3x^2 - 6x + 8$

By calculator, $P(1) = P(-2) = 0$. So, $(x - 1)(x + 2)$ is a factor of $P(x)$.

Only option B satisfies this.

10. $\boxed{\text{C}}$

A. ✗. There is no $f(x)$ defined in the question.

B. ✗. Coefficient of $x^2 = -2^2 = -4 < 0$. The graph open downwards.

C. ✓. $y = -4(x - 1.5)^2 + 23$. The axis of symmetry is $x = 1.5$.

D. ✗. When $x = 0$, $y = 23 - (-3)^2 = 14 \neq 23$.

11. $\boxed{\text{D}}$

$$\text{Amount} = 7500\left(1 + \frac{4\%}{2}\right)^{4\times 2}$$

$$\approx \$8787$$

12. A

$$\frac{3}{2a} = \frac{4}{3b} = \frac{7}{5c} \quad \Rightarrow \quad \frac{\left(\frac{3}{2}\right)}{a} = \frac{\left(\frac{4}{3}\right)}{b} = \frac{\left(\frac{7}{5}\right)}{c}$$

So, $a : b : c = \frac{3}{2} : \frac{4}{3} : \frac{7}{5}$. Since they are positive numbers, $a > c > b$.

13. B

Let $y = a + \frac{b}{x}$, where a and b are non-zero constants.

$$\begin{cases} 1 = a + b \\ 3 = a + \frac{b}{2} \end{cases}$$

Solving, we have $a = 5$ and $b = -4$.

$7 = 5 - \frac{4}{x}$

$x = -2$

14. B

Put $n = 3$ into $a_n = 2a_{n-2} + a_{n-1}$.

$$a_3 = 2a_1 + a_2$$
$$a_3 = 2(2) + 5$$
$$a_3 = 9$$

Put $n = 4$, $n = 5$, $n = 6$ and $n = 7$ into $a_n = 2a_{n-2} + a_{n-1}$.

$a_4 = 2a_2 + a_3$	$a_5 = 2a_3 + a_4$	$a_6 = 2a_4 + a_5$	$a_7 = 2a_5 + a_6$
$a_4 = 2(5) + 9$	$a_5 = 2(9) + 19$	$a_6 = 2(19) + 37$	$a_7 = 2(37) + 75$
$a_4 = 19$	$a_5 = 37$	$a_6 = 75$	$a_7 = 149$

15. C

Let the required height be h cm.

$$\pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2(2r)$$
$$h + \frac{4r}{3} = 2r$$
$$h = \frac{2r}{3}$$

16. D

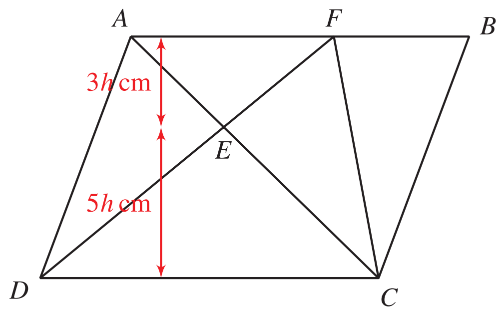
$\triangle AFE \sim \triangle CDE$ (ratio 3 : 5)

Let $AF = 3$ cm, then $CD = 5$ cm and $BF = 2$ cm.

$$\frac{(2)(3h+5h)}{2} = 16$$

$$h = 2$$

Area of $\triangle CDE = \frac{(5)(5\times 2)}{2} = 25\text{ cm}^2$



17. A

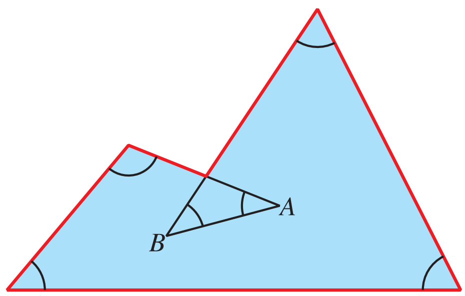
Consider the shaded pentagon as shown.

