| Solution | Marks |
|---|---|
| **REG-2324-MOCK-SET 7-MATH-EP(M2)** | |
| **Suggested solutions** | |
| 1. $\frac{d}{dx}\left(\sqrt{1-x}\right) = \lim_{h\to 0}\frac{\sqrt{1-(x+h)}-\sqrt{1-x}}{h}$ | 1M |
| $= \lim_{h\to 0}\frac{(1-x-h)-(1-x)}{h[\sqrt{1-x-h}+\sqrt{1-x}]}$ | 1M |
| $= \lim_{h\to 0}\frac{-1}{\sqrt{1-x-h}+\sqrt{1-x}}$ | 1M |
| $= \frac{-1}{2\sqrt{1-x}}$ | 1A |
| 2. (a) $\frac{C_5^n a^5}{C_3^n a^3} = \frac{3a^2}{5}$ | 1M |
| $\frac{\frac{n!}{5!(n-5)!}\times a^2}{\frac{n!}{3!(n-3)!}} = \frac{3a^2}{5}$ | 1M |
| $\frac{a^2(n-3)(n-4)}{20} = \frac{3a^2}{5}$ | |
| $(n-3)(n-4) = 12$ | 1A |
| $n^2 - 7n = 0$ | |
| $n = 7$ or $0$ (rejected) | 1A |
| (b) $(1+ax)^7 = 1 + 7ax + 21a^2x^2 + \ldots$ | |
| $\left(2+\frac{1}{x}\right)^2 = 4 + \frac{4}{x} + \frac{1}{x^2}$ | |
| Constant term $= 1(4) + 7a(4) + 21a^2(1)$ | 1M |
| $= 21a^2 + 28a + 4$ | 1A |
| 3. (a) $\int x^3 \ln x \, dx = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 \, dx$ | 1M+1A |
| $= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + \text{constant}$ | 1A |
| (b) $y = \int x^3 \ln x \, dx$ | 1M |
| $= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$ | |
| The curve $C$ passes through $(1, 1)$. | |
| $1 = \frac{1}{4}(1)^4 \ln 1 - \frac{1}{16}(1)^4 + C$ | 1M |
| $C = \frac{17}{16}$ | |
| Required equation is $y = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + \frac{17}{16}$. | 1A |

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| 4. (a) $\cot A = 3\tan B$ | |
| $\dfrac{\cos A}{\sin A} = \dfrac{3\sin B}{\cos B}$ | 1M |
| $\cos A\cos B = 3\sin A\sin B$ | |
| $2\cos(A+B) - \cos(A-B)$ | |
| $= 2(\cos A\cos B - \sin A\sin B) - (\cos A\cos B + \sin A\sin B)$ | 1M |
| $= \cos A\cos B - 3\sin A\sin B$ | |
| $= 0$ | |
| Thus, $2\cos(A+B) = \cos(A-B)$. | 1 |
| (b) $\cot\left(x + \dfrac{11\pi}{15}\right) = 3\tan\left(x - \dfrac{4\pi}{15}\right)$ | |
| $2\cos\left[\left(x+\dfrac{11\pi}{15}\right) + \left(x - \dfrac{4\pi}{15}\right)\right] = \cos\left[\left(x+\dfrac{11\pi}{15}\right) - \left(x - \dfrac{4\pi}{15}\right)\right]$ | 1M |
| $2\cos\left(2x + \dfrac{7\pi}{15}\right) = -1$ | |
| $\cos\left(2x + \dfrac{7\pi}{15}\right) = -\dfrac{1}{2}$ | |
| $2x + \dfrac{7\pi}{15} = \dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$ | |
| $x = \dfrac{\pi}{10}$ or $\dfrac{13\pi}{30}$ | 1A+1A |
| 5. (a) $10\ln(0 - ke) = 0^2 + 10$ | 1M |
| $\ln(-ke) = 1$ | |
| $-ke = e^1$ | |
| $k = -1$ | 1A |
| (b) $10\ln(x-y) = x^2 + 10$ | |
| $\dfrac{10}{x-y}\left(1 - \dfrac{dy}{dx}\right) = 2x$ | 1M |
| At $A\ (0,\ -e)$, | |
| $\dfrac{10}{0+e}\left(1 - \dfrac{dy}{dx}\right) = 0$ | 1M |
| $\dfrac{dy}{dx} = 1$ | 1A |
| Required equation is $y = x - e$. | 1A |

