M2REG-TRIG-2425-ASM-SET 5-MATH
Suggested solutions

1. (a) $y = \cos\left(x + \frac{\pi}{8}\right)\sin\left(x - \frac{\pi}{24}\right)$

$= \frac{1}{2}\left[\sin\left(2x + \frac{\pi}{12}\right) + \sin\frac{-\pi}{6}\right]$ 1M

$= \frac{1}{2}\sin\left(2x + \frac{\pi}{12}\right) - \frac{1}{4}$ 1

(b) Maximum value of $y = \frac{1}{2}(1) - \frac{1}{4}$ 1M

$= \frac{1}{4}$ 1A

Minimum value of $y = \frac{1}{2}(-1) - \frac{1}{4}$

$= -\frac{3}{4}$ 1A

2. (a) By considering the sum of roots, $\sin\alpha + \sin\beta = \frac{7}{12}$. 1A

By considering the product of roots, $\sin\alpha\sin\beta = \frac{1}{12}$. 1A

(b) $\cos(\alpha+\beta)\cos(\alpha-\beta) = \frac{1}{2}(\cos 2\alpha + \cos 2\beta)$ 1M

$= \frac{1}{2}(1 - 2\sin^2\alpha + 1 - 2\sin^2\beta)$ 1M

$= 1 - \sin^2\alpha - \sin^2\beta$

$= 1 + 2\sin\alpha\sin\beta - (\sin\alpha + \sin\beta)^2$ 1M

$= 1 + 2\left(\frac{1}{12}\right) - \left(\frac{7}{12}\right)^2$

$= \frac{119}{144}$ 1A

3. (a) $\sin(A+B)\sin(A-B) = \frac{1}{2}(-\cos 2A + \cos 2B)$ 1M

$= \frac{1}{2}(-1 + 2\sin^2 A + 1 - 2\sin^2 B)$ 1M

$= \sin^2 A - \sin^2 B$ 1

(b) By (a),

$\sin^2 kx - \sin^2(k-1)x = \sin[kx + (k-1)x]\sin[kx - (k-1)x]$ 1M

$= \sin[(2k-1)x]\sin x$ 1A

4. $\dfrac{\cos 7\theta - \cos 5\theta + \cos 3\theta - \cos\theta}{\sin 7\theta - \sin 5\theta + \sin 3\theta - \sin\theta}$

$= \dfrac{-2\sin 6\theta \sin\theta - 2\sin 2\theta \sin\theta}{2\cos 6\theta \sin\theta + 2\cos 2\theta \sin\theta}$ 1M+2A

$= -\dfrac{\sin 6\theta + \sin 2\theta}{\cos 6\theta + \cos 2\theta}$

$= -\dfrac{2\sin 4\theta \cos 2\theta}{2\cos 4\theta \cos 2\theta}$ 1M

$= -\tan 4\theta$ 1

5. $\cos A + \cos B - \cos C$

$= 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2} - \left(1 - 2\sin^2\dfrac{C}{2}\right)$ 2M

$= 2\cos\dfrac{\pi - C}{2}\cos\dfrac{A-B}{2} + 2\sin\dfrac{C}{2}\sin\dfrac{\pi-(A+B)}{2} - 1$

$= 2\sin\dfrac{C}{2}\cos\dfrac{A-B}{2} + 2\sin\dfrac{C}{2}\cos\dfrac{A+B}{2} - 1$ 1M

$= 2\sin\dfrac{C}{2}\left[\cos\dfrac{(A+B)}{2} + \cos\dfrac{A-B}{2}\right] - 1$

$= 2\sin\dfrac{C}{2}\left(2\cos\dfrac{A}{2}\cos\dfrac{B}{2}\right) - 1$ 1M

$= 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\sin\dfrac{C}{2} - 1$ 1

6. $\sin 3A + \sin 3B + \sin 3[\pi - (A+B)]$

$= 2\sin\dfrac{3}{2}(A+B)\cos\dfrac{3}{2}(A-B) + \sin 3(A+B)$ 2M

$= 2\sin\dfrac{3}{2}(A+B)\cos\dfrac{3}{2}(A-B) + 2\sin\dfrac{3}{2}(A+B)\cos\dfrac{3}{2}(A+B)$ 1M

$= 2\sin\dfrac{3}{2}(A+B)\left[\cos\dfrac{3}{2}(A-B) + \cos\dfrac{3}{2}(A+B)\right]$

$= 2\sin\dfrac{3}{2}(\pi - C)\left(2\cos\dfrac{3A}{2}\cos\dfrac{3B}{2}\right)$ 1M

$= 4\sin\left(\dfrac{3\pi}{2} - \dfrac{3C}{2}\right)\cos\dfrac{3A}{2}\cos\dfrac{3B}{2}$