The reflex angle is equal to $\angle A + \angle B + 180°$.

By considering the sum of angles of the pentagon,

$$\text{(required sum)} + 180° = (5-2)180°$$

$$\text{required sum} = 360°$$



18. C

$\triangle AFG \sim \triangle AED$

$$\frac{FG}{DE} = \frac{AG}{AD}$$

$$\frac{FG}{5} = \frac{\frac{1}{2}\sqrt{5^2+12^2}}{12}$$

$$FG = \frac{65}{24}$$

19. C

Radius $= 2 \times 1 = 2$ cm

$\cos\angle ACM = \frac{1}{2}$

$\angle ACM = 60°$

Required area

$$= \pi(2)^2 \times \frac{60°}{360°} - \frac{(1)(\sqrt{2^2-1^2})}{2}$$

$$\approx 1.23\text{ cm}^2$$

20. A

$CE = h\cos\alpha$ and $BE = CE = h\cos\alpha$

$AB = BE\sin\beta = h\cos\alpha\sin\beta$

21. A

$\angle COD = \angle BOC = \angle AOB = 70^\circ$

$\angle AOD = 360^\circ - 70^\circ \times 3 = 150^\circ$

$\angle ACD = \dfrac{\angle AOD}{2} = \dfrac{150^\circ}{2} = 75^\circ$

22. B

Let O be the centre of the circle, and N be the mid-point of AB such that $ON \perp AB$.

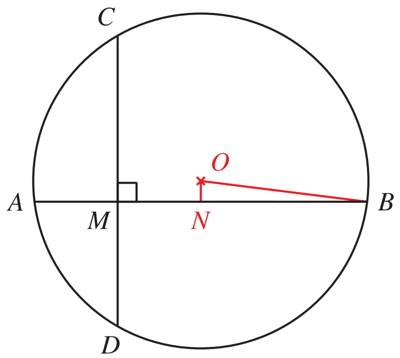
$\triangle AMD \sim \triangle CMB$

$\dfrac{AM}{MD} = \dfrac{CM}{MB}$

$CM = 4$

$NB = \dfrac{2+6}{2} = 4$ and $ON = 4 - \dfrac{4+3}{2} = 0.5$

Radius $= \sqrt{4^2 + 0.5^2} \approx 4.03$



23. C

Coordinates of centre are $(-5,\ 7)$.

Since centre is a point equidistant from A and B, it lies on the locus of P.

$2(-5) + 7 + k = 0$

$k = 3$

24. A

$\angle AOC = 202^\circ - 22^\circ = 180^\circ$. A, O and C are collinear.

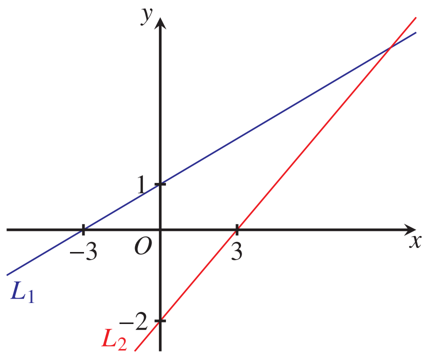
$\angle BOC = 202^\circ - 172^\circ = 30^\circ$

Required distance $= 6\sin 30^\circ = 3$

25. B

Assign reasonable values to the intercepts.

L_1:
$(0,\ 1) \rightarrow b = -2$
$(-3,\ 0) \rightarrow a = \frac{2}{3}$
L_2:
$(0,\ -2) \rightarrow d = -2$
$(3,\ 0) \rightarrow c = -\frac{2}{3}$
The result follows.



26. D

Consider the x-intercept of two straight lines,

$$\frac{-4}{2} = -\frac{2}{m}$$
$$m = 1$$

Two straight lines are perpendicular to each other,

$$2 \times \frac{-1}{n} = -1$$
$$n = 2$$

27. A

C: $x^2 + y^2 - 6x - 2y + \frac{5}{3} = 0$

A. ✓. Centre $(3,\ 1)$ and radius $= \sqrt{3^2 + 1^2 - \frac{5}{3}} = \frac{5}{\sqrt{3}} < 3$.
The circle lies on the right of the y-axis.