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|---|---|
| 6. (a) Radius of water surface $= h\tan\dfrac{\pi}{3} = \sqrt{3}h$ cm | 1M |
| $A = \pi(\sqrt{3}h)\left(\sqrt{(\sqrt{3}h)^2 + h^2}\right)$ | 1M |
| $= 2\sqrt{3}\pi h^2$ | 1 |
| (b) When the volume of water in the vessel is $216\pi$ cm$^2$, | |
| $216\pi = \dfrac{1}{3}\pi(\sqrt{3}h)^2 h$ | 1M |
| $h = 6$ | |
| Using the result of (a), | |
| $A = 2\sqrt{3}\pi h^2$ | |
| $\dfrac{\mathrm{d}A}{\mathrm{d}t} = 4\sqrt{3}\pi h\dfrac{\mathrm{d}h}{\mathrm{d}t}$ | 1M |
| $-12\pi = 4\sqrt{3}\pi(6)\dfrac{\mathrm{d}h}{\mathrm{d}t}$ | 1M |
| $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -\dfrac{1}{2\sqrt{3}}$ | 1A |
| Required rate is $-\dfrac{1}{2\sqrt{3}}$ cm/s. | |
| 7. (a) $(\det A)^2 - 3\det A + 2 = 0$ | |
| $\det A = 1$ or $2$ | 1A |
| (b) $B^2 = \begin{pmatrix} x & x-1 \\ 1 & x+1 \end{pmatrix}\begin{pmatrix} x & x-1 \\ 1 & x+1 \end{pmatrix}$ | |
| $= \begin{pmatrix} x^2+x-1 & x(x-1)+(x-1)(x+1) \\ x+(x+1) & (x-1)+(x+1)^2 \end{pmatrix}$ | |
| $= \begin{pmatrix} x^2+x-1 & 2x^2-x-1 \\ 2x+1 & x^2+3x \end{pmatrix}$ | 1A |
| (c) $\begin{vmatrix} x & x-1 \\ 1 & x+1 \end{vmatrix} = x(x+1)-(x-1)$ | |
| $= x^2+1$ | 1A |
| $\begin{vmatrix} x^2+x-1 & 2x+1 \\ 2x^2-x-1 & x^2+3x \end{vmatrix} - 3\begin{vmatrix} x & x-1 \\ 1 & x+1 \end{vmatrix} + 2 = 0$ | |
| $\begin{vmatrix} x^2+x-1 & 2x^2-x-1 \\ 2x+1 & x^2+3x \end{vmatrix} - 3\begin{vmatrix} x & x-1 \\ 1 & x+1 \end{vmatrix} + 2 = 0$ | 1M |
| $(\det B)^2 - 3(\det B) + 2 = 0$ | |
| $\det B = 1$ or $2$ | 1M |
| $x^2 + 1 = 1$ or $2$ | |
| $x = 0$ or $-1$ or $1$ | 1A+1A |

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|---|---|
| 8. (a) When $n = 1$, | |
| L.H.S. $= \dfrac{1}{2} + \dfrac{1}{2(3)} = \dfrac{2}{3}$ and R.H.S. $= \dfrac{1+1}{2+1} = \dfrac{2}{3} =$ L.H.S. | |
| It is true for $n = 1$. | 1 |
| Assume $\displaystyle\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k+1}{k+2}$, where $k \in \mathbf{Z}^+$. | 1 |
| $\displaystyle\sum_{r=1}^{k+2} \frac{1}{r(r+1)}$ | |
| $\displaystyle= \sum_{r=1}^{k+1} \frac{1}{r(r+1)} + \frac{1}{(k+2)(k+3)}$ | |
| $\displaystyle= \frac{k+1}{k+2} + \frac{1}{(k+2)(k+3)}$ | 1M |
| $\displaystyle= \frac{(k+1)(k+3)+1}{(k+2)(k+3)}$ | |
| $\displaystyle= \frac{(k+2)^2}{(k+2)(k+3)}$ | |
| $\displaystyle= \frac{k+2}{k+3}$ | 1 |
| It is true for $n = k+1$. | |
| By M.I., it is true $\forall n \in \mathbf{Z}^+$. | 1 |
| (b) $\displaystyle\sum_{r=100}^{2019} \left(\frac{4}{r+1} \times \frac{25}{r}\right) = 100\sum_{r=1}^{2019} \frac{1}{r(r+1)} - 100\sum_{r=1}^{99} \frac{1}{r(r+1)}$ | 1M |
| $\displaystyle= 100 \times \frac{2018+1}{2018+2} - 100 \times \frac{98+1}{98+2}$ | 1M |
| $\displaystyle= \frac{96}{101}$ | 1A |

| Solution | Marks |
|---|---|
| 9. (a) $x^2+2x+5=(x+1)^2+4$ | |
| Let $x+1=2\tan\theta$. Then $dx=2\sec^2\theta\,d\theta$. | |
| $\displaystyle\int_0^1 \frac{1}{x^2+2x+5}\,dx=\int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}}\frac{2\sec^2\theta}{4\tan^2\theta+4}\,d\theta$ | 1M |
| $\displaystyle=\frac{1}{2}\int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}}d\theta$ | |
| $\displaystyle=\frac{1}{2}\Big[\theta\Big]_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}}$ | 1M |
| $\displaystyle=\frac{\pi}{8}-\frac{1}{2}\tan^{-1}\frac{1}{2}$ | 1A |
| (b) (i) $\displaystyle\frac{2\tan\theta}{1+\tan^2\theta}=\frac{2\times\frac{\sin\theta}{\cos\theta}}{\sec^2\theta}$ | |
| $=2\sin\theta\cos\theta$ | |
| $=\sin 2\theta$ | 1 |
| $\displaystyle\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}$ | |
| $=\cos^2\theta-\sin^2\theta$ | |
| $=\cos 2\theta$ | 1 |
| (ii) Let $t=\tan\theta$. Then $dt=\sec^2\theta\,d\theta=(1+\tan^2\theta)\,d\theta$. | 1M |
| $\displaystyle\int_0^{\frac{\pi}{4}}\frac{1}{\sin 2\theta+2\cos 2\theta+3}\,d\theta$ | |
| $\displaystyle=\int_0^1\frac{1}{\frac{2t}{1+t^2}+2\times\frac{1-t^2}{1+t^2}+3}\left(\frac{1}{1+t^2}\right)dt$ | 1M |
| $\displaystyle=\int_0^1\frac{1}{2t+2(1-t^2)+3(1+t^2)}\,dt$ | |
| $\displaystyle=\int_0^1\frac{1}{t^2+2t+5}\,dt$ | |
| $\displaystyle=\frac{\pi}{8}-\frac{1}{2}\tan^{-1}\frac{1}{2}$ | 1A |
| (c) Let $u=\dfrac{\pi}{4}-\theta$. Then $du=-\,d\theta$. | 1M |
| $\displaystyle\int_0^{\frac{\pi}{4}}\frac{1}{\sin 2\theta+2\cos 2\theta+3}\,d\theta=-\int_{\frac{\pi}{4}}^{0}\frac{1}{\sin\left(\frac{\pi}{2}-2u\right)+2\cos\left(\frac{\pi}{2}-2u\right)+3}\,du$ | |
| $\displaystyle=\int_0^{\frac{\pi}{4}}\frac{1}{\cos 2u+2\sin 2u+3}\,du$ | |
| $\displaystyle=\int_0^{\frac{\pi}{4}}\frac{1}{\cos 2\theta+2\sin 2\theta+3}\,d\theta$ | 1 |

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| (d) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{3(\sin 2\theta+\cos 2\theta+2)}{(\sin 2\theta+2\cos 2\theta+3)(\cos 2\theta+2\sin 2\theta+3)}\,\mathrm{d}\theta$ | |
| $\displaystyle=\int_0^{\frac{\pi}{4}} \frac{(\sin 2\theta+2\cos 2\theta+3)+(\cos 2\theta+2\sin 2\theta+3)}{(\sin 2\theta+2\cos 2\theta+3)(\cos 2\theta+2\sin 2\theta+3)}\,\mathrm{d}\theta$ | |
| $\displaystyle=\int_0^{\frac{\pi}{4}} \frac{1}{\cos 2\theta+2\sin 2\theta+3}\,\mathrm{d}\theta+\int_0^{\frac{\pi}{4}} \frac{1}{\sin 2\theta+2\cos 2\theta+3}\,\mathrm{d}\theta$ | 1M |
| $\displaystyle=2\int_0^{\frac{\pi}{4}} \frac{1}{\sin 2\theta+2\cos 2\theta+3}\,\mathrm{d}\theta$ | |
| $\displaystyle=2\left(\frac{\pi}{8}-\frac{1}{2}\tan^{-1}\frac{1}{2}\right)$ | 1M |
| $\displaystyle=\frac{\pi}{4}-\tan^{-1}\frac{1}{2}$ | 1A |

| Solution | Marks |
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| 10. (a) $a = 1$ and $b = -2$. | 1A+1A |
| (b) (i) $y = x + \dfrac{1}{2(x+1)} + \dfrac{1}{2(x-1)}$ | |
| $\dfrac{dy}{dx} = 1 - \dfrac{1}{2(x+1)^2} - \dfrac{1}{2(x-1)^2}$ | 1A |
| $\dfrac{d^2y}{dx^2} = \dfrac{1}{(x+1)^3} + \dfrac{1}{(x-1)^3}$ | 1A |
| (ii) When $\dfrac{dy}{dx} = 0$, | |
| $1 - \dfrac{1}{2(x+1)^2} - \dfrac{1}{2(x-1)^2} = 0$ | 1M |
| $2(x+1)^2(x-1)^2 - (x-1)^2 - (x+1)^2 = 0$ | |
| $2x^4 - 6x^2 = 0$ | |
| $x^2 = 0$ or $3$ | |
| $x = 0$ or $\pm\sqrt{3}$ | |

| $x$ | $x < -\sqrt{3}$ | $-\sqrt{3} < x < -1$ | $-1 < x < 0$ | $0 < x < 1$ | $1 < x < \sqrt{3}$ | $x > \sqrt{3}$ |
|---|---|---|---|---|---|---|
| $\dfrac{dy}{dx}$ | + | − | − | − | − | + |

1M

| Solution | Marks |
|---|---|
| Maximum point is $\left(-\sqrt{3}, -\dfrac{3\sqrt{3}}{2}\right)$. | 1A |
| Minimum point is $\left(\sqrt{3}, \dfrac{3\sqrt{3}}{2}\right)$. | 1A |
| (c) Required area | |
| $= \displaystyle\int_2^4 \left[\left(x + \frac{1}{2(x+1)} + \frac{1}{2(x-1)}\right) - x\right] dx$ | 1M+1A |
| $= \displaystyle\int_2^4 \left(\frac{1}{2(x+1)} + \frac{1}{2(x-1)}\right) dx$ | |
| $= \left[\dfrac{1}{2}\ln\lvert x+1\rvert + \dfrac{1}{2}\ln\lvert x-1\rvert\right]_2^4$ | 1M |
| $= \dfrac{\ln 5}{2}$ | 1A |

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| 11. (a) (i) $\Delta = \begin{vmatrix} 1 & 1 & 2 \\ -3 & -2 & k \\ 1 & k+1 & 9 \end{vmatrix} = -k^2 - 6k + 7$ | 1A |
| $= -(k+7)(k-1)$ | |
| Since $(E)$ has a unique solution, we have $\Delta \neq 0$. | 1M |
| Thus, we have $k < -7$ or $-7 < k < 1$ or $k > 1$. | 1A |
| (ii) When $k = 1$, the augmented matrix of $(E)$ is | |
| $\left(\begin{array}{ccc\|c} 1 & 1 & 2 & a \\ -3 & -2 & 1 & b \\ 1 & 2 & 9 & c \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 2 & a \\ 0 & 1 & 7 & b+3a \\ 0 & 1 & 7 & c-a \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 2 & a \\ 0 & 1 & 7 & b+3a \\ 0 & 0 & 0 & c-b-4a \end{array}\right)$ | 1M+1A |
| Since $(E)$ is consistent, we have $c - b - 4a = 0$. | 1A |
| When $k = 1$ and $c - b - 4a = 0$, the augmented matrix of $(E)$ is | |
| $\left(\begin{array}{ccc\|c} 1 & 1 & 2 & a \\ 0 & 1 & 7 & b+3a \\ 0 & 0 & 0 & 0 \end{array}\right)$ | |
| Thus, the solution set of $(E)$ is $\{(-2a - b + 5t,\ 3a + b - 7t,\ t) : t \in \mathbb{R}\}$. | 1A |
| (b) (i) When $k = 1$, $a = -1$, $b = n$ and $c = 0$, $(E)$ becomes the first three equations of $(T)$. | 1M |
| Since $(T)$ is consistent, the first three equations are also consistent. | |
| $0 - n - 4(-1) = 0$ | 1M |
| $n = 4$ | 1A |
| (ii) The solution set of the first three equations is $\{(-2 + 5t,\ 1 - 7t,\ t) : t \in \mathbb{R}\}$. | |
| $(-2 + 5t) + (1 - 7t) - t^2 = 0$ | 1M |
| $-t^2 - 2t - 1 = 0$ | |
| $t = -1$ | 1A |
| Thus, we have $x = -7$, $y = 8$ and $z = -1$. | 1A |

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| 12. (a) (i) $\overrightarrow{AB} = -6\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ | |
| $\overrightarrow{BC} = 5\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ | |
| $\overrightarrow{AC} = -\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ | 1A+1A |
| (ii) Required vector $= \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\overrightarrow{AC} \cdot \overrightarrow{AC}} \overrightarrow{AC}$ | 1M |
| $= \dfrac{(-6)(-1) + (1)(-2) + (7)(2)}{1^2 + 2^2 + 2^2} (-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ | |
| $= -2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ | 1A |
| (iii) Let $M$ be a point on the line $AC$ such that $BM \perp AC$. | |
| $\overrightarrow{AM} = -2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ | |
| $\overrightarrow{BM} = \overrightarrow{AM} - \overrightarrow{AB}$ | |
| $= 4\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}$ | |
| Required distance $= BM$ | |
| $= \sqrt{4^2 + 5^2 + 3^2}$ | 1M |
| $= 5\sqrt{2}$ | 1A |
| (b) Note that $B$, $M$ and $D$ are collinear. | |
| Let $\overrightarrow{BD} = k\overrightarrow{BM}$, where $k$ is a real number. | 1M |
| $\overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{BC}$ | |
| $= (4k - 5)\mathbf{i} + (-5k + 3)\mathbf{j} + (-3k + 5)\mathbf{k}$ | |
| Note that $CD \perp AB$. | |
| $0 = (-6\mathbf{i} + \mathbf{j} + 7\mathbf{k}) \cdot [(4k - 5)\mathbf{i} + (-5k + 3)\mathbf{j} + (-3k + 5)\mathbf{k}]$ | 1M |
| $= 68 - 50k$ | |
| $k = \dfrac{34}{25}$ | 1A |
| Denote the origin by $O$. | |
| $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$ | 1M |
| $= (-5\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \left(\dfrac{136}{25}\mathbf{i} - \dfrac{34}{5}\mathbf{j} - \dfrac{102}{25}\mathbf{k}\right)$ | |
| $= \dfrac{11}{25}\mathbf{i} - \dfrac{29}{5}\mathbf{j} - \dfrac{2}{25}\mathbf{k}$ | |
| The coordinates of $D$ are $\left(\dfrac{11}{25}, -\dfrac{29}{5}, -\dfrac{2}{25}\right)$. | 1A |



**END OF PAPER**