$= -4\cos\dfrac{3C}{2}\cos\dfrac{3A}{2}\cos\dfrac{3B}{2}$ 1M

$= -4\cos\dfrac{3A}{2}\cos\dfrac{3B}{2}\cos\dfrac{3C}{2}$ 1

7. (a) $\dfrac{\sin\alpha - \sin\beta}{\cos\alpha - \cos\beta} = \dfrac{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}{-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}$ 1M

$= -\cot\dfrac{\alpha+\beta}{2}$ 1

(b) $3\sin\alpha - 4\cos\beta = 3\sin\beta - 4\cos\alpha$

$\dfrac{3(\sin\alpha - \sin\beta)}{4(\cos\alpha - \cos\beta)} = -1$

$-\dfrac{3}{4}\cot\dfrac{\alpha+\beta}{2} = -1$ 1M

$\cot\dfrac{\alpha+\beta}{2} = \dfrac{4}{3}$ 1A

Since α and β are acute, the angle $\dfrac{\alpha+\beta}{2}$ is also acute.

$\sin\dfrac{\alpha+\beta}{2} = \dfrac{3}{\sqrt{3^2+4^2}}$ 1M

$= \dfrac{3}{5}$ 1A

8. (a) $\sin\dfrac{(m+4)\pi}{8} - \sin\dfrac{m\pi}{8} = 2\cos\dfrac{m\pi+2\pi}{8}\sin\dfrac{\pi}{4}$ 1M

$= \sqrt{2}\cos\dfrac{(m+2)\pi}{8}$ 1A

(b) $\cos\dfrac{3\pi}{8} + \cos\dfrac{7\pi}{8} + \ldots + \cos\dfrac{2015\pi}{8}$

$= \dfrac{1}{\sqrt{2}}\left(\sin\dfrac{5\pi}{8} - \sin\dfrac{\pi}{8}\right) + \dfrac{1}{\sqrt{2}}\left(\sin\dfrac{9\pi}{8} - \sin\dfrac{5\pi}{8}\right) + \ldots + \dfrac{1}{\sqrt{2}}\left(\sin\dfrac{2017\pi}{8} - \sin\dfrac{2013\pi}{8}\right)$ 1M

$= \dfrac{1}{\sqrt{2}}\left(\sin\dfrac{2017\pi}{8} - \sin\dfrac{\pi}{8}\right)$

$= \dfrac{1}{\sqrt{2}}\left(\sin\left(252\pi + \dfrac{\pi}{8}\right) - \sin\dfrac{\pi}{8}\right)$ 1M

$= 0$ 1A

9. When $n = 1$,

L.H.S. $= \cos x$

R.H.S. $= \dfrac{\sin 2x}{2\sin x} = \dfrac{2\sin x\cos x}{2\sin x} = \cos x =$ L.H.S.

It is true for $n = 1$. 1

Assume $\cos x + \cos 3x + \cos 5x + \ldots + \cos(2k-1)x = \dfrac{\sin 2kx}{2\sin x}$, where $k \in \mathbb{Z}^+$. 1M

$\cos x + \cos 3x + \cos 5x + \ldots + \cos(2k-1)x + \cos(2k+1)x$

$= \dfrac{\sin 2kx}{2\sin x} + \cos(2k+1)x$ 1M

$= \dfrac{1}{2\sin x}[\sin 2kx + 2\cos(2k+1)x\sin x]$

$= \dfrac{1}{2\sin x}[\sin 2kx + \sin(2k+2)x + \sin(-2kx)]$ 1M

$= \dfrac{\sin(2k+2)x}{2\sin x}$

It is true for $n = k+1$.

By M.I., it is true $\forall n \in \mathbb{Z}^+$. 1

10. $$2\cos A + \cos 2B + \cos 2C = 0$$

$$2\cos A + 2\cos(B+C)\cos(B-C) = 0$$ 1M

$$\cos A + \cos(\pi - A)\cos(B-C) = 0$$

$$\cos A - \cos A\cos(B-C) = 0$$ 1M

$$\cos A[1 - \cos(B-C)] = 0$$

$$\cos A = 0 \quad \text{or} \quad \cos(B-C) = 1$$ 1A

$$A = \frac{\pi}{2} \quad \text{or} \quad B - C = 0$$

$$A = \frac{\pi}{2} \quad \text{or} \quad B = C$$ 1A

Thus, $\triangle ABC$ is a right-triangle or $\triangle ABC$ is an isosceles triangle with $B = C$. 1M

$\triangle ABC$ can be a right-angled triangle with $A = \dfrac{\pi}{2}$ but $B \neq C$.

The claim is disagreed. 1A

 END OF PAPER