B. ✗. Sub $(0,\ 0)$ into L.H.S. of equation of C, L.H.S. $= 0 + 5 = 5 > 0$.
$(0,\ 0)$ lies outside C.

C. ✗. Centre is at $(3,\ 1)$.

D. ✗. Area $= \left(\frac{5}{\sqrt{3}}\right)^2 \pi = \frac{25\pi}{3} < \frac{25\pi}{2}$.

28. C

Required probability

$= 1 - \left(\frac{4}{7}\right)^2$

$= \frac{33}{49}$

29. B

Average of 60 and 70 is 65.

If there are more boys than girls, the mean score will be closer to the mean of boys.

The mean score will lie between 60 and 65.

Mathematical explanation

Let the ratio of number of boys to girls be $1 : \beta$, where $0 < \beta < 1$.

Mean $= \frac{60(1) + 70(\beta)}{1+\beta} = 60 + \frac{10\beta}{1+\beta} = 65 + \frac{5\beta - 5}{1+\beta}$

Since $\frac{10}{1+\beta} > 0 > \frac{5\beta-5}{1+\beta}$, the mean of the test marks lies between 60 and 65.

Only option B satisfies this.

30. A

Median $= \frac{(20+n)+25}{2} \le 24$ and Interquartile range $= (30+n) - (10+m) \ge 18$

$n \le 3$

$n - m \ge -2$

$m - n \le 2$

I. ✓. $m \le n + 2 \le 3 + 2 = 5$ and $m \ge 0$.

II. ✓. From the stem-and-leaf diagram, $n \ge 1$. Combine with $n \le 3$, we have $1 \le n \le 3$.

III. ✗. It is possible that $m = n = 1$ such that all conditions are satisfied.

31. B

$5\log_2 \alpha + 5\log_2 \beta = \frac{10}{3}$

$\log_2 \alpha\beta = \frac{2}{3}$

$\alpha\beta = 2^{\frac{2}{3}}$

32. C

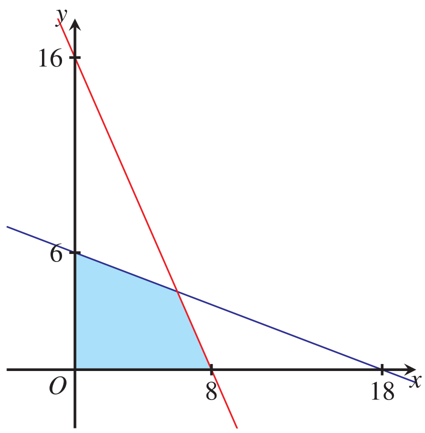
$2 - 2^3 + 2^4 + 4 \times 2^5 = 138 = 10001010_2$ and $5 \times 2^{10} = 1010000000000_2$.

Only option C satisfies this.

33. C

Straight line	x-intercept	y-intercept
$x + 3y = 18$	18	6
$2x + y = 16$	8	16
$x = 0$	0	
$y = 0$		0

$3x - y + 16$ is larger when x is larger and y is smaller.
$3x - y + 16$ attains its maximum at the bottom right corner, $(8,\ 0)$.
Required value $= 3(8) - 0 + 16 = 40$



34. D

Check the value of $g(0.5)$.

A. ✗. $g(0.5) = f\left(\frac{0.5}{2}\right) - 1 = f(0.25) - 1$
B. ✗. $g(0.5) = f(1 + 1) = f(2)$
C. ✗. $g(0.5) = f\left(\frac{0.5}{2} - 1\right) = f(-0.75)$
D. ✓. $g(0.5) = f(1) - 1 = 0 - 1 = -1$

35. C

$$\frac{2}{1 - \sin\theta} - \frac{2}{1 + \sin\theta} = 4$$
$$2(1 + \sin\theta) - 2(1 - \sin\theta) = 4(1 - \sin\theta)(1 + \sin\theta)$$
$$4\sin^2\theta + 4\sin\theta - 4 = 0$$
$$\sin\theta \approx 0.618 \quad \text{or} \quad -1.62$$
$$\theta \approx 38.2^\circ \quad \text{or} \quad 142^\circ$$

36. D

Let O be the centre of the circle. $AQOP$ is a square of length 9 cm.

$\angle ACB = \angle ABC = 45^\circ$, $QC = BP = \dfrac{9}{\tan 45^\circ} = 9\text{ cm}$,

$CO = BO = \dfrac{9}{\sin 45^\circ} = 9\sqrt{2}\text{ cm}$

Perimeter $= (9+9) + (9+9) + (9\sqrt{2} + 9\sqrt{2}) \approx 61\text{ cm}$

37. B

$\angle BCA = \angle BAC = \dfrac{180^\circ - 30^\circ}{2} = 75^\circ$

$\angle CDE = \angle ACB = 75^\circ$

$\angle DCE = 180^\circ - 2 \times 75^\circ = 30^\circ$

$\angle ECA = 180^\circ - 75^\circ - 30^\circ = 75^\circ$

$\angle EFA = 180^\circ - 75^\circ = 105^\circ$

38. C

Use the calculator program to check the intersections.

A. ✗. Centre of this circle is at $(-3,\ 1)$.

B. ✗. MATH ERROR $\Rightarrow$ no intersections.

C. ✓. Only one intersection $\Rightarrow$ circle touches the line.

D. ✗. MATH ERROR $\Rightarrow$ no intersections.

39. D

The straight line $5x + 2y = 2b$ passes through $B\,(0,\ b)$.

Slope of straight line $5x + 2y = 2b$ is $-\dfrac{5}{2}$, and the inclination is around 112°.

Denote the angle between the line $5x + 2y = 2b$ and the y-axis by θ. We have $\theta \approx 21.8^\circ$.

$\angle OBA = 2\theta \approx 43.6^\circ$ and $\tan(2\theta) = \dfrac{a}{b} = \dfrac{20}{21}$.

So, $a : b = 20 : 21$.

40. B

Using calculator CMPLX mode,

$$\frac{-9i^{2019}}{i - i^{2020}} = \frac{9i}{i-1} = \frac{9}{2} - \frac{9}{2}i$$

41. A

Let the first term and common ratio be a and r respectively.

$$\frac{ar^6}{ar^2} = \frac{48}{3}$$
$$r^4 = 16$$
$$r = \pm 2$$

I. ✓. 5th term $= 3 \times r^2 = 12$.

II. ✗. The sequence has negative terms when $r = -2$.

III. ✗. Sum to infinity does not exist.

42. A

$$\text{Required probability} = \frac{7!3!}{9!} = \frac{1}{12}$$

43. B

Since BV is perpendicular to the plane VAC, $\angle BVA = \angle BVC = 90°$.

$AB = BC = \sqrt{6^2 + 8^2} = 10\,\text{cm}$ and $BM = BN = 5\,\text{cm}$

$\triangle BMN$ is equilateral. So, $MN = 5\,\text{cm}$.

$\angle VBM = \tan^{-1}\frac{8}{6}$

$$VM^2 = 6^2 + 5^2 - (2)(6)(5)\cos\angle VBM$$
$$VM = 5$$

So, $VM = VN = MN = 5\,\text{cm}$, and the required area is $\frac{1}{2}(5)^2 \sin 60° = \frac{25\sqrt{3}}{4}\,\text{cm}^2$.

44. A

Probability of getting at least 2 heads $= 1 - \left(\frac{1}{2}\right)^4 - C^4_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 = \frac{11}{16}$

$$\text{Required probability} = \frac{C^3_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 \times \frac{1}{2}}{\frac{11}{16}} = \frac{3}{11}$$

45. D

The last 5 terms of the sequence can be obtained by multiplying 2^7 to each term of the first five terms.

Required variance $= 10 \times (2^7)^2 = 163\,840